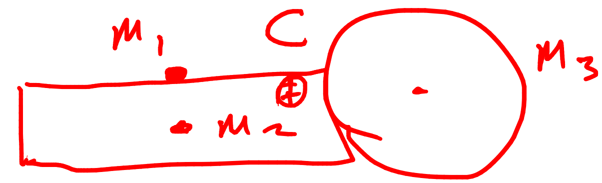
Announcements: — not discussing Midterm 2 Retake or Final until Wednesday.

— Prof Ertekin on travel

Today: — Rigid body kinematics



Recap: Center of Mass

$\oplus$ = COM

$$m = \sum_i m_i \qquad \vec{r}_c = \frac{1}{m} \sum_i m_i r_i$$

$$m = \iiint_{\mathcal{B}} \rho \, dV \qquad \vec{r}_c = \frac{1}{m} \iiint_{\mathcal{B}} \rho \vec{r} \, dV \qquad [\rho] = kg/m^3$$

$$m = \iint_{\mathcal{B}} \rho \, dA \qquad \vec{r}_c = \frac{1}{m} \iint_{\mathcal{B}} \rho \vec{r} \, dA \qquad [\rho] = kg/m^2$$

$$m = \int_{\mathcal{B}} \rho \, d\ell \qquad \vec{r}_c = \frac{1}{m} \int_{\mathcal{B}} \rho \vec{r} \, d\ell \qquad [\rho] = kg/m$$

Recap: Rigid Body Kinetics

$\vec{F} = m\vec{a}$ $\longrightarrow$

C = CoM.

$$\boxed{\begin{aligned} \sum \vec{F} &= m\vec{a}_c \\ \sum M_{cz} &= I_{cz}\,\alpha_z \end{aligned}}$$

OR

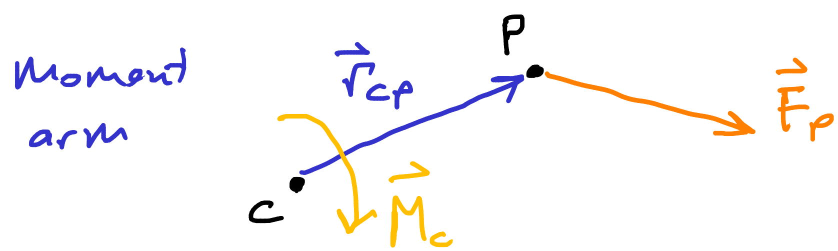
$$\boxed{\sum M_{oz} = I_{oz}\,\alpha_z}$$

O = fixed point

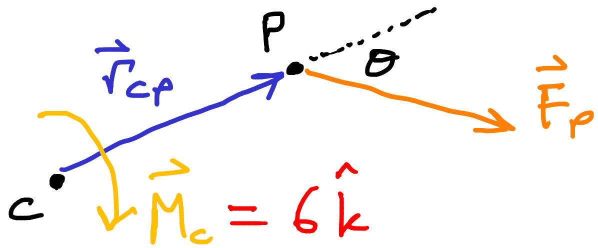
I_{cz} = moment of inertia about z-axis through C (resistance to rotation)

$$\boxed{\vec{M}_c = \vec{r}_{cp} \times \vec{F}_p}$$

moment about C due to force at P



$$\vec{M}_c = \vec{r}_{cp} \times \vec{F}_p$$



$M_c =$

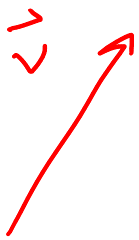
A. $r_{cp} F_p \cos\theta$

B. $r_{cp} F_p \sin\theta$

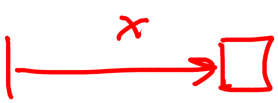
C. $-r_{cp} F_p \cos\theta$

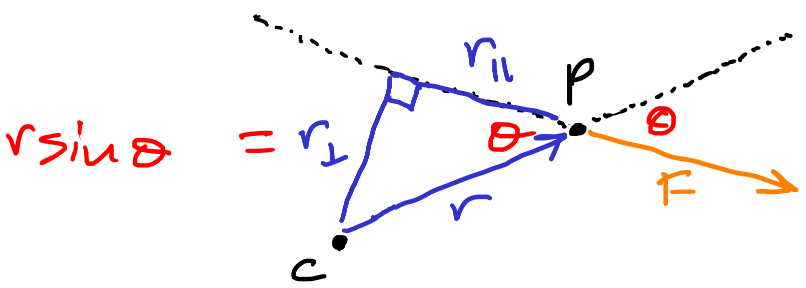
D. $-r_{cp} F_p \sin\theta$

$$\vec{M}_c = M_{cz}\hat{k}$$



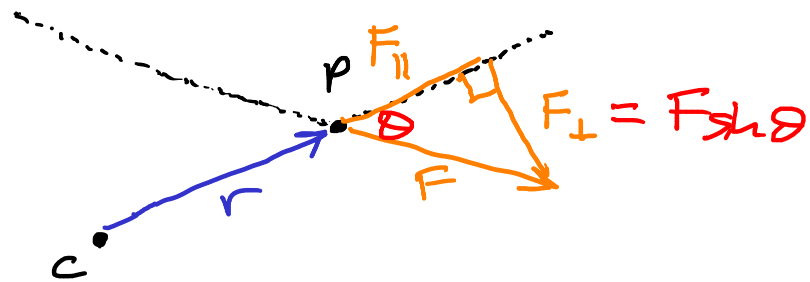
$$M_{cz} = -r F \sin\theta$$





$\parallel$ "parallel"

$\perp$ "orthogonal"



$M_c =$

A. $rF_\perp$

B. $rF_\parallel$

C. $r_\perp F$

D. $r_\parallel F$

$$M_c = r F \sin\theta = r_\perp F$$

$$\underbrace{r \sin\theta}_{r_\perp}$$

moment is caused by:

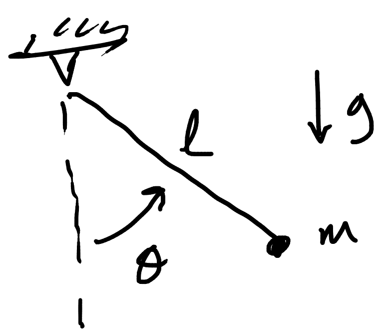
— orth. force on moment arm

or — force on orth moment arm

Recap: Kinetics method

1. FBD
2. Kinematics: find $\vec{a}/\vec{\alpha}$ and additional constraints
3. Newton: $F = ma$ (also $M = I\alpha$)
4. Algebra: solve for F/M or a/α

Recap: Point-mass pendulum

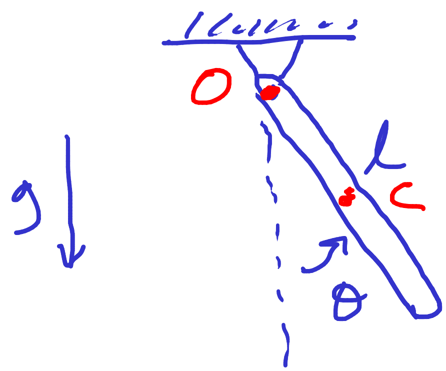


Find: ang. acc. and string tension.

Ans: $\ddot{\theta} = -\frac{g}{\ell}\sin\theta$

$T = mg\cos\theta + m\ell\dot{\theta}^2$

Ex Rigid-rod pendulum



Rigid rod, length ℓ, mass m, gravity g

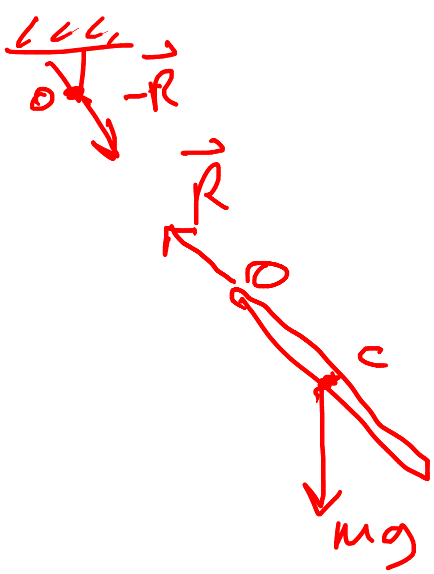
Find: ang. acc. as function of θ, $\dot{\theta}$

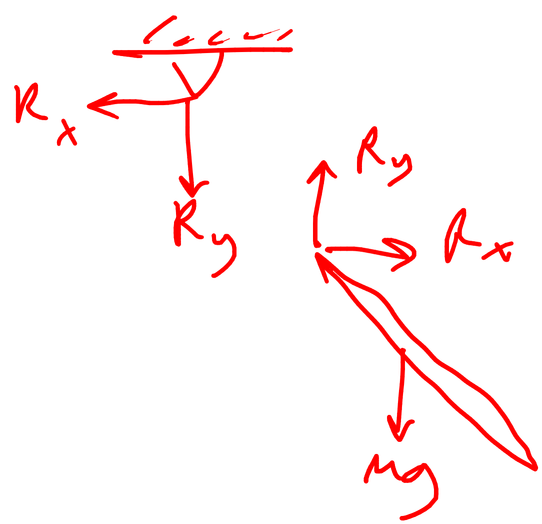
① FBD

How many bodies? A. 1 B. 2 C. 3 D. 4 E. 5

How many points? A. 1 B. 2 C. 3 D. 4 E. 5

How many force arrows? A. 1 B. 2 C. 3 D. 4 E. 5

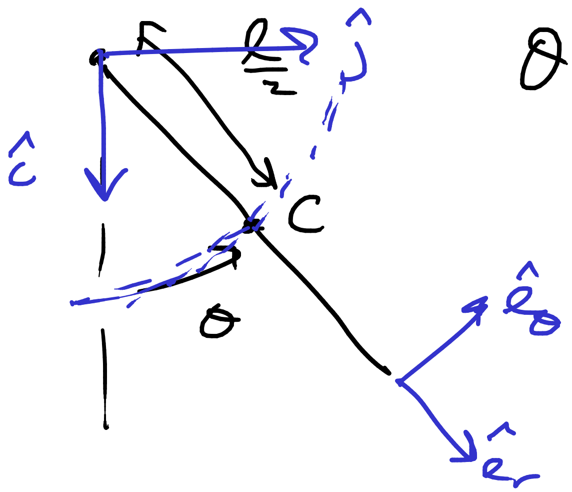




only physical forces and only once.

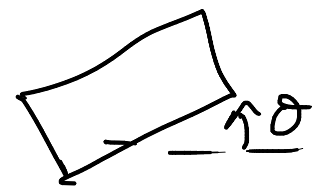
② kinematics: $\vec{a} / \vec{\alpha}$

$\alpha_z = \ddot{\theta}$



θ, ω, α

$\theta = \dot{\theta}$, $\omega = \ddot{\theta}$



$$\vec{a}_c = \frac{l}{2}\ddot{\theta}\hat{e}_\theta - \frac{l}{2}\dot{\theta}^2\hat{e}_r$$

③ Newton

$\sum \vec{F} = m a_c$ $\qquad \vec{R} + mg\hat{\imath} = -m\frac{l}{2}\dot{\theta}^2\hat{e}_r + m\frac{l}{2}\ddot{\theta}\,\hat{e}_\theta$ $\qquad$ (A)

$\vec{R} = R_r\hat{e}_r + R_\theta\hat{e}_\theta$

$\sum M_{cz} = I_{cz}\,\alpha$ $\qquad I_{cz} = \frac{1}{12}ml^2$

$\vec{M}_c = \vec{r}_{co} \times \vec{R}$

$= \left(-\frac{l}{2}\hat{e}_r\right) \times \left(R_r\hat{e}_r + R_\theta\hat{e}_\theta\right)$

$= \frac{l}{2}R_\theta\,\hat{k}$

$-\frac{l}{2}R_\theta = \frac{1}{12}ml^2\ddot{\theta}$ $\qquad$ (B)

④ Algebra

$(A)_\theta \Rightarrow$ $\quad R_\theta - mg\sin\theta = m\frac{l}{2}\ddot{\theta}$ $\qquad$ (C)

$(B) \Rightarrow$ $\quad -\frac{l}{2}R_\theta = \frac{1}{12}ml^2\ddot{\theta}$ $\qquad$ (D)

$l(C) + 2(D) \Rightarrow$ $\quad -mgl\sin\theta = \frac{2}{3}ml^2\ddot{\theta}$

$$\boxed{\ddot{\theta} = -\frac{3}{2}\frac{g}{l}\sin\theta}$$