TAM 212. Midterm 1. Mar. 5, 20145.
Discussion 'Quiz'

- There are 20 questions, each worth 5 points.
- You must not communicate with other students during this test.
- No electronic devices allowed.
- This is a 2 hour exam.
- Do not turn this page until instructed to do so.
- There are several different versions of this exam.

1. Fill in your information:

Full Name: ____________________________

UIN (Student Number): ____________________________

NetID: ____________________________

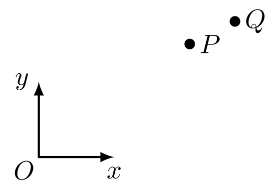
2. Circle your discussion section:

	Monday	**Tuesday**	**Wednesday**	**Thursday**
8–9				
9–10		ADC (260 MEB)		ADK (260 MEB)
10–11		ADD (335 MEB)		
11–12		ADE (335 MEB)		ADL (153 MEB)
12–1	ADA (243 MEB)	ADF (335 MEB)	ADJ (335 MEB)	ADN (260 MEB)
1–2				
2–3				
3–4				
4–5			ADO (260 MEB)	
5–6	ADB (260 MEB)	ADH (260 MEB)	ADM (243 MEB)	

3. Fill in the following answers on the Scantron form:

91. A
92. A
93. A
94. A
95. D
96. C

1/1. (1 point) Points P and Q are moving in circular paths around the origin O with angular velocities ω_P and ω_Q.



The two particles are moving with the same speed. Which statement is true?

A. $|\omega_P| \leq \frac{1}{2}|\omega_Q|$

B. $\frac{1}{2}|\omega_Q| < |\omega_P| \leq |\omega_Q|$

C. ★ $|\omega_Q| < |\omega_P| \leq 2|\omega_Q|$

D. $2|\omega_Q| < |\omega_P|$

Solution. If the particles both have speed v then we can observe from the diagram that ω_P is a little bit higher than ω_Q.

More precisely, taking absolute values to ignore the direction, we have:

$$|\omega_P| = \frac{v}{r_P}$$
$$|\omega_Q| = \frac{v}{r_Q}$$
$$|\omega_P| = \frac{r_Q}{r_P}|\omega_Q|.$$

From the diagram, $\frac{r_Q}{r_P} \approx 1.25$, so ω_P is larger than $|\omega_Q|$, but smaller than $2|\omega_Q|$.

2/1. (1 point) A particle moves so that its position vector in the Cartesian basis is given by

$$\vec{r} = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + t\,\hat{k} \text{ m}.$$

Using cylindrical coordinates, what is the angular component of velocity v_θ at $t = \pi/4$ s?

A. $v_\theta < -1$ m/s

B. -1 m/s $\leq v_\theta < 0$ m/s

C. $v_\theta = 0$ m/s

D. 0 m/s $\leq v_\theta < 1$ m/s

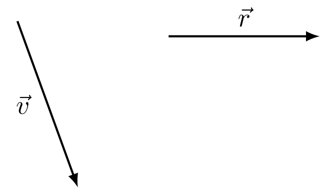
E. ★ 1 m/s $\leq v_\theta$

Solution. If we realize that the $\hat{\imath}, \hat{\jmath}$ motion is uniform circular motion with constant $R = 1$ m and $\dot{\theta} = 1$ rad/s, then we known that $v_\theta = R\dot{\theta} = 1$ m/s.

Alternatively, $\vec{r} = \frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath} + \frac{\pi}{4}\,\hat{k}$, so $\hat{e}_r = \frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath}$ and $\hat{e}_\theta = -\frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath}$. Then:

$$\begin{aligned}
\vec{r}(t) &= \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + t\,\hat{k} \\
\vec{v}(t) &= -\sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + \hat{k} \\
\vec{v}(\pi/4) &= -\frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath} + \hat{k} \\
v_\theta &= \vec{v} \cdot \hat{e}_\theta \\
&= \left(-\frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath}\right) \cdot \left(-\frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath} + \hat{k}\right) \\
&= 1\text{m/s}.
\end{aligned}$$

3/1. (1 point) The position vector $\vec{r}$ and velocity $\vec{v}$ for a single particle P are shown below at a particular instant.



Which statement about $\dot{r}$ is true at this instant?

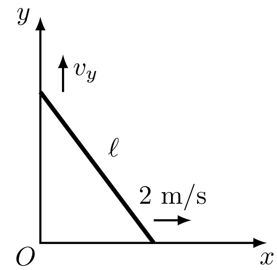
A. ★ $\dot{r} > 0$

B. $\dot{r} = 0$

C. $\dot{r} < 0$

Solution. $\vec{v} \cdot \vec{r} > 0$ so $\vec{v}$ has a positive component in the $\vec{r}$ direction, meaning that $\vec{r}$ is getting longer and $\dot{r} > 0$.

4/1. (1 point) A ladder leaning against the wall has a fixed length of $\ell = 5$ m. The bottom of the ladder is 3 m from the wall and is moving along the ground away from the wall at a speed of 2 m/s. What is the vertical component of the velocity v_y of the top of the ladder, assuming it remains in contact with the wall?



A. ★ $v_y < -1$ m/s

B. -1 m/s $\leq v_y < 0$ m/s

C. $v_y = 0$ m/s

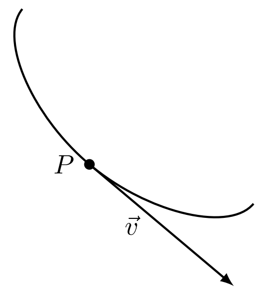
D. 0 m/s $< v_y < 1$ m/s

E. 1 m/s $< v_y$

Solution. Taking x to be the horizontal coordinate of the bottom of the ladder, and y the vertical coordinate of the top of the ladder, we have $x = 3$ and $y = 4$ at the instant shown. Then:

$$\begin{aligned}
\ell^2 &= x^2 + y^2 \\
0 &= 2x\dot{x} + 2y\dot{y} \\
\dot{y} &= -\frac{x}{y}\dot{x} \\
v_y &= -\frac{3}{4}2 \\
&= -1.5 \text{ m/s.}
\end{aligned}$$

5/1. (1 point) A point P is moving around a curve and at a given instant has position and velocity $\vec{v}$ as shown.



Which direction is the closest to the direction of the normal basis vector $\hat{e}_n$ at the instant shown?

A. $\swarrow$

B. $\searrow$

C. $\nwarrow$

D. ★ $\nearrow$

Solution. $\hat{e}_n$ is inwards to the curve.
