Announcements: Final Exam Plan.

Wed, May 13, 1:30-4:30

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25 A. Paper based final — like Midterm 1 — MC

75 B. CTF — with retries for partial credit.

Today: - Finish rod-pendulum example.

- Car dynamics

- Moments of inertia

Recap: Rigid Body Kinetics

$$\boxed{\begin{aligned}\sum \vec{F} &= m\vec{a}_c \\ \sum M_{cz} &= I_{cz}\,\alpha_z\end{aligned}}$$

OR

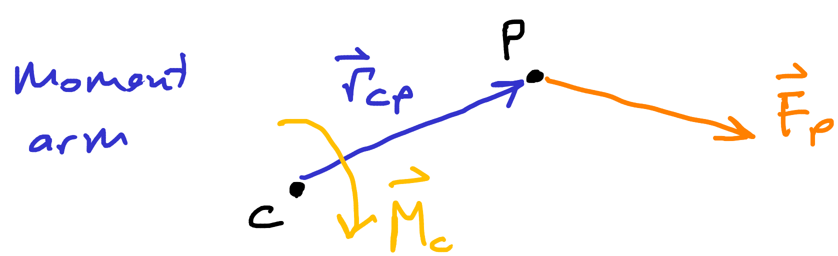
$$\boxed{\sum M_{oz} = I_{oz}\,\alpha_z}$$

O = fixed point

I_{cz} = moment of inertia about z-axis through C
(resistance to rotation)

$$\boxed{\vec{M}_c = \vec{r}_{cp} \times \vec{F}_p}$$

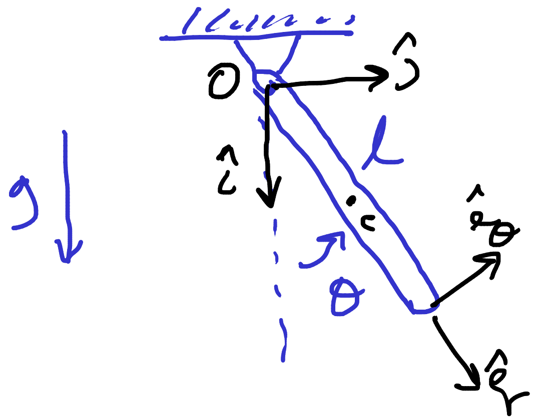
moment about C
due to force at P



Recap: Kinetics method

1. FBD
2. Kinematics: find $\vec{a}/\vec{\alpha}$ and additional constraints
3. Newton: $F = ma$ (also $M = I\alpha$)
4. Algebra: solve for F/M or a/α

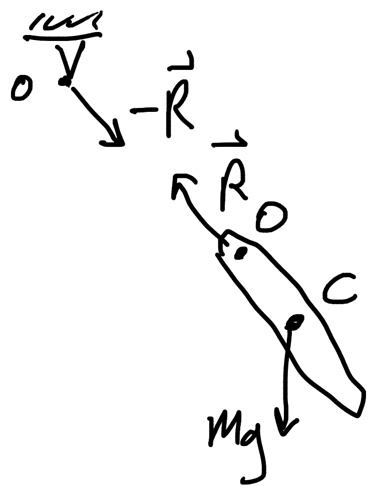
Ex Rigid-rod pendulum



Rigid rod, length ℓ, mass m, gravity g

Find: ang. acc. as function of θ, $\dot{\theta}$

① FBD



② Kinematics

$$\alpha_z = \ddot{\theta}$$

$$\vec{a}_c = -\frac{\ell}{2}\dot{\theta}^2 \hat{e}_r + \frac{\ell}{2}\ddot{\theta}\hat{e}_\theta$$

③ Newton about C

$\sum \vec{F} = m a_c$

$$\vec{R} + mg\hat{c} = -m\frac{l}{2}\dot{\theta}^2\hat{e}_r + m\frac{l}{2}\ddot{\theta}\hat{e}_\theta \qquad (A)$$

$$R_r\hat{e}_r + R_\theta\hat{e}_\theta$$

$\sum M_{cz} = I_{cz}\alpha$

$$I_{cz} = \frac{1}{12}ml^2$$

$$\vec{M}_c = \vec{r}_{co} \times \vec{R}$$

$$= \left(-\frac{l}{2}\hat{e}_r\right) \times \left(R_r\hat{e}_r + R_\theta\hat{e}_\theta\right)$$

$$= \frac{l}{2}R_\theta\hat{k}$$

$$-\frac{l}{2}R_\theta = \frac{1}{12}ml^2\ddot{\theta} \qquad (B)$$

④ Algebra

$(A)_\theta \Rightarrow \quad R_\theta - mg\sin\theta = m\frac{l}{2}\ddot{\theta} \qquad (C)$

$(B) \Rightarrow \quad -\frac{l}{2}R_\theta = \frac{1}{12}ml^2\ddot{\theta} \qquad (D)$

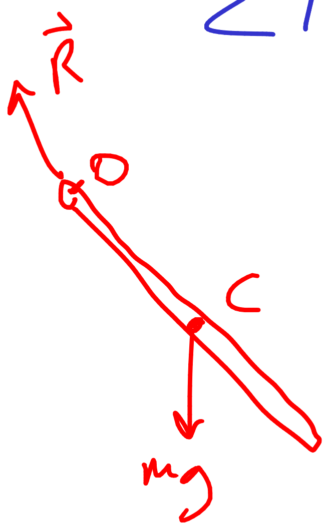
$l(C) + 2(D) \Rightarrow \quad -mgl\sin\theta = \frac{2}{3}ml^2\ddot{\theta}$

$$\boxed{\ddot{\theta} = -\frac{3}{2}\frac{g}{l}\sin\theta} \qquad (E)$$

Alternative solution using fixed-point O

③ Newton

$$\sum M_{Oz} = I_{Oz}\,\alpha \qquad I_{Oz} = \frac{1}{3}ml^2$$



$$\vec{M}_O = \vec{r}_{OC} \times \vec{F}_g$$
$$= \left(\frac{l}{2}\hat{e}_r\right) \times \left(mg\,\hat{i}\right)$$
$$= \left(\frac{l}{2}\hat{e}_r\right) \times \left(mg\cos\theta\,\hat{e}_r - mg\sin\theta\,\hat{e}_\theta\right)$$
$$= -mg\frac{l}{2}\sin\theta\,\hat{k}$$

$$-mg\frac{l}{2}\sin\theta = \frac{1}{3}ml^2\ddot{\theta}$$

④ Algebra

$$\boxed{\ddot{\theta} = -\frac{3}{2}\frac{g}{l}\sin\theta}$$

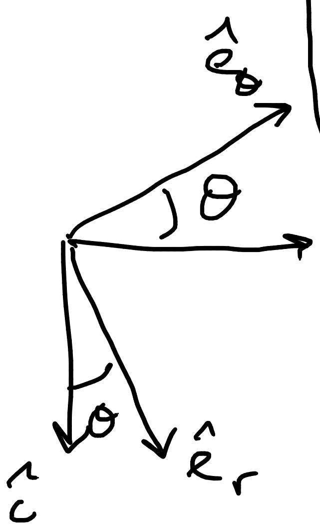
Find $\vec{R}$

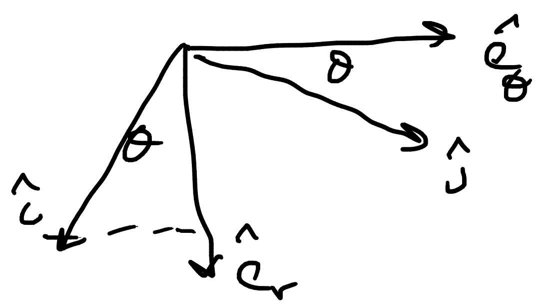
$$\vec{R} + mg\hat{i} = -m\frac{l}{2}\dot{\theta}^2\hat{e}_r + m\frac{l}{2}\ddot{\theta}\hat{e}_\theta$$

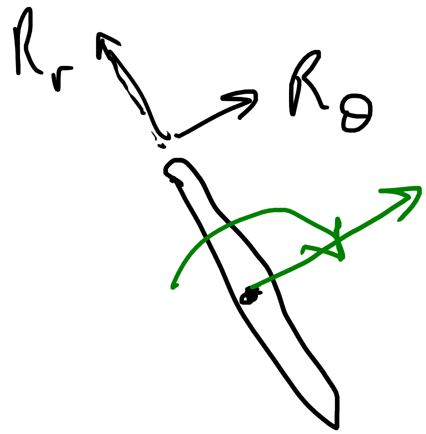
$$R_r\hat{e}_r + R_\theta\hat{e}_\theta = \vec{R} = -mg(\cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta) - \frac{ml}{2}\dot{\theta}^2\hat{e}_r + \frac{ml}{2}\left(-\frac{3}{2}\frac{g}{l}\sin\theta\right)\hat{e}_\theta$$

$$R_r = -mg\cos\theta - \frac{ml}{2}\dot{\theta}^2$$

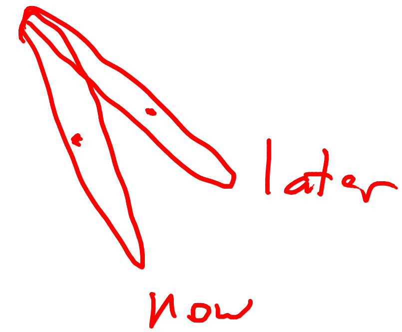
$$R_\theta = mg\sin\theta - \frac{3}{4}mg\sin\theta = \frac{1}{4}mg\sin\theta$$







$R_\theta > 0$

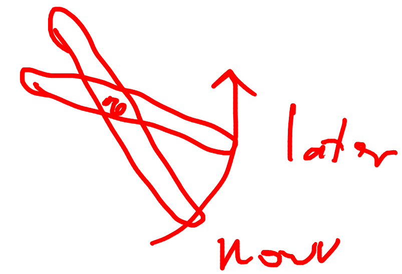


swinging up:

C moving up

A. rot. CW

B. rot CCW



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rotation rate

A. speeding up

B. slowing down

Moment is CW

α is CW

ω is CCW

ω is dec. in mag.

$\omega_z > 0$ $\quad \dot{\omega}_z = \alpha_z < 0$