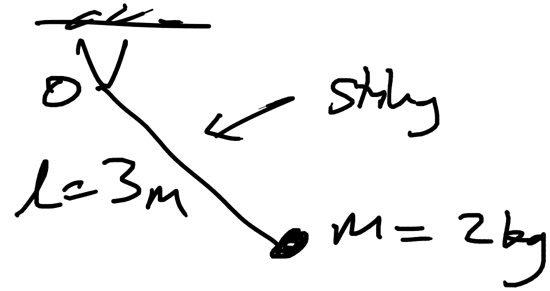
- Discussion optional M–W
- office hours Sunday 3rd → Thursday 14th
  4–6 pm Grainger.

Moments of Inertia

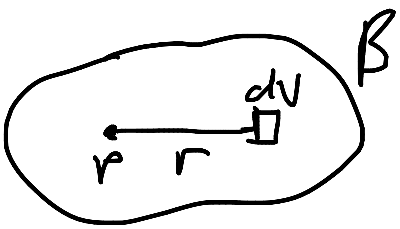
point mass

$I_{Oz} = mr^2$

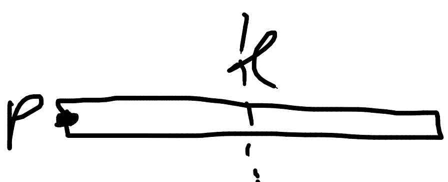


$I_{Oz} = 18 \text{ kg m}^2$

body



$$I_{Pz} = \iiint_B \rho r^2 dV \qquad r^2 = x^2 + y^2$$

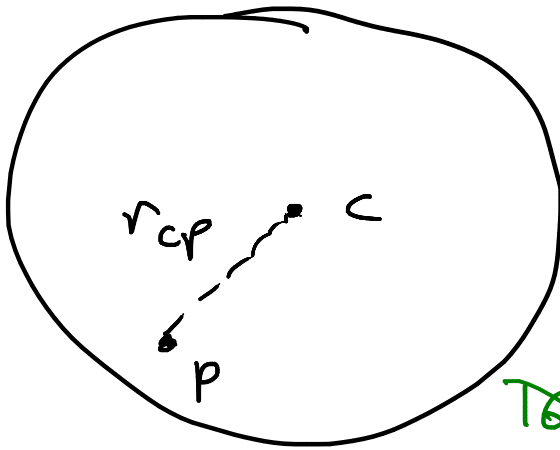


rod $I_{Pz} = \frac{1}{3} m l^2$

Q. broomstick, rotating about end, cut in half, what is the change in I_{Pz}?

A. same. B. $\frac{1}{2}$. C. $\frac{1}{4}$ D. $\frac{1}{8}$ E. $\frac{1}{16}$

parallel axis thm



CoM

$\underbrace{3 \cdot 2^2}_{12}$ 16   4

$$I_{pz} = I_{cz} + m r_{cp}^2$$

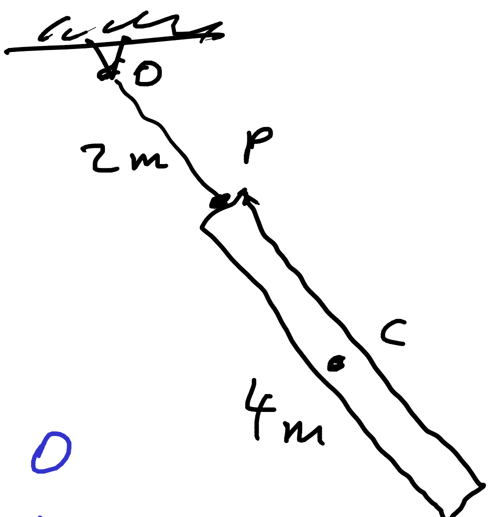
$$\boxed{I_{oz} = \underset{4}{I_{cz}} + \underset{3}{m}\, r_{co}^2}$$

M = A. 1   B. 2   C. 3   D. 4   E. 5 kg

16

rod: $I_p = \frac{1}{3} m l^2$

Ex



$$I_{oz} = \cancel{I_{pz}} + \cancel{m r_{po}^2}$$

~~16~~   ~~3, 2^2~~

$I_{pz} = 16$ kg m^2

$$\boxed{I_{oz} = \text{A. } 16 \quad \text{B. } 22 \quad \text{C. } 28 \quad \text{D. } 38 \quad \text{rod} \quad \text{E. } 52 \text{ kg m}^2}$$

I_{cz} = A. 0   B. 4   C. 12   D. 14   E. 28 kg m^2

$\frac{1}{12} m l^2$

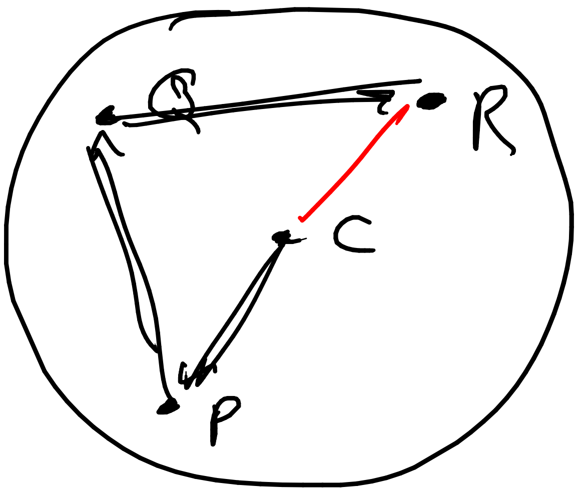
∥ axis C → O

r_{co} = A. 0   B. 2   C. 4   D. 6   E. 8 m

Parallel axis thm:

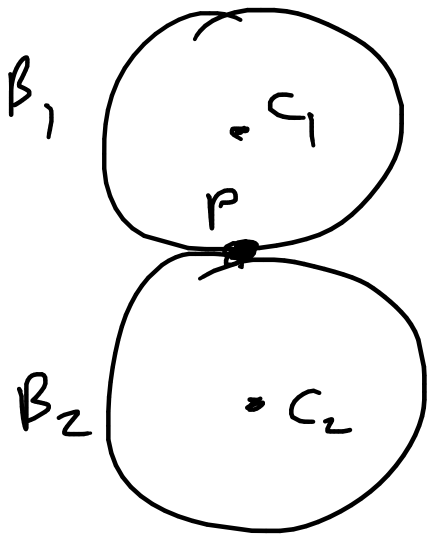
$$\boxed{I_{pz} = I_{cz} + m r_{cp}^2}$$

must be COM



Additive thm

Additive thm



$m_1 = 1\,kg$

$r_1 = 2\,m$

$m_2 = 1\,kg$

$r_2 = 2\,m$

$$I^{total}_{?} \neq I^1_{c_1 z} + I^2_{c_2 z}$$

$$I^1_{c_1 z} = 2\ kg\,m^3$$

$$I^2_{c_2 z} = 2\ kg\,m^2$$

$$I^{total} \neq 4\ kg\,m^2$$

$\frac{1}{2}mr^2$

$I^{total}_{pz} =$

A. 4 $kg\,m^2$

B. 6

C. 8

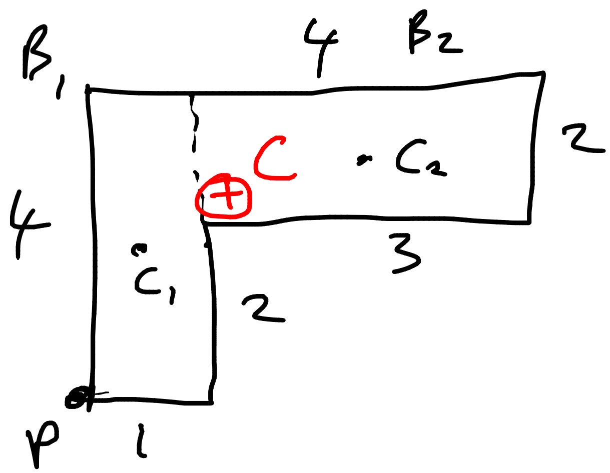
D. 10

E. 12

$$\begin{cases} I^1_{pz} = I^1_{c_1 z} + m r^2_{c_1 p} \\ I^2_{pz} = I^2_{c_2 z} + m r^2_{c_2 p} \end{cases}$$

$$\boxed{I^{total}_{pz} = I^1_{pz} + I^2_{pz}}$$ ← same p, same z

Computing MoI of composite bodies about C



- MoIs $I^1_{C_1 z}$, $I^2_{C_2 z}$
- parallel axis

$$I^1_{Cz}, \quad I^2_{Cz}$$

- additive thm

$$I^{total}_{Cz} = I^1_{Cz} + I^2_{Cz}$$

$$\sum \vec{M} = I \vec{\alpha}$$

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} I_x & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix}$$

$$I = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{zz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$