TAM 212: Class 39

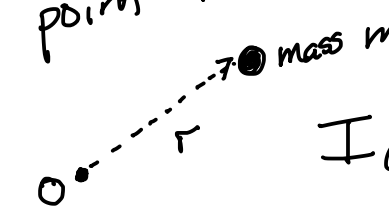
Moments of Inertia - resistance to rotational acceleration.

Euler's 2nd Law: $\sum M_{C,z} = I_{C,z} \alpha_z$

looking up in tables

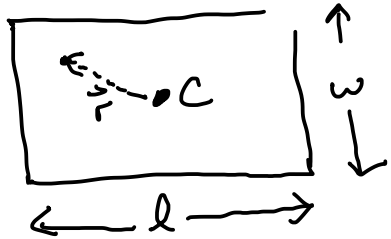
How to calculate?

ex) point mass



$I_{O,z} = mr^2$

ex) Rigid Body



C = center of mass

ρ = mass density

r = distance from rotation axis

For R.B.

$$I_{c,z} = \int \rho r^2 \, dA$$

$$= \int_{-l/2}^{l/2} \int_{-w/2}^{w/2} \rho (x^2 + y^2) \, dx \, dy$$

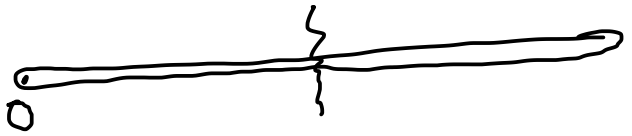
$$= \ldots\ldots \quad \frac{1}{12} m (l^2 + w^2)$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$r^2 = x^2 + y^2$$

Note: $m = \rho lh$

Ex) Rod of length l



$$I_{O,z} = \frac{1}{3} m l^2$$

$I'_{O,z}$

idiocue question: $I'_{O,z} = N \, I_{O,z}$

A) $N = 1/8$

B) $N = 1/4$

c) $N = 1/2$

D) $N = 1$

mass $m \to \frac{1}{2} m$

length $l \to \frac{1}{2} l$

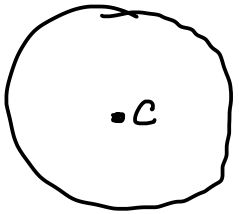
$$I_{O,z} = \frac{1}{3} m l^2$$

$$I'_{O,z} = \frac{1}{3} \left(\frac{1}{2} m\right) \left(\frac{1}{2} l\right)^2$$

$$= \frac{1}{8} \left(\frac{1}{3} m l^2\right)$$

Parallel Axis Theorem: to obtain moment of inertia about $I_{o,z}$ about point O, in terms of $I_{c,z}$ Moment of Inertia about COM

ex) disk

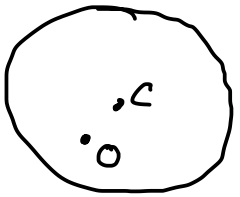


$I_{c,z}$

A) $<$

B) $>$

C) $=$



$I_{o,z}$

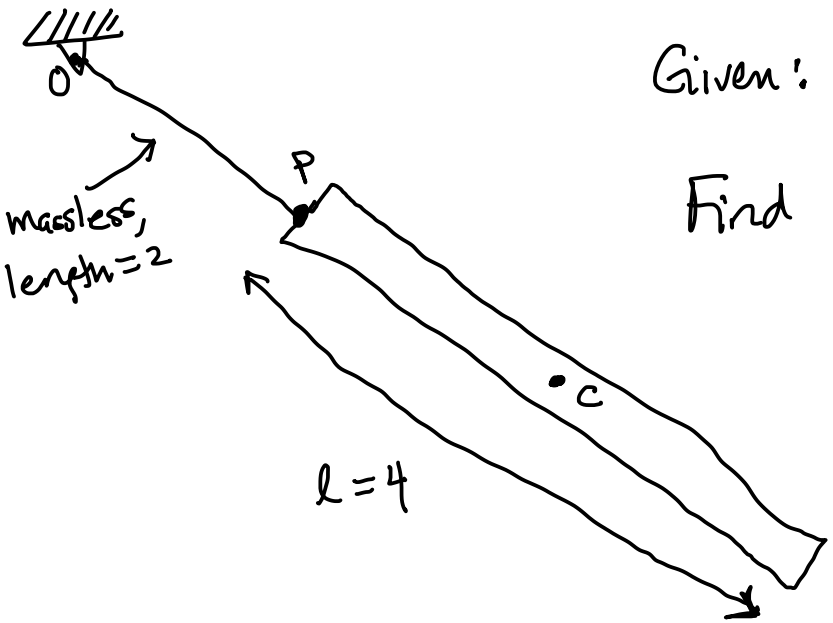
parallel axis theorem:

$$I_{o,z} = I_{c,z} + mr_{co}^2$$

↑ MUST BE C.O.M.

total mass of body

Chocolate Question:



Given: $I_{P,z} = 16 \text{ kg m}^2$

Find $I_{O,z}$ =

A) 16
B) 22
C) 28
D) 38
E) 52

STEP 1) Find Mass

$$I_{P,z} = \frac{1}{3} m l^2$$

$$16 = \frac{1}{3} m (4)^2$$

$$m = 3 \text{ kg}$$

$$I_{O,z} = I_{C,z} + m r_{CO}^2$$

$$I_{P,z} = I_{C,z} + m r_{CP}^2$$