Pure bending

Take a flexible strip, such as a thin ruler, and apply equal forces with your fingers as shown. Each hand applies a couple or moment (equal and opposite forces a distance apart). The couples of the two hands must be equal and opposite. Between the thumbs, the strip has deformed into a circular arc. For the loading shown here, just as the deformation is uniform, so the internal bending moment is uniform, equal to the moment applied by each hand.
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Geometry of deformation

We assume that "plane sections remain plane" → All faces of "grid elements" remain at 90° to each other, hence

\[ \gamma_{xy} = \gamma_{xz} = 0 \]

Therefore,

\[ \tau_{xy} = \tau_{xz} = 0 \]

Shear strain
Shear stress

No external loads on y or z surfaces:

\[ \sigma_y = \sigma_z = \tau_{yz} = 0 \]

Thus, at any point of a slender member in pure bending, we have a **state of uniaxial stress**, since \( \sigma_x \) is the only non-zero stress component

For positive moment, \( M > 0 \) (as shown in diagram):

- Segment \( AB \) decreases in length \( \sigma_x < 0 \) and \( \epsilon_x < 0 \)
- Segment \( A'B' \) increases in length \( \sigma_x > 0 \) and \( \epsilon_x > 0 \)

Hence there must exist a surface parallel to the upper and lower where

\[ \sigma_x = 0 \text{ and } \epsilon_x = 0 \]

This surface is called **NEUTRAL AXIS**
Geometry of deformation

DE is the **Neutral Axis** \((E_x = 0)\)

\(\varepsilon_x(y)\) - strain field as a function of \(y\)

\(L_{JK} = \) length line JK

\[ \varepsilon_x = \frac{\Delta L_{JK}}{L_{JK}_{\text{initial}}} = \frac{L_{JK}_{\text{final}} - L_{JK}_{\text{initial}}}{L_{JK}_{\text{initial}}} \]

\[ \varepsilon_x = \frac{(y-\rho)\theta - \rho \theta}{\rho} \]

\[\Rightarrow \quad \varepsilon_x = -\frac{y}{\rho}\]
Constitutive and Force Equilibrium

Constitutive relationship:
\[ \varepsilon_x = \frac{-y}{\rho} \]
\[ \sigma_x = E\varepsilon_x = -\frac{Ey}{\rho} \]

Force equilibrium:

\[ \sum F_x = 0 \]
\[ \sum dF = \sum \sigma_x \cdot dA = \int -\frac{E}{\rho} \sigma_y \cdot dA \]

\[ \Rightarrow -\frac{E}{\rho} \sum y \cdot dA = 0 \]

\[ \Rightarrow \text{N.A. is at the centroid} \ (\bar{y} = 0) \]