



This calculation documents the internal forces in the basketball hoop structure during a slam dunk.

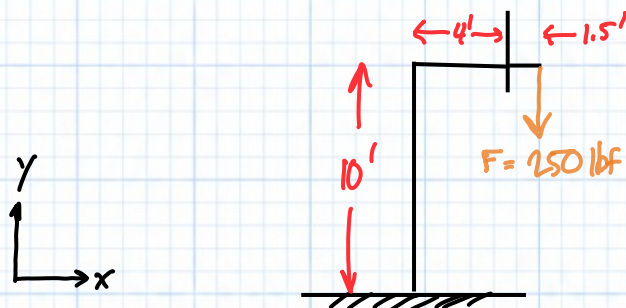
(1) Idealization of components and loads on the system.

The system shown on the right consists of a vertical pole, a horizontal pole, the backboard, and rim. Consider both poles and rim as beams. The rim is considered as beam in XY plane shown below.



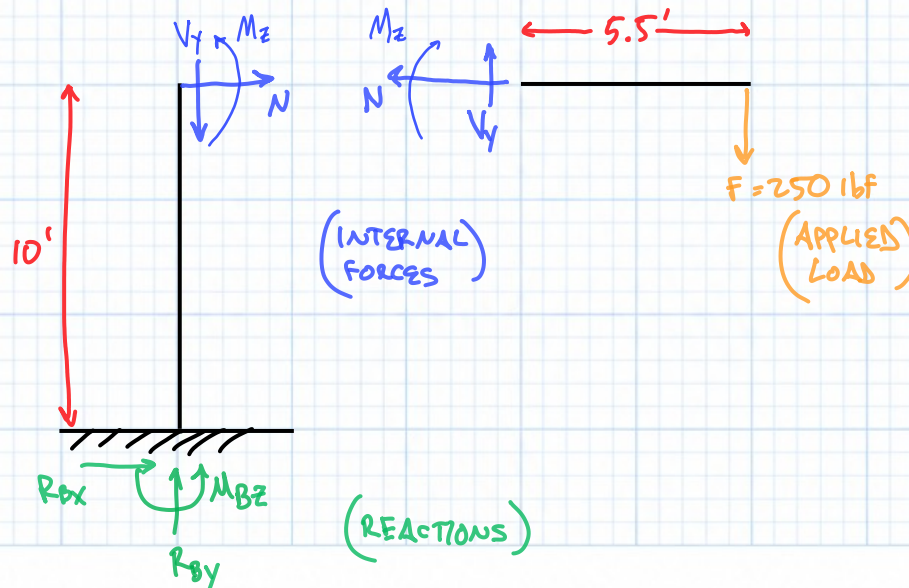
The loads on the system include Bradley (250 lbf), and the self-weight of components. For this analysis assume the component self-weight is negligible, only static loading (no dynamics), the force from Bradley has only a vertically downward component, the force is applied at outer end of the rim. There are no loads on the backboard.

This idealization of the problem is:



Height of basketball hoop is typically 10 ft, diameter of rim is 1.5 ft, 4ft cantilever approximated from photo. Consider the rim as an extension of the horizontal pole here. (assume full continuity).

2) Free Body Diagram. Here, the rim and horizontal pole are shown as continuous, and separate diagrams are drawn in order to show internal forces at the connection between components.





3) Use static equilibrium to solve for the reactions at the base.

$$\sum F_x: N = 0$$

$$\sum F_y: -F + V_y = 0 \quad V_y = F = 250 \text{ lbf}$$

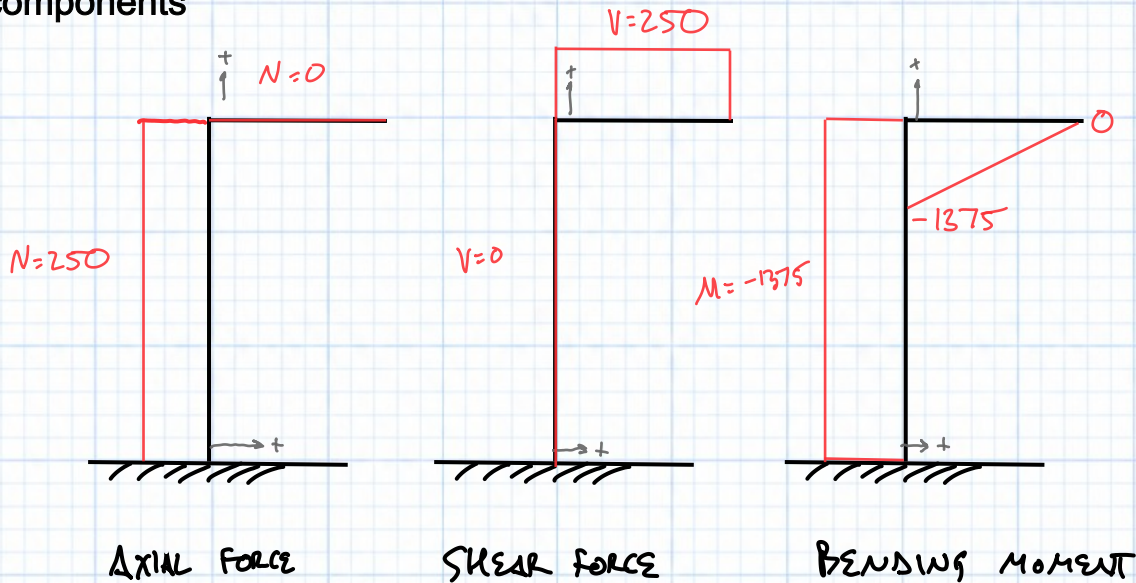
$$\sum M_z: -M_z - (5.5)F = 0 \quad M_z = -5.5F = -1375 \text{ ft-lbf}$$

$$\sum F_y: -V_y + R_{By} = 0 \quad R_{By} = V_y = 250 \text{ lbf}$$

$$\sum M: M_{Bz} + M_z = 0 \quad M_{Bz} = 1375 \text{ ft-lbf}$$

$$\sum F_x: R_{Bx} = 0$$

4) Member internal force diagrams for the both horizontal and vertical components

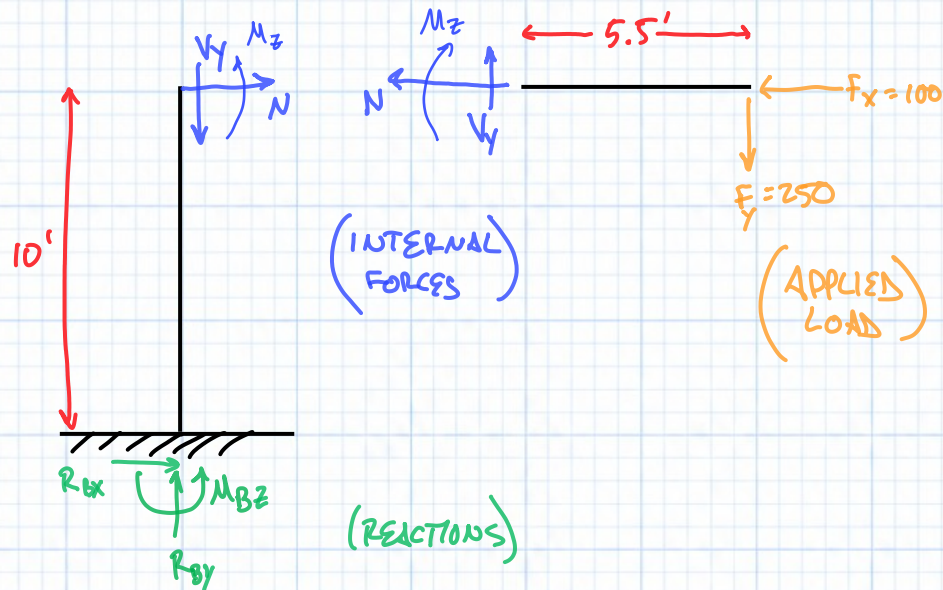


Positive internal force directions are indicated with $\rightarrow +$

Note that there is no axial force in the horizontal beam and no shear force in the vertical beam. This is because the applied force is assumed to have only a vertical component.



Note that if the assumed loading includes a lateral component in the plane of structure, the internal forces would be different. For example, assume a 100 lbf component towards the hoop:



$$R_{By} = 250 \text{ lbf}$$

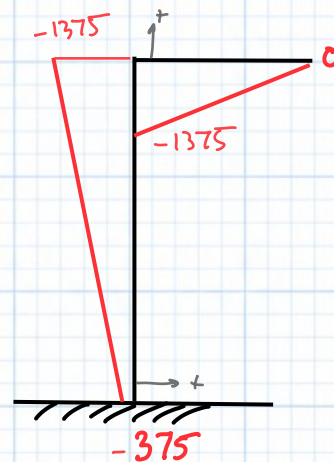
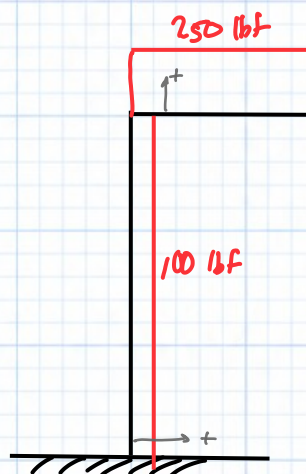
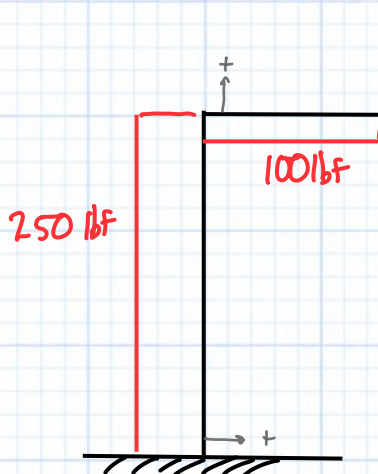
$$V_y = 250 \text{ lbf}$$

$$R_{Bx} = 100 \text{ lbf}$$

$$N = -100 \text{ lbf}$$

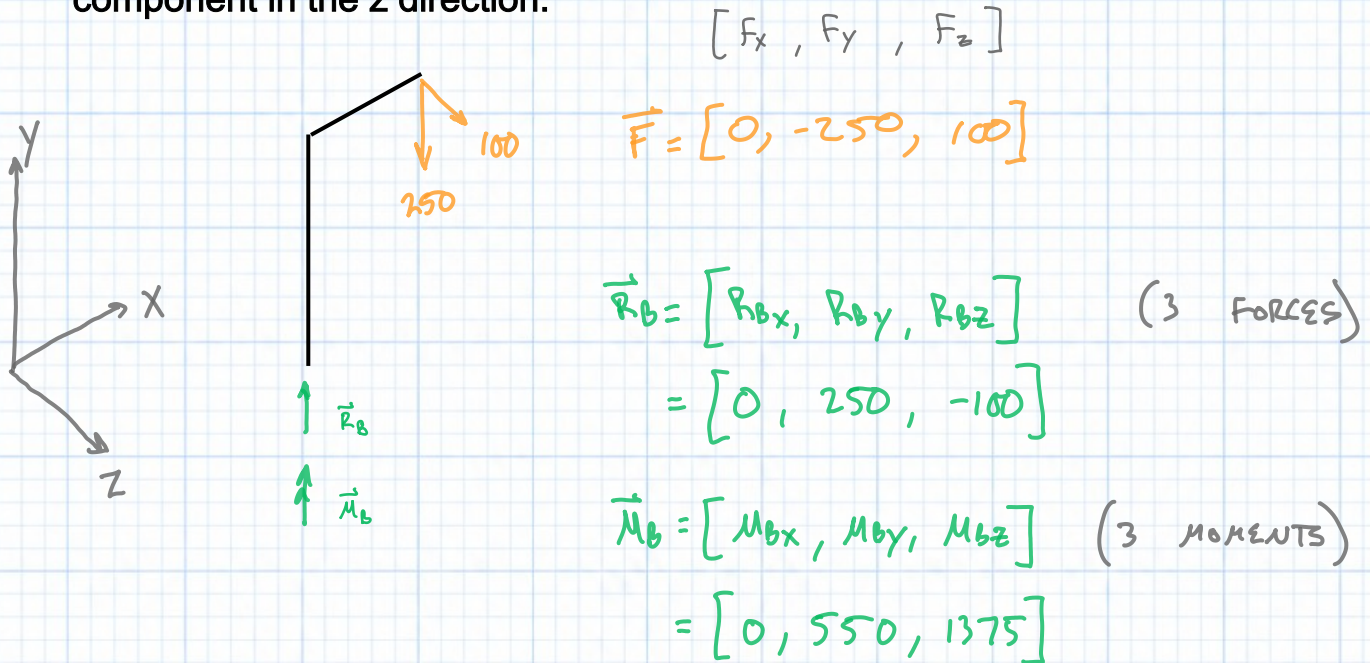
$$M_{Bz} = 375 \text{ ft} \cdot \text{lbf}$$

$$M_z = -1375 \text{ ft} \cdot \text{lbf}$$

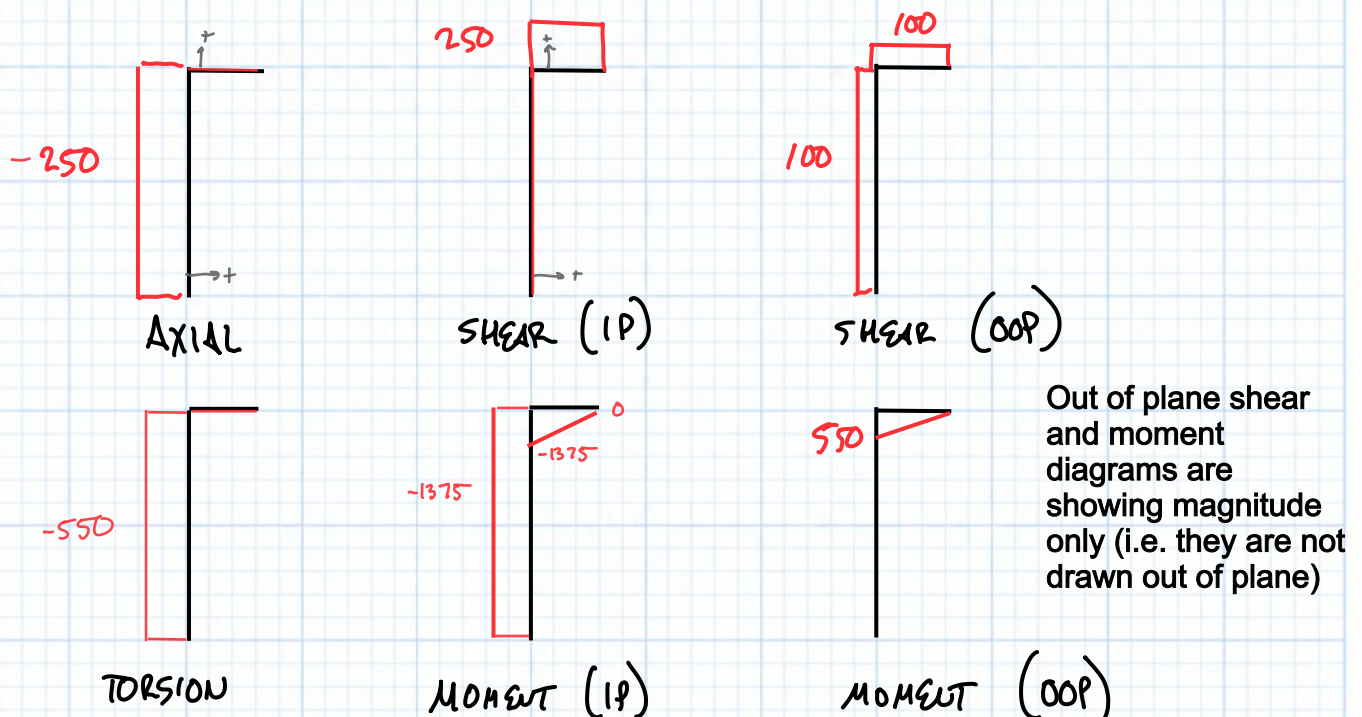




Note that if the assumed loading includes a lateral component in the out of plane direction, the problem is now 3D. For instance, assume a 100lbf component in the z direction:



This 3D loading now requires 6 internal force diagrams (IP: in-plane, OOP: out-of-plane where in-plane refers to the XY plane):





5) Where is the hoop most likely to fail and why?

To be complete, we would need to check the entire load path, from the applied load at the rim to the ground. This would include the rim, rim to horizontal pole connection, horizontal pole, vertical pole, and connection to ground. Locations of discontinuities, such as connections, are common failure locations. We do not have enough information about the rim connection to evaluate it here, so we'll focus on the pole.

The idealized model used here (with only vertical loading) shows a constant bending moment + axial force in the vertical pole, a constant shear + linear moment in the horizontal pole. If the combination of shear and bending is governing, it would fail at the connection between vertical and horizontal members (at the bend in the pipe). If the combination of axial and bending is governing, failure is equally likely along the vertical member. Looking at the actual structure, there is a connection between the rim and horizontal pole, as well as a connection between the vertical pole and the ground that could also be locations of stress concentrations (from the discontinuity of the connections).

6a) What happened?

The force from a person hanging on the rim lead to member internal forces higher than the structure could support.

6b) Why did it fail?

The (component scale) member forces equated to (material scale) stresses that exceeded the capacity of the material at a given location.

6c) Isn't it supposed to be designed so it can handle any weight that can be applied on it?

Designing for "any" weight is impractical, and would be excessively expensive. It should be designed for forces likely to occur plus some margin of safety, but forces that are likely to occur can be difficult to anticipate.



University of Illinois Urbana-Champaign
Theoretical and Applied Mechanics

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