D9: Crane Beam Design

Name: Md Sharier Nazim

Checked By: Nusrat

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1 Objective

Determine the plate thickness a of a welded built-up steel I-beam (three identical $6a \times a$ plates), for the Newmark Lab crane so that the factor of safety is at least 2 against (1) normal stress from the peak bending moment, (2) shear stress from the peak shear force, and (3) shear stress in the welds.

2 Given Data

Quantity	Symbol	Value
Crane rated capacity	F	20 tons = 40,000 lb = 40 kip
Beam length	L	50 ft = 600 in
Normal stress capacity	σ_Y	36 ksi
Shear stress capacity	τ_Y	18 ksi
Required factor of safety	FS	2
Plate dimensions	–	$6a \times a$ (three plates)

Allowable stresses:

$$\sigma_{all} = \frac{\sigma_Y}{\text{FS}} = \frac{36 \text{ ksi}}{2} = 18 \text{ ksi}, \qquad \tau_{all} = \frac{\tau_Y}{\text{FS}} = \frac{18 \text{ ksi}}{2} = 9 \text{ ksi}.$$

3 Assumptions

A1. Each beam is simply supported (pin at one end, roller at the other) on the wall rails. The span is taken as the full beam length, $L = 50$ ft.

A2. The cart rides midway between the two parallel beams, so each beam carries half the rated load:

$$P = \frac{F}{2} = \frac{40 \text{ kip}}{2} = 20 \text{ kip}.$$

A3. The cart load is modeled as a single concentrated force P at position u.

A4. The beam self-weight is neglected.
Note: If self-weight is considered, it can be included as a uniform distributed load $w = \gamma A$, with unit weight of steel $\gamma = 490 \text{ lb/ft}^3 = 0.2836 \times 10^{-3} \text{ kip/in}^3$.

A5. The material is linear elastic, deflections are small, and plane sections remain plane. Therefore $\sigma = Mc/I$ and $\tau = VQ/(It)$ apply.

A6. Each weld spans the full web thickness a at the flange–web junction. The weld metal is at least as strong as the base metal (given), so $\tau_{all,weld} = 9$ ksi.

4 Cross-Section Properties

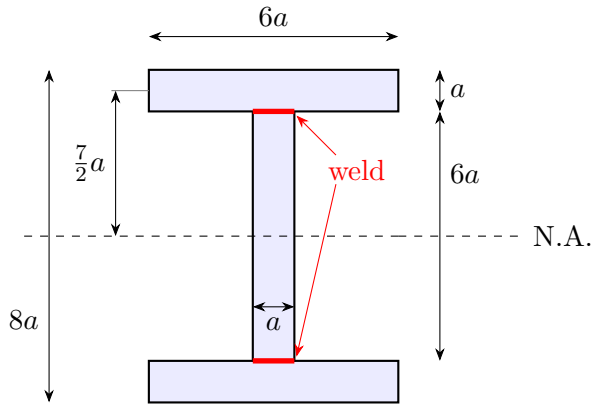


Figure 1: Built-up I-section: three $6a \times a$ plates. Welds in red; N.A. = neutral axis. The dimension $\frac{7}{2}a$ locates each flange centroid relative to the N.A.

Area

$$A = 3\,(6a)(a) = 18a^2.$$

Neutral axis

The section is symmetric top to bottom, so the neutral axis is at mid-height:

$$\bar{y} = \frac{8a}{2} = 4a \text{ from the bottom,} \qquad c = 4a \text{ (distance to extreme fiber).}$$

Moment of inertia

Use the parallel-axis theorem $I = \sum \left(I_{c,i} + A_i d_i^2\right)$, where d_i is the distance from each plate's centroid to the N.A.

Plate	$I_{c,i} = \dfrac{bh^3}{12}$	A_i	d_i	$A_i d_i^2$
Top flange ($b = 6a$, $h = a$)	$\dfrac{6a \cdot a^3}{12} = 0.5a^4$	$6a^2$	$\frac{6a}{2} + \frac{a}{2} = 3.5a$	$6a^2(3.5a)^2 = 73.5a^4$
Web ($b = a$, $h = 6a$)	$\dfrac{a(6a)^3}{12} = 18a^4$	$6a^2$	0	0
Bottom flange ($b = 6a$, $h = a$)	$0.5a^4$	$6a^2$	$3.5a$	$73.5a^4$
Sum	$19a^4$			$147a^4$

$$I = 19a^4 + 147a^4 = \boxed{166\,a^4}$$

First moment of area Q

$Q = A'\,\bar{y}'$, where A' is the area above the cut and $\bar{y}'$ is the distance from the centroid of A' to the N.A.

At the weld (flange–web junction): the area above the cut is the top flange only.

$$Q_{weld} = (6a \cdot a)\,(3.5a) = 21a^3.$$

At the neutral axis (maximum Q): the area above the cut is the top flange plus the upper half of the web (height $3a$, centroid $1.5a$ above the N.A.).

$$Q_{NA} = \underbrace{(6a^2)(3.5a)}_{\text{flange}} + \underbrace{(a \cdot 3a)(1.5a)}_{\text{half web}} = 21a^3 + 4.5a^3 = 25.5a^3.$$

At both cuts the width is $t = a$ (web thickness).

5 Free-Body Diagram and Support Reactions

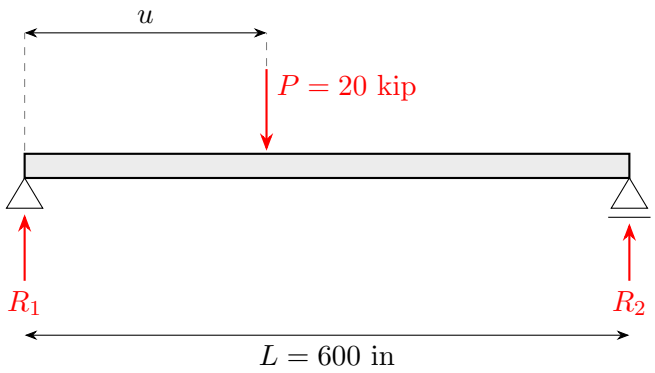


Figure 2: Free-body diagram of one crane beam with the cart load P at position u.

Referring to Fig. 2:

Moments about the left support (R_1 has no moment arm there):

$$\sum M_1 = 0: \quad R_2\,L - P\,u = 0 \quad \Rightarrow \quad R_2 = \frac{Pu}{L}.$$

Vertical force balance:

$$\sum F_y = 0: \quad R_1 + R_2 - P = 0 \quad \Rightarrow \quad R_1 = P - \frac{Pu}{L} = P\left(1 - \frac{u}{L}\right).$$

6 Internal Shear and Bending Moment

Cut the beam at position x and apply equilibrium to the left segment. Sign convention: positive V acts downward on the cut face of the left segment, and positive M is sagging.

Region 1, $0 < x < u$ (left of the cart):

$$\sum F_y = 0: \; V = R_1 = P\left(1 - \frac{u}{L}\right), \qquad \sum M_{cut} = 0: \; M = R_1\,x = P\left(1 - \frac{u}{L}\right)x.$$

Region 2, $u < x < L$ (right of the cart):

$$V = R_1 - P = -\frac{Pu}{L}, \qquad M = R_1\,x - P(x-u) = Pu\left(1 - \frac{x}{L}\right).$$

These results are plotted in Fig. 3.

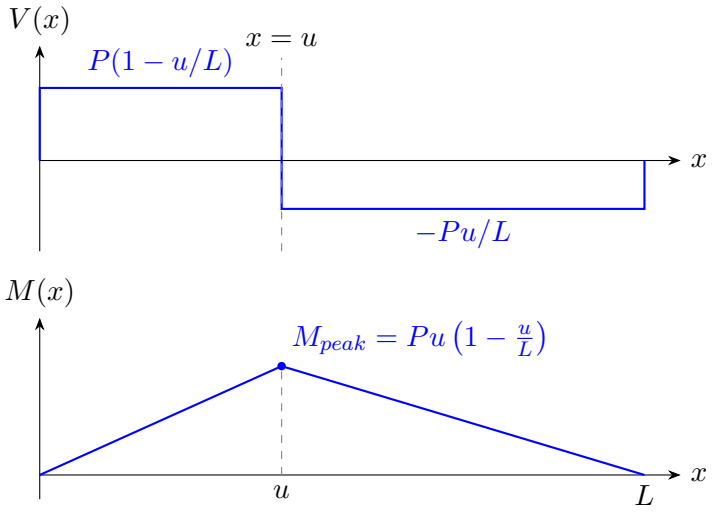


Figure 3: Shear (top) and bending moment (bottom) diagrams for an arbitrary cart position u.

Bending moment

The moment diagram is a triangle that peaks under the cart ($x = u$):

$$M(u) = P\,u\left(1 - \frac{u}{L}\right) = P\left(u - \frac{u^2}{L}\right).$$

To find the cart position that maximizes it, set the derivative to zero:

$$\frac{dM}{du} = P\left(1 - \frac{2u}{L}\right) = 0 \quad \Rightarrow \quad u = \frac{L}{2}.$$

The second derivative $d^2M/du^2 = -2P/L < 0$, so this is a maximum. Substituting:

$$M_{max} = P \cdot \frac{L}{2}\left(1 - \frac{1}{2}\right) = \frac{PL}{4} = \frac{(20\text{ kip})(600\text{ in})}{4} = \boxed{3000\text{ kip}\cdot\text{in}}\ (= 250\text{ kip}\cdot\text{ft}).$$

Shear force

For a given u, the shear magnitudes in the two regions are

$$|V_1| = P\left(1 - \frac{u}{L}\right), \qquad |V_2| = \frac{Pu}{L}.$$

$|V_1|$ is largest as $u \to 0$ (cart at the left support), and $|V_2|$ is largest as $u \to L$ (cart at the right support). Either way,

$$V_{max} = P = \boxed{20\text{ kip}}.$$

7 Check 1 – Normal Stress from Peak Bending Moment

The maximum bending stress is at the extreme fiber ($c = 4a$) of the midspan section:

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{(3000)(4a)}{166a^4} = \frac{12000}{166\,a^3} = \frac{72.29}{a^3}\text{ ksi} \quad (a \text{ in inches}).$$

Requirement $\sigma_{max} \le \sigma_{all}$:

$$\frac{72.29}{a^3} \le 18 \quad \Rightarrow \quad a^3 \ge \frac{72.29}{18} = 4.016\text{ in}^3 \quad \Rightarrow \quad \boxed{a \ge 1.59\text{ in}}.$$

8 Check 2 – Shear Stress from Peak Shear Force

The transverse shear stress is largest at the neutral axis, where Q is largest:

$$\tau_{max} = \frac{V_{max}\,Q_{NA}}{I\,t} = \frac{(20)(25.5a^3)}{(166a^4)(a)} = \frac{510}{166\,a^2} = \frac{3.072}{a^2}\text{ ksi}.$$

Requirement $\tau_{max} \le \tau_{all}$:

$$\frac{3.072}{a^2} \le 9 \quad \Rightarrow \quad a^2 \ge 0.341\text{ in}^2 \quad \Rightarrow \quad \boxed{a \ge 0.58\text{ in}}.$$

9 Check 3 – Shear Stress in the Welds

The welds join the flanges to the web and carry the horizontal shear at that cut. Use $Q_{weld} = 21a^3$ and weld width $t = a$:

$$\tau_{weld} = \frac{V_{max}\, Q_{weld}}{I\, t} = \frac{(20)(21a^3)}{(166a^4)(a)} = \frac{420}{166\, a^2} = \frac{2.530}{a^2} \text{ ksi}.$$

Requirement $\tau_{weld} \le \tau_{all}$:

$$\frac{2.530}{a^2} \le 9 \quad \Rightarrow \quad a^2 \ge 0.281 \text{ in}^2 \quad \Rightarrow \quad \boxed{a \ge 0.53 \text{ in}}.$$

10 Design Selection and Verification

Check	Minimum a (in)
(1) Bending	1.59
(2) Shear at N.A.	0.58
(3) Weld shear	0.53

Selected design: $\boxed{a = 1.625 \text{ in } (1\tfrac{5}{8} \text{ in plate})}$. This is the next standard plate thickness above 1.59 in.

- Flange plates: $6a = 9.75$ in wide $\times$ 1.625 in thick.
- Web plate: $6a = 9.75$ in tall $\times$ 1.625 in thick.
- Overall depth: $8a = 13$ in.

Verification with $a = 1.625$ in:

$$I = 166(1.625)^4 = 166(6.973) = 1157.5 \text{ in}^4, \qquad c = 4(1.625) = 6.5 \text{ in},$$
$$Q_{NA} = 25.5(1.625)^3 = 109.4 \text{ in}^3, \qquad Q_{weld} = 21(1.625)^3 = 90.1 \text{ in}^3.$$

$$\sigma_{max} = \frac{(3000)(6.5)}{1157.5} = 16.85 \text{ ksi} \quad \Rightarrow \text{ FS} = \frac{36}{16.85} = 2.14,$$
$$\tau_{max} = \frac{(20)(109.4)}{(1157.5)(1.625)} = 1.16 \text{ ksi} \quad \Rightarrow \text{ FS} = \frac{18}{1.16} = 15.5,$$
$$\tau_{weld} = \frac{(20)(90.1)}{(1157.5)(1.625)} = 0.96 \text{ ksi} \quad \Rightarrow \text{ FS} = \frac{18}{0.96} = 18.8.$$

Failure mode	Stress (ksi)	Capacity (ksi)	FS	FS ≥ 2?
(1) Bending, Mc/I	16.85	36	2.14	✓
(2) Shear at N.A., VQ/It	1.16	18	15.5	✓
(3) Weld shear, VQ/It	0.96	18	18.8	✓

All three failure modes have FS ≥ 2, so $a = 1.625$ in is an acceptable design.