TAM 251 - Discussion 10

Built-up wooden cantilever: reference solution

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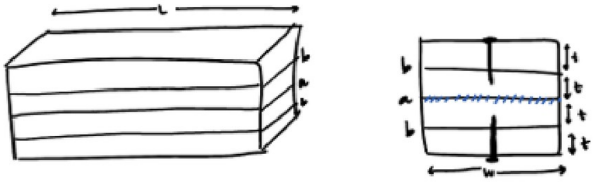
Given: P = 12 kN, L = 3000 mm; up to four 240 × 60 mm planks.

Allowables: wood bending 10 MPa; wood shear 1.5 MPa; glue shear 2.0 MPa.
Nails: 2.0 kN per nail; 200 available. Use N and mm (1 N/mm² = 1 MPa).

Assume full composite action, negligible self-weight, and adequate fixed-end anchorage. The load acts in the web plane; local bearing, deflection and stability are not checked.

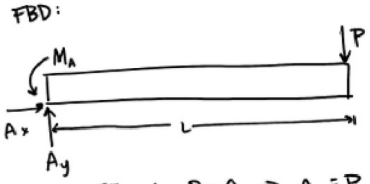
1. Internal loads and a first trial

Trial: stack all four planks flat (240 × 240 mm overall). Glue the middle interface; use 50 nails at each outer interface. The handwritten work shows the load diagrams and bending check (I is in m⁴).



Finding shear, bending moments

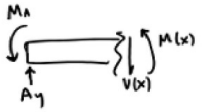
FBD:



$\Sigma F_y: A_y - P = 0 \Rightarrow A_y = P$

$\Sigma F_x: A_x = 0$

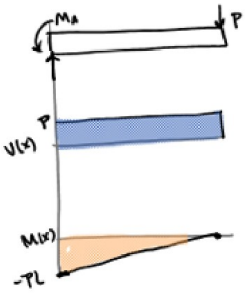
$\Sigma M_0: M_A - PL = 0 \Rightarrow M_A = PL$



$\Sigma F_y: A_y - V(x) = 0$
$\Rightarrow V(x) = A_y = P$

$\Sigma M_0: M_A - V(x)\cdot x + M(x) = 0$
$M(x) = V(x)\cdot x - M_A$
$M(x) = Px - PL = P(x - L)$

Sketch diagrams:



NB: $V(x) = P$ everywhere
$|M(x)|$ is max @ $x = 0$
$M(x) = -PL$

For this configuration:

$I = \frac{1}{12}bh^3 = \frac{1}{12}w(4t)^3 =$ 0.000276

Check Max Normal Stress in the wood
NB: it occurs @ $c = 2t$

$\sigma = \frac{Mc}{I}$

plug in Mmax

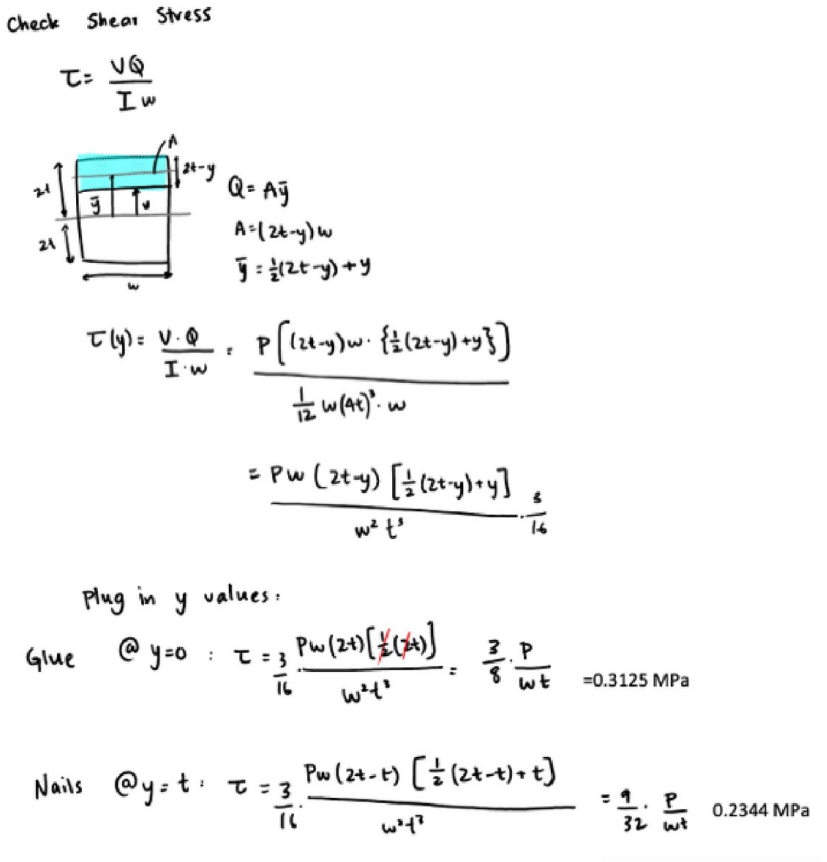
$\sigma_{max} = \frac{(-PL)(2t)}{\frac{1}{12}wt^3} = 15.625$ MPa

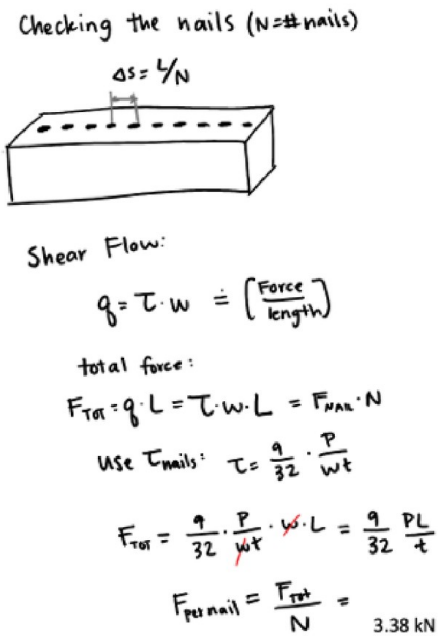
$V_{max} = 12{,}000\,N\,;\, M_{max} = PL = 36{,}000{,}000\,N\,mm.$

The four-layer trial fails in bending: 15.625 MPa > 10 MPa.

2. Four-layer trial: wood shear and connections

Continue the same trial. In the handwritten work, interface a is the central glue joint; interface b is either outer nailed joint. Each nailed joint has N = 50 nails.





At the neutral axis / central glue joint:

$\tau_{wood,max} = 0.3125\,MPa < 1.5\,MPa\,;\ \tau_{glue} = 0.3125\,MPa < 2.0\,MPa\,.$

At each outer nailed interface:

$q = \frac{VQ}{I} = 56.25\,N/mm\,;\ s = L/50 = 60\,mm\,.$

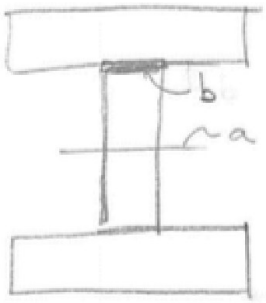
$F_{nail} = qs = 3375\,N = 3.375\,kN > 2.0\,kN\,.$

The 0.2344 MPa in the handwritten work is the nominal interface shear used to obtain q = τw; compare the force per nail, not this interface stress, with the stated nail capacity.

Conclusion: the trial passes the wood-shear and glue checks, but fails bending and nail capacity. Adding nails cannot correct the insufficient bending section modulus. Use a deeper cross section.

3. One acceptable design: three-plank I-section

Use one plank for each flange and one vertical plank for the web. Continuously glue both flange–web interfaces along the full 3000-mm length. The fourth plank is not needed.



Top and bottom flanges: 240 × 60 mm each.
Web: 60 × 240 mm; overall height = 360 mm.
Neutral axis at mid-height; c = 180 mm.
Flange centroid: 150 mm from the neutral axis.
In the sketch, a marks the neutral axis; b marks a joint.
Each glue joint is 60 mm wide (not 240 mm).

Section property (parallel-axis theorem):

$$I_z = \frac{60(240^3)}{12} + 2\left[\frac{240(60^3)}{12} + (240)(60)(150^2)\right]$$

$$I_z = 725{,}760{,}000\, mm^4.$$

(1) Maximum bending stress in the wood

$$\sigma_{w,max} = \frac{M_{max}\, c}{I_z} = \frac{(36{,}000{,}000)(180)}{725{,}760{,}000} = 8.929\, MPa < 10\, MPa.$$

(2) Maximum shear stress in the wood (at the neutral axis)

$$Q_{NA} = (240)(60)(150) + (60)(120)(60) = 2{,}592{,}000\, mm^3.$$

$$\tau_{w,max} = \frac{(12{,}000)(2{,}592{,}000)}{(725{,}760{,}000)(60)} = 0.714\, MPa < 1.5\, MPa.$$

(3) Shear in each continuous glue joint

$$Q_g = (240)(60)(150) = 2{,}160{,}000\, mm^3.$$

$$q_g = \frac{V\, Q_g}{I_z} = 35.714\, N/mm;\ \tau_g = \frac{35.714}{60} = 0.595\, MPa < 2.0\, MPa.$$

Both joints must carry this shear flow; do not divide it between the two joints.

Selected design: three planks with continuous glue at both joints. All three specified strength requirements are satisfied under the stated assumptions. No nails are needed.

Optional nails-only alternative (same I-section): use one row at each joint with 50-mm spacing. Each nail carries qs = 1.786 kN < 2.0 kN. Using 60 nails per joint gives 120 total (≤ 200). Nail embedment, edge distances and slip require separate detailing.