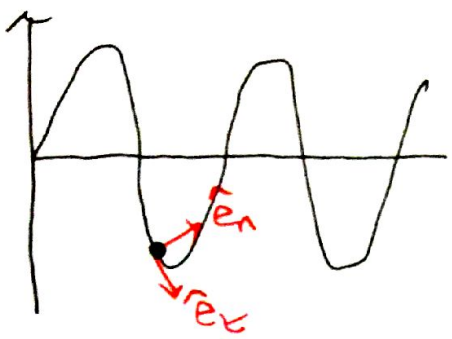
8 MAY 2015

Praire Learn 8-14 - "easy way"



$m = 6$ kg

$y(x) = 2\sin\left(\frac{3}{2}x\right)$

$x = 3$, speed $v = 2$, $\dot{v} = 5$

moving to RIGHT

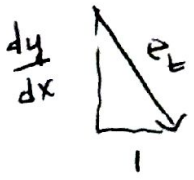
Find force of wire on the particle:

Method of Assumed Motion)

position $\vec{r}(x) = x(t)\,\hat{i} + y(t)\,\hat{j}$

$= x\,\hat{i} + 2\sin\left(\frac{3}{2}x\right)\hat{j}$

tang/norm coordinates:
at current position



$$\hat{e}_t = \hat{i} + 2\cdot\frac{3}{2}\cos\left(\frac{3}{2}x\right)\hat{j}\Big|_{x=3}$$

$$= \left(\hat{i} + 3\cos\left(\frac{9}{2}\right)\hat{j}\right)\Big/\sqrt{1+9\cos^2\left(\frac{9}{2}\right)}$$

$$\hat{e}_n = \left(-3\cos\left(\frac{9}{2}\right)\hat{i} + \hat{j}\right)\Big/\sqrt{1+9\cos^2\left(\frac{9}{2}\right)}$$

$\vec{v} = v\,\hat{e}_t = \dot{s}\,\hat{e}_t$ $\qquad \dot{s} = 2$ (given)

$\vec{a} = \ddot{s}\,\hat{e}_t + \frac{\dot{s}^2}{\rho}\,\hat{e}_n$ $\qquad \ddot{s} = \dot{v} = 5$ (given)

still need ρ = radius of curvature of path

$$\rho = \left|\frac{\left(1+\left(dy/dx\right)^2\right)^{3/2}}{\left(d^2y/dx^2\right)}\right|$$

$$dy/dx = 3\cos\left(\tfrac{3}{2}x\right) = 3\cos(9/2)$$

$$d^2y/dx^2 = -\frac{9}{2}\sin\left(\tfrac{3}{2}x\right) = -\frac{9}{2}\sin\left(\tfrac{9}{2}\right)$$

$$\downarrow \text{ at } x = 3$$

$$\Rightarrow \quad \rho = 0.3765 \quad \text{meters}$$

Thus $\vec{a} = \ddot{s}\,\hat{e}_L + \frac{\dot{s}^2}{\rho}\hat{e}_n$

$$= \ddot{s}\left(\frac{\hat{\imath} + 3\cos\left(\frac{9}{2}\right)\hat{\jmath}}{1.18318}\right) + \frac{\dot{s}^2}{\rho}\left(\frac{-3\cos\left(\frac{9}{2}\right)\hat{\imath} + \hat{\jmath}}{1.18318}\right)$$

$$= \frac{\hat{\imath}}{1.18318}\left[\ddot{s} - \frac{3\dot{s}^2}{\rho}\cos\left(\frac{9}{2}\right)\right] + \frac{\hat{\jmath}}{1.18318}\left[3\ddot{s}\cos\left(\frac{9}{2}\right) + \frac{\dot{s}^2}{\rho}\right]$$

$$= (11.7179\,\hat{\imath} + 7.46116\,\hat{\jmath})/(1.18318)$$

Total Force on Particle $\vec{F} = \vec{F}_g + \vec{F}_w$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_g + \vec{F}_w = 6(11.7179\hat{\imath} + 7.46116\hat{\jmath})/(1.18318)$$

$$-6(9.8\hat{\jmath}) + \vec{F}_w = \left[6(11.7179\hat{\imath}) + 6(7.46116\hat{\jmath})\right]/1.183$$

$$\boxed{\vec{F}_w = 59.4225\,\hat{\imath} + 96.6361\,\hat{\jmath}}$$