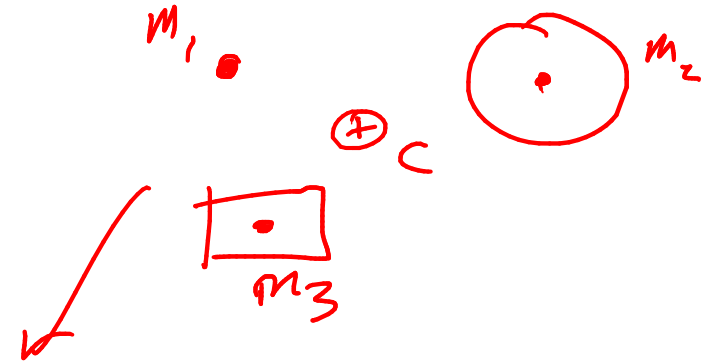


Announcements :

- not discussing Midterm 2 Retake or Final until Wednesday.
- Prof Ertekin on travel

Today: - Rigid body kinematics

Recap : Center of Mass



$$m = \sum_i m_i$$

$$\vec{r}_c = \frac{1}{m} \sum_i m_i \vec{r}_i$$

$$m = \iiint_{\mathcal{B}} \rho dV$$

$$\vec{r}_c = \frac{1}{m} \iiint_{\mathcal{B}} \rho \vec{r} dV$$

$$[\rho] = \text{kg/m}^3$$

$$m = \iint_{\mathcal{B}} \rho dA$$

$$\vec{r}_c = \frac{1}{m} \iint_{\mathcal{B}} \rho \vec{r} dA$$

$$[\rho] = \text{kg/m}^2 \quad \leftarrow$$

$$m = \int_{\mathcal{B}} \rho dl$$

$$\vec{r}_c = \frac{1}{m} \int_{\mathcal{B}} \rho \vec{r} dl$$

$$[\rho] = \text{kg/m}$$

Recap: Rigid Body Kinetics

$$\sum \vec{F} = m\vec{a}_c$$

$$\sum M_{cz} = I_{cz} \alpha_z$$

OR

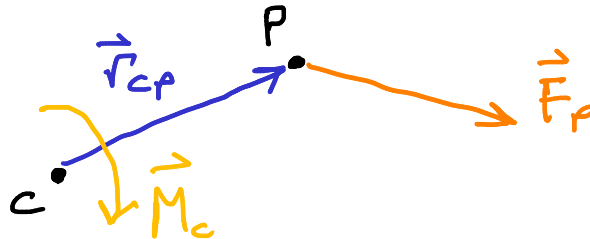
$$\sum M_{Oz} = I_{Oz} \alpha_z$$

$O = \text{fixed point}$

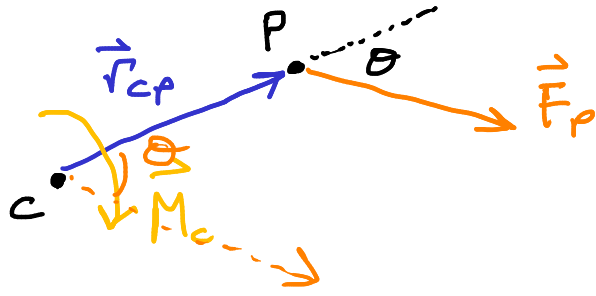
I_{cz} = moment of inertia about z -axis through C
(resistance to rotation)

$$\vec{M}_c = \vec{r}_{cp} \times \vec{F}_p$$

moment about C
due to force at P



$$\vec{M}_c = \vec{r}_{cp} \times \vec{F}_p$$



$$\vec{M}_c = M_{cz} \hat{k}$$

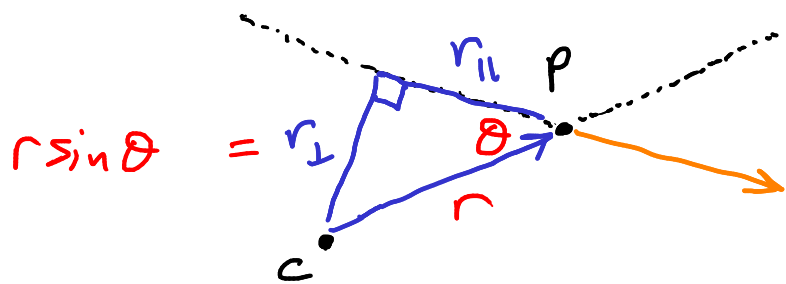
$$M_c =$$

- A. $r_{cp} F_p \cos \theta$
- B. $r_{cp} F_p \sin \theta$**
- C. $-r_{cp} F_p \cos \theta$
- D. $-r_{cp} F_p \sin \theta$

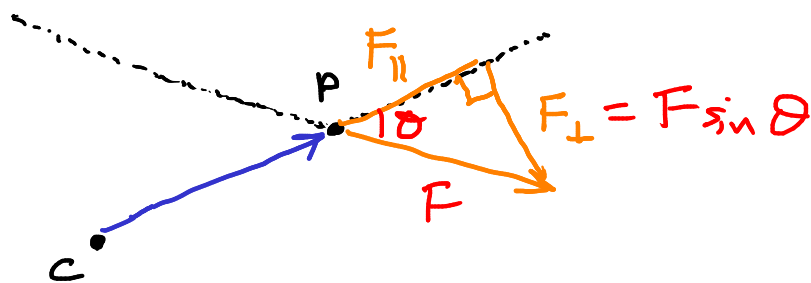
$$M_{cz} =$$

~~$M_{cz} =$~~

$M_{cz} = I_{cz} \alpha_z$



$\parallel$ "parallel"
 $\perp$ "orthogonal"



$$M_c = A. r_{\perp} F_{\perp}$$

$$B. r_{\parallel} F_{\parallel}$$

$$C. r_{\perp} F_{\parallel}$$

$$D. r_{\parallel} F_{\perp}$$

E. NOT A.

$$M_c = r F \sin \theta$$

$$= r F_{\perp}$$

$$\underline{\underline{or}} \quad M_c = r_{\perp} F$$

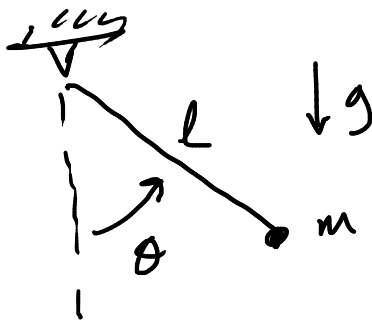
— Force orth. to moment arm that produces moment.

— Force applied to orth. moment arm produces moment.

Recap: Kinetics method

- ① FBD
- ② Kinematics: find $\vec{a}/\vec{\alpha}$ and additional constraints
- ③ Newton: $F = ma$ (also $M = I\alpha$)
- ④ Algebra: solve for F/M or a/α

Recap: Point-mass pendulum

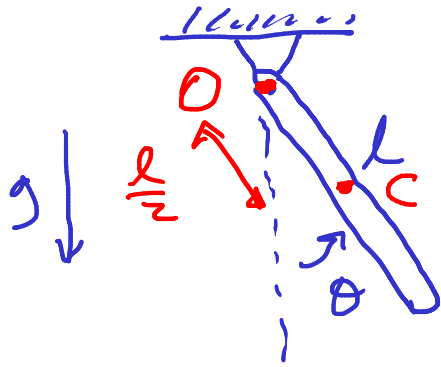


Find: ang. acc. and string tension.

Ans: $\ddot{\theta} = -\frac{g}{l} \sin \theta$

$$T = mg \cos \theta + ml \dot{\theta}^2$$

Ex Rigid-rod pendulum



Rigid rod, length l , mass m , gravity g

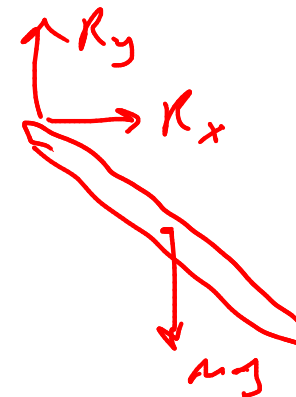
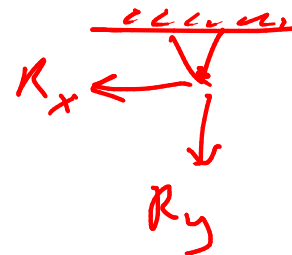
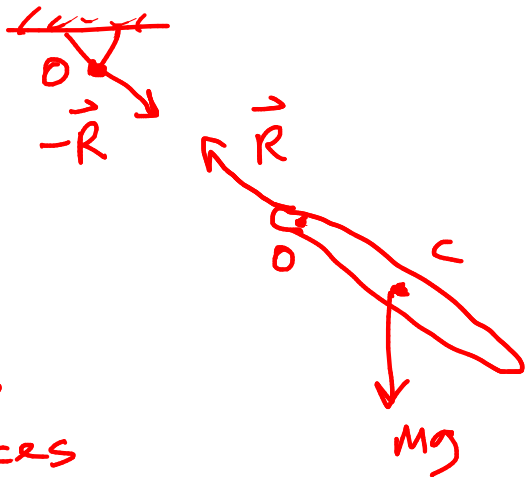
Find: ang. acc. as function of $\theta, \dot{\theta}$

① FBD

How many bodies? A.1 B.2 C.3 D.4 E.5

How many points? A.1 B.2 C.3 D.4 E.5

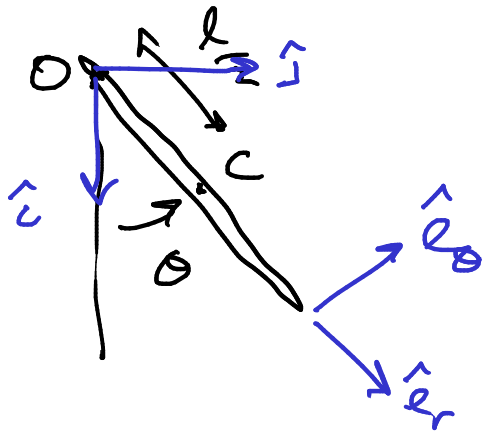
How many ^{force} arrows? A.1 B.2 C. 3 D.4 E. 5



physical
moments
and forces

② $\vec{a}/\vec{z}$ about C

$$\vec{a}_c = -\frac{l}{2} \ddot{\theta} \hat{e}_r + \frac{l}{2} \ddot{\theta} \hat{e}_\theta$$



3

Newton

$$\sum \vec{F} = m\vec{a}_c$$

$$\vec{R} + mg\hat{c} = -m\frac{l}{2}\ddot{\theta}^2\hat{e}_r + m\frac{l}{2}\ddot{\theta}\hat{e}_\theta \quad (A)$$

$$\nwarrow R_r\hat{e}_r + R_\theta\hat{e}_\theta$$

$$\sum M_{cz} = I_{cz} \alpha$$

$$I_{cz} = \frac{1}{12}ml^2$$

$$\vec{M}_c = \vec{r}_{co} \times \vec{R}$$

$$= \left(-\frac{l}{2}\hat{e}_r\right) \times (R_r\hat{e}_r + R_\theta\hat{e}_\theta)$$

$$= \frac{l}{2}R_\theta\hat{k}$$

$$-\frac{l}{2}R_\theta = \frac{1}{12}ml^2\ddot{\theta}$$

(B)

4 Algebra

$$(A)_\theta \Rightarrow R_\theta - mg\sin\theta = m\frac{l}{2}\ddot{\theta} \quad (C)$$

$$(B) \Rightarrow -\frac{l}{2}R_\theta = \frac{1}{12}ml^2\ddot{\theta} \quad (D)$$

$$l(C) + 2(D) \Rightarrow -mg\sin\theta = \frac{2}{3}ml\ddot{\theta}$$

$$\boxed{\ddot{\theta} = -\frac{3}{2}\frac{g}{l}\sin\theta}$$

$$\dot{\theta} = -\frac{g}{r}\sin\theta$$

$$= -\frac{3}{2}\frac{g}{l}\sin\theta$$