Chapter 7: Transverse Shear

Chapter Objectives

✓ Determine shear stress in a prismatic beam
✓ Determine shear flow in a built-up beam

Glulam beam?

beam made of multiple boards, such as "glulam" beam

uniform beam of constant cross-section

Bending failure usually begins near the half-height of the beam. The fracture often runs longitudinally and between boards.
Symmetry of shear stresses

- When shearing stresses are exerted on the vertical faces of an element, equal stresses must be exerted on the horizontal faces.
- Longitudinal shearing stresses must exist in any member subjected to transverse loading.

At a point in material under loading:

Consider stresses acting on the \( z-x \) face:

\[ \sigma_{xz} = 0 \]

\[ \nabla M_0 = 0 \]
acting on the $z-x$ plane

$\zeta = 0$

$-\sigma_x \frac{dy}{2} + \sigma_x dy \frac{dz}{2}$

$-\sigma_z \frac{dx}{2} + \sigma_z dx \frac{dz}{2}$

$\tau_{zx} \cdot dx \cdot dy \cdot dz - c_{xz} \cdot dz \cdot dy \cdot dx = 0$

divide by $dx \cdot dy \cdot dz$

$\tau_{zx} = \tau_{xz}$

$\zeta_{xx} = \zeta_{zz}$

$\tau_{xx} = \tau_{yy} = \tau_{zz}$

ALWAYS TRUE

-in solids
-in fluids

$\tau_{xy} = \tau_{yx}$

$\tau_{yz} = \tau_{zy}$

Stress Inventory

Up to 3 normal stresses
$\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$

Up to 3 shear stresses
$\tau_{xy} = \tau_{yx} = \tau_{xz}$
$\tau_{yz} = \tau_{zy} = \tau_{zx}$
Shear stress in beams

- Transverse loading applied to a beam results in normal and shearing stresses in transverse sections.

- Distribution of normal and shearing stresses satisfies

\[
\begin{align*}
F_x &= \int \sigma_x dA = 0 \\
F_y &= \int \tau_{xy} dA = V \\
F_z &= \int \tau_{xz} dA = 0 \\
M_x &= \int (v \tau_{xz} - z \tau_{xy}) dA = 0 \\
M_y &= \int z \sigma_y dA = 0 \\
M_z &= \int (-y \sigma_x) dA = M
\end{align*}
\]

- When shearing stresses are exerted on the vertical faces of an element, equal stresses must be exerted on the horizontal faces.

- Longitudinal shearing stresses must exist in any member subjected to transverse loading.
Transverse loading of beams

Shear forces due to transverse loading creates corresponding **longitudinal** shear stresses which will act along **longitudinal** planes of the beam.
Transverse loading of beams

When a transverse shear load is applied, it tends to cause warping of the cross section. Therefore, when a beam is subject to moments and shear forces, the cross section will not remain plane as assumed in the derivation of the bending stress formula.

However, we can assume that the warping due to the transverse shear stresses is small enough that it can be neglected, which is particularly true for slender beams.
Longitudinal shear forces in beams

Consider a prismatic beam subjected to transverse loading.

\[ V(x) = \frac{dM}{dx} \]

\[ \sigma_C(y) = -\frac{M_c \cdot y}{I_z} = -\frac{M(0) \cdot y}{I_z} \]

\[ \sigma_D(y) = -\frac{M_b \cdot y}{I_z} = -\frac{M(x + \Delta x) \cdot y}{I_z} \]

\[ \varepsilon_{F_x} = 0 \]

but the force caused by \( \int \sigma_b \cdot dA \)

is clearly larger in magnitude than \( \int \sigma_c \cdot dA \)

\( \Rightarrow \) Must be some shear force \( \Delta H \) on the bottom of this material element.

\[ Q(y) \text{ is first mom. of area of } A \text{ w.r.t. the Neutral Axis} \]

\[ Q(y) = \bar{y}_1 \cdot A \]

\[ \Delta H + \frac{\Delta M}{I_z} \int y \cdot dA = 0 \]

\[ \Delta H + \frac{M_c}{I} \int y \cdot dA - \frac{M_b}{I} \int y \cdot dA = 0 \]

\[ \Delta H + \frac{M_c}{I} \bar{y}_1 \cdot A = 0 \]

\[ \Delta H = \frac{\Delta M \cdot Q(y)}{I_z} \]

Divide by \( \Delta x \)

\[ \frac{\Delta H}{\Delta x} = \frac{-\Delta M \cdot Q(y)}{I_z} \]
Shear stress

Shear stress caused by $\Delta H$ is $q /$ the width of the beam (out of the page) at $y = y_1$

$$\tau_{xy} = \frac{V(x) \cdot Q(y)}{I_z \cdot t(y)}$$

**Average Shear Stress**

$q = \text{Shear flow} = \text{force/length acting in the longitudinal axis of the beam}$

$V = \text{transverse shear force in the beam}$

$Q = A \cdot \bar{y}$ of the shaded area above $y_1$

$I = \text{2nd mom of area of the entire cross-section}$

$$\tau_{avg} = \frac{\Delta H}{\Delta A} = \frac{\Delta H}{\Delta x \cdot t(y)} = \frac{V \cdot Q}{I \cdot t(y)}$$

$\tau_{avg} = \text{average shear stress along the width of the cut plane of interest}$
Shear stress at free edges

\[ \tau = 0 \]

No shear traction on upper or lower faces

\[ \Rightarrow \tau_{x-y} = \tau_{y-x} \]

\[ \Rightarrow \tau = 0 \]

in the cross-section at top & bottom edges

\[ \tau_{\text{max}} \text{ will occur at the Neutral Axis} \]
Shear Stress Distribution (rectangular cross-section)

\[ \tau = \frac{V \cdot Q}{I_z} \]

\[ V = \sqrt{I_z} \]

\[ I_z = \frac{bh^3}{12} \]

\[ \frac{b}{2} \cdot \frac{1}{2} \left( \frac{1}{2}h - y \right) \cdot b \]

\[ = \frac{b}{2} \left( \frac{1}{4}h - y \right) \]

\[ \frac{b}{2} \left( \frac{1}{4}h - y \right) \]

\[ \text{max. where } y = 0 \text{ (N.A.)} \]

\[ \tau = \frac{V(x) - b \cdot \left( \frac{h^2}{4} - y^2 \right)}{\left( \frac{bh^3}{12} \right) \cdot b} \]

\[ \tau = \frac{V(x)}{L} \]

\[ \text{for a rectangular cross section} \]

\[ \tau \text{ (parabolic distribution)} \]

\[ \text{max at } y = 0 \text{ (N.A.)} \]
Shear Stress Distribution: American Standard (S-beam) and wide-flange (W-beam) beams

Wide-flange beam
Shear-stress distribution is parabolic but has a jump at the flange-to-web junctions.

\[
\tau = \frac{V}{I} t
\]

\(\tau\) small in flanges because \(t\) large
\(\tau\) large in web, b.c. \(t\) small

Notice the symmetry about N.A.: \(Q_1 = Q_2\)
Example 1

Knowing that the vertical shear in the beam is $V = 400 \text{ N}$, determine the average shear stress at points A and B.

\[ \tau = \frac{VQ}{I \cdot t} \]

\[ I = 2I_1 + I_2 \]
\[ I_1 = \frac{a^4}{12}, \quad I_2 = \frac{a \cdot (3a)^3}{12} = \frac{9a^4}{4} \]
\[ I = 2 \cdot \frac{a^4}{12} + \frac{9a^4}{4} = 2 \cdot \frac{a^4}{12} + \frac{9a^4}{4} = \frac{2a^4}{12} + \frac{9a^4}{4} = \frac{2a^4 + 27a^4}{12} = \frac{29a^4}{12} \]

\[ A = a^2, \quad B = a^2 + \frac{a}{2} = a \]

\[ \tau_B = \frac{V \cdot a^3}{(2 \cdot \frac{a^4}{12}) \cdot a} = 12 \cdot \frac{V}{2a^2} \text{ length}^2 \]

\[ Q_A = \tau_A = Q_3 + Q_4 \]
\[ Q_3 = a^2 \cdot a = a^3 (= Q_B) \]
\[ Q_4 = A_4 \cdot \bar{y}_4 = \left(\frac{a}{2} \cdot 3a\right) \cdot \frac{1}{9} a = \frac{3a^3}{8} \]
\[ Q_A = a^3 + \frac{3a^3}{8} = \frac{11a^3}{8} \]

\[ \tau_A = V \cdot \frac{(\frac{11a^3}{8})}{(2 \cdot \frac{a^4}{12}) (3a)} = \frac{V}{a^2} \cdot \frac{11/4}{29/4} = \frac{11V}{58a^2} \]
Shear Flow in Built-up Beams

Consider the built-up beam below where the section is composed of 4 rectangular segments glued to one another.

How can we calculate the shear stress in the glued segments?
Example 2

A beam is made of four planks glued together. Knowing that the vertical shear in the beam is $V = 500$ N, determine the minimum required shear strength $\tau_g$ for the glue.
Built up beams with fasteners (bolts or nails)

Unlike glue, fasteners supply resistance to longitudinal shear forces at fixed internals.

Fasteners are typically spaced at a constant interval $\Delta s$ along the length of the beam.

If we know the shear flow $q$, how much load does each fastener carry?
Example 3  A beam is made of four planks, nailed together as shown. If each nail can support a shear force of 30 lb, determine the maximum spacing $s$ of the nails at B and at C so that the beam will support the force of 80lb.
Example 4

A square box beam is constructed from four planks as shown. Knowing that the spacing between nails is 1.5 in. and the beam is subjected to a vertical shear of magnitude $V = 600 \text{ lb}$, determine the shearing force in each nail.