Chapter 2: Strain

Chapter Objectives

✔ Understand the concepts of normal and shear strain
✔ Apply the concept to determine the strain for various types of problems
DEFORMATION: change in length or shape of a body when forces are applied (or change in temperature)

Rubber membrane subject to tension

Undeformed

Deformed
Extensional strain

Change in length of a member divided by its original length (i.e., deformation per unit length)

\[ \epsilon = \frac{\delta}{L} = \frac{L_{\text{final}} - L_{\text{initial}}}{L_{\text{initial}}} \]

Undeformed configuration

Deformed configuration

Uniform strain along member AB

Strain is dimensionless!

Recall point-wise definition of stress:

\[ \sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \]

Similarly, we have a point-wise definition of strain:

\[ \epsilon = \lim_{\Delta x \to 0} \frac{\Delta \delta}{\Delta x} = \frac{d\delta}{dx} \]
True vs Engineering Strain

We just defined “engineering strain”, \( \epsilon = \frac{\delta}{L_i} \)

“True strain” accounts for change in length of the bar as strain increases
True vs Engineering Strain

For $L_i = 10$

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\epsilon_{\text{eng}} = \frac{\delta}{L_i}$</th>
<th>$\epsilon_{\text{true}} = \ln \left( \frac{L_f}{L_i} \right)$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.001</td>
<td>0.00099</td>
<td>0.05%</td>
</tr>
<tr>
<td>0.05</td>
<td>0.005</td>
<td>0.00498</td>
<td>0.25%</td>
</tr>
<tr>
<td>0.1</td>
<td>0.01</td>
<td>0.00995</td>
<td>0.5%</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.0953</td>
<td>4.9%</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>0.4054</td>
<td>23.3%</td>
</tr>
</tbody>
</table>
Part of a control linkage of an airplane consists of a rigid member CDB and a flexible cable AB. If a force is applied at the end D of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point D. Originally the cable is unstretched.

**Method 1: Trigonometry**
Example

Part of a control linkage of an airplane consists of a rigid member CDB and a flexible cable AB. If a force is applied at the end D of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point D. Originally the cable is unstretched.

**Method 2: Assume rotations are small**
Shear Strain

Axial loads: change in length
Shear loads: change in angle/shape

Shear strain = Change in angle that was originally at 90 degrees \( \left( \frac{\pi}{2} \right) \)

\[ = \gamma \] (for now, we consider shear strain magnitudes only)
Example

The rectangular plate is deformed into the shape shown by the dashed lines.

Determine

a) the average normal strain along diagonal BD

b) the average shear strain at corner B
Measurement of Strain

• **Direct measurement:**
  - Initial and final lengths of some section of the specimen are measured, perhaps by some handheld device such as a ruler.
  - Axial strain computed directly by following formula:

\[
\epsilon = \frac{\delta}{L} = \frac{L_{\text{final}} - L_{\text{initial}}}{L_{\text{initial}}}
\]

- Accurate measurements of strain in this way may require a fairly large initial length.
Measurement of Strain

- **Contact Extensometer:**
  - A clip-on device that can measure very small deformations
  - Two clips attach to a specimen before testing
  - The clips are attached to a transducer body
  
  \[ \varepsilon = \frac{\delta}{L} = \frac{L_{\text{final}} - L_{\text{initial}}}{L_{\text{initial}}} \]

  - The transducer outputs a voltage
  - Changes in voltage output are converted to strain

A tensile test in the Materials Testing Instructional Laboratory, Talbot Lab, UIUC

High-temperature contact extensometer. From instron.com
Measurement of Strain

- Strain gages
  - Small electrical resistors whose resistance changes with strain
  - Change in resistance can be converted to strain measurement
  - Often sold as “rosettes,” which can measure normal strain in two or more directions
  - Can be bonded to test specimen
Measurement of Strain

- **Digital Image Correlation (DIC)**
  - Image placed on surface of test specimen
  - Image may consist of speckles or some regular pattern
  - Deformation of image tracked by digital camera
  - Image analysis used to determine multiple strain components

DIC system analyzing a notch fracture test, from trilion.com

Strain field in a notch fracture test, as measured using DIC.
From barthelat-lab.mcgill.ca