

Chapter 8: Combined Loading

Chapter Objectives

- ✓ Determine stresses developed in thin-walled pressure vessels
- ✓ Determine stresses developed in a member's cross section when axial load, torsion, bending and shear occur simultaneously.

Thin-walled pressure vessels

- Inner radius: r
- Wall thickness: t
- Thin walls: $\frac{r}{t} \geq 10$
- Assume that stress distribution in thin wall is uniform
- Pressure vessel contains fluid under pressure p



Cylindrical vessels

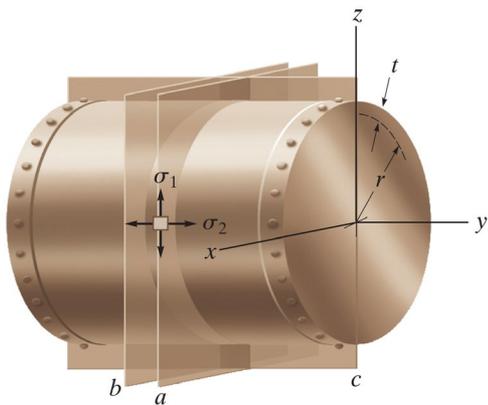


Spherical vessels

Pressure vessel failures



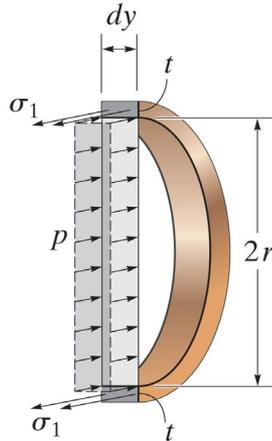
Cylindrical vessels



(a)

Figure: 08_01a

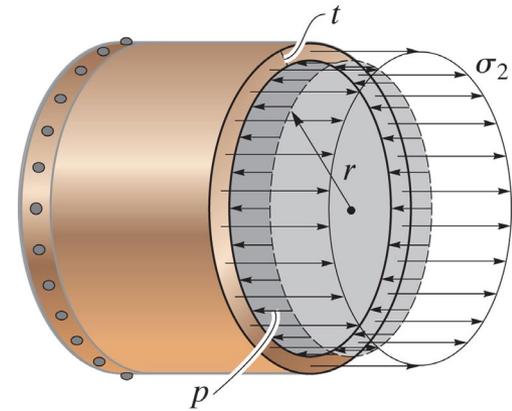
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(b)

Figure: 08_01b

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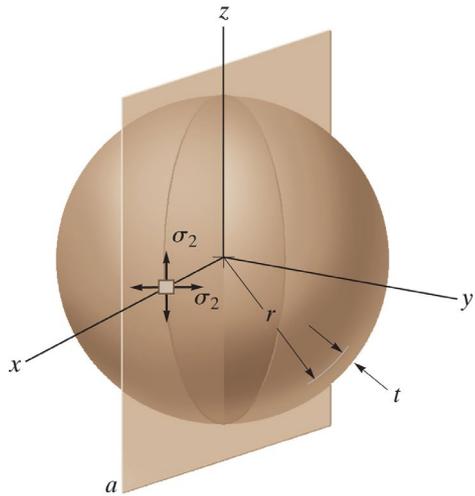


(c)

Figure: 08_01c

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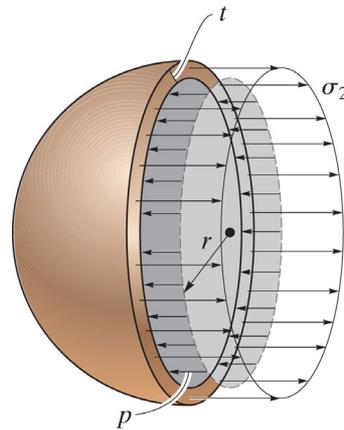
Spherical vessels



(a)

Figure: 08_02a

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(b)

Figure: 08_02b

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Example: A spherical pressure vessel of $D = 900$ -mm outer diameter is to be fabricated from a steel having an ultimate tensile stress $\sigma_U = 400$ MPa. Knowing that a factor of safety of $FS=4.0$ is desired and that the gage pressure can reach $p=3.5$ MPa, determine the smallest wall thickness that should be used.

Review of stress analysis

- **Normal force P**

leads to uniform normal stress $\sigma = \frac{P}{A}$

- **Torsional moment T (circular shafts)**

leads to shear stress varying in the cross-section $\tau = \frac{T \rho}{J}$

- **Shear force V**

leads to shear stress varying in the cross-section $\tau = \frac{V Q}{I t}$

- **Bending moment M**

leads to normal stress varying in the cross-section $|\sigma| = \frac{M c}{I}$

- **Internal pressure p in a spherical or cylindrical vessel**

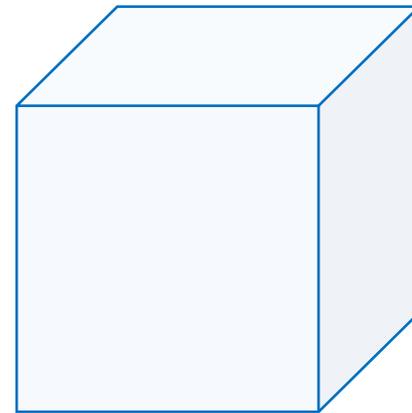
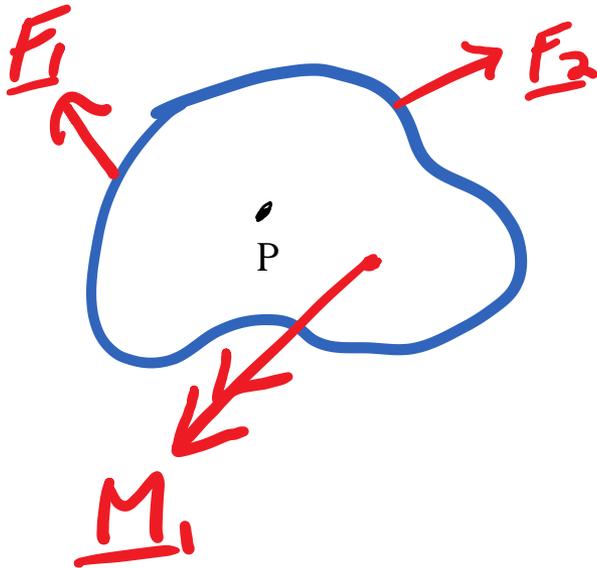
leads to hoop stress $\sigma_h = \frac{pr}{t}$ and axial stress $\sigma_a = \frac{pr}{2t}$

State of stress in a solid

Consider a solid body subjected to externally applied loads

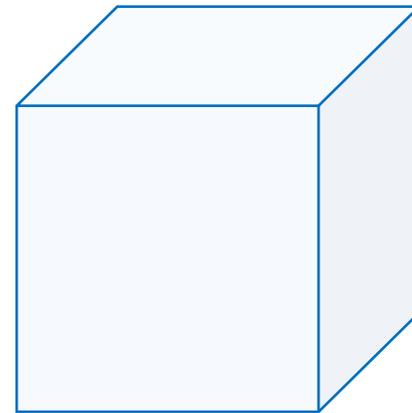
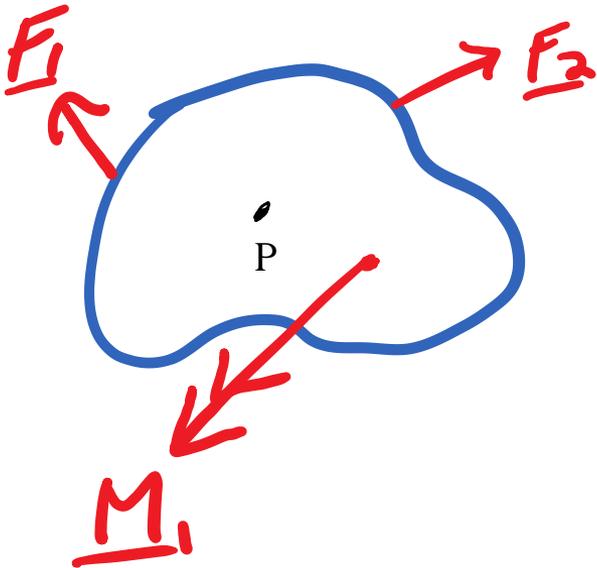
Suppose we wish to know about the stresses being caused at a particular point of interest, point P.

We can draw a small (infinitesimal) cube of material around the point of interest and note the infinitesimal forces that act on each face. In general, the force acting on each face can be broken into 3 vector components, one of which is normal to the face and two of which act in the plane of the face



State of stress in a solid

We can convert each of these forces into stresses.



Stress in 2 and 3 dimensions

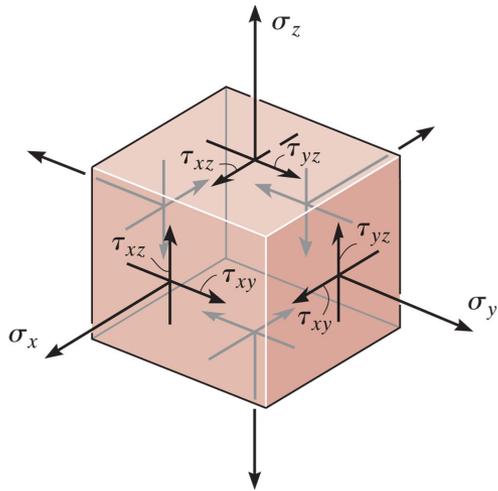
Normal stresses: $\sigma_x, \sigma_y, \sigma_z$

Shear stresses: $\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{xz} = \tau_{zx}$

→ equalities due to requirement of zero net moment in equilibrium

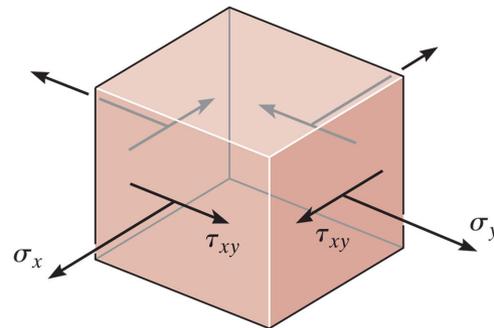
First index: indicates face normal direction

Second index: indicates force direction on that face



General state of stress

(a)

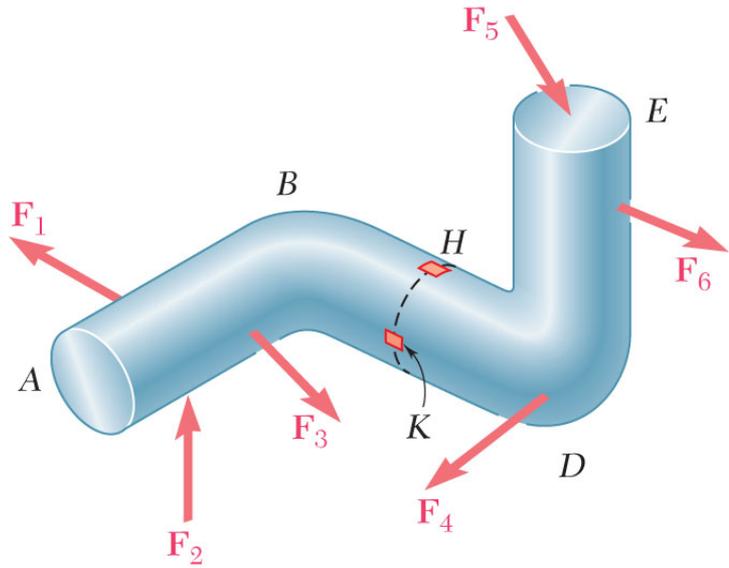


Plane stress

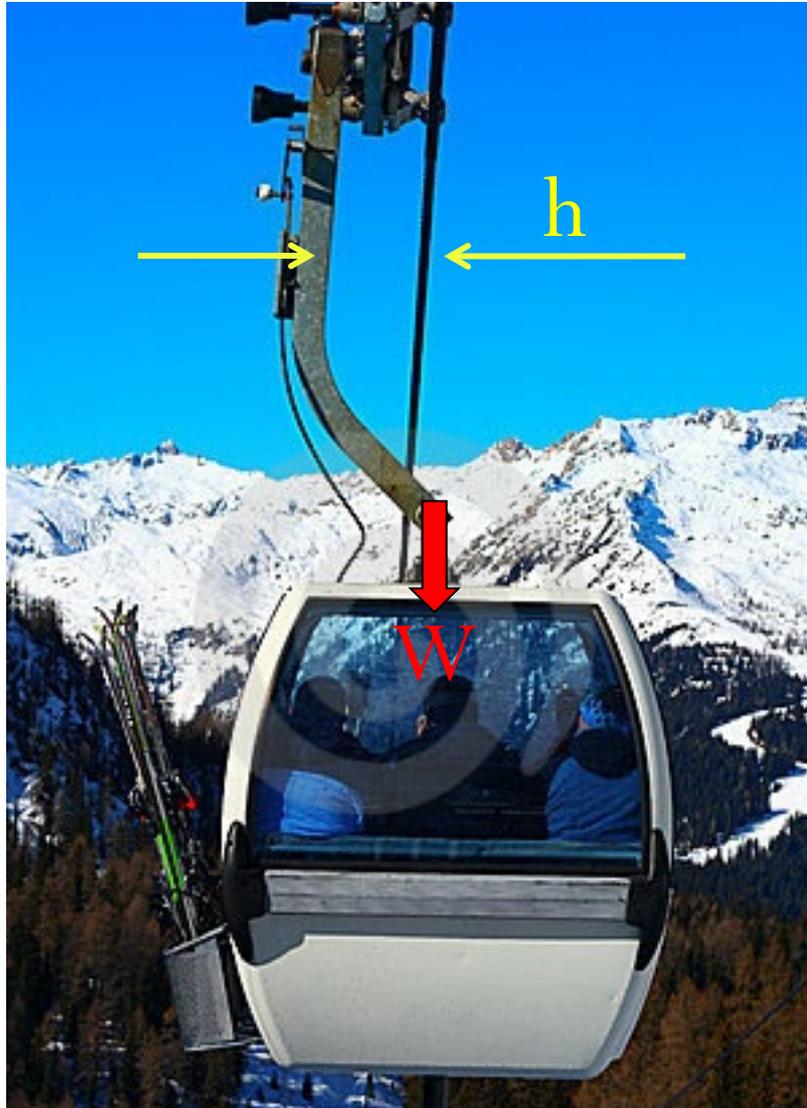
(b)

Figure: 09_01

Stresses under combined loading



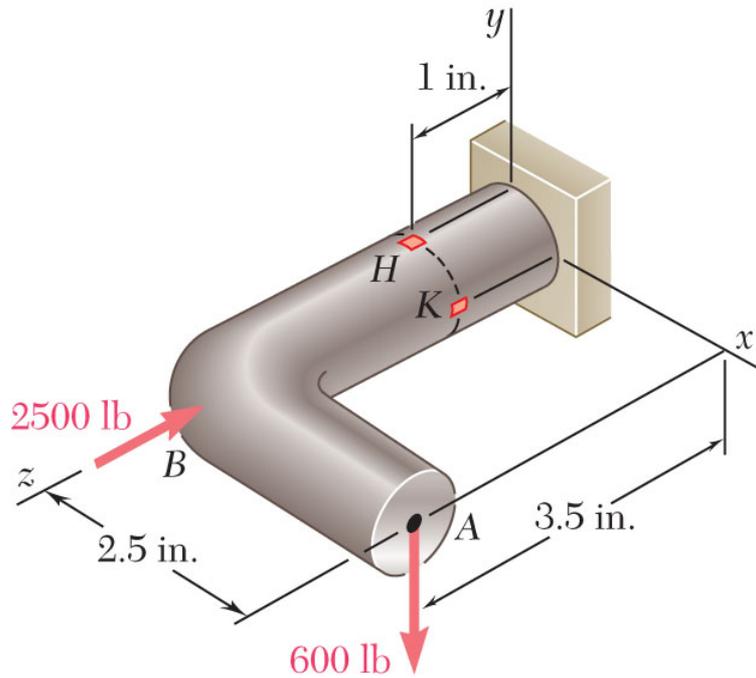
Jet Ski Gondola



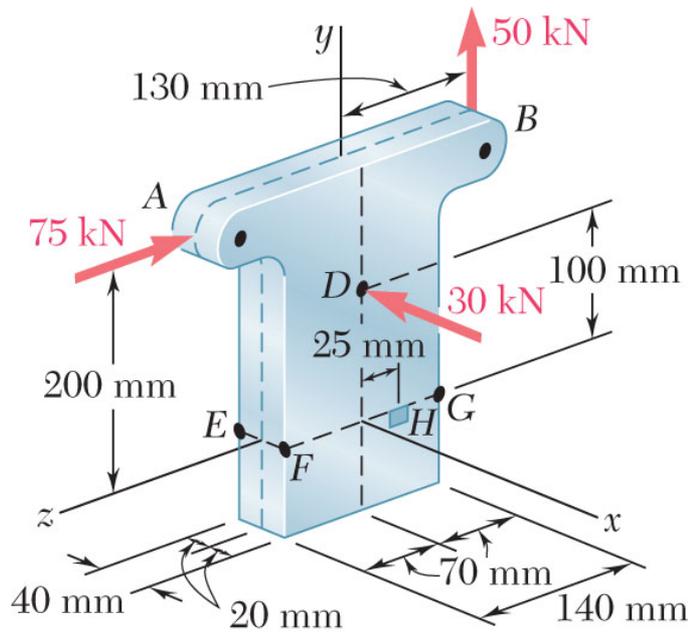
A gondola on a ski lift is supported by a bent arm, as indicated in the figure. The arm has a square cross-sectional area with side length b and is offset by a distance h from the line of action of the weight force W . Find an expression for the minimum and maximum tensile stresses in the straight section of the arm.

Example

Find the state of stress at points H and K



Example



Three forces are applied to a short steel post as shown. Determine the state of stress at point H on the outer surface of the post.