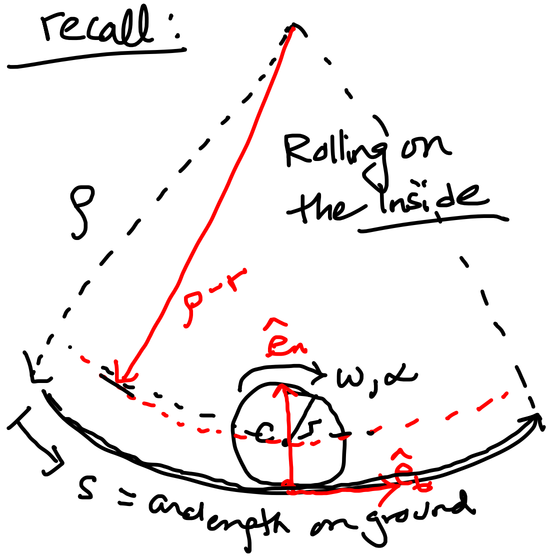
TAM 212 : Class 29

recall:



s = distance covered

$\dot s$ = ground speed of rolling wheel

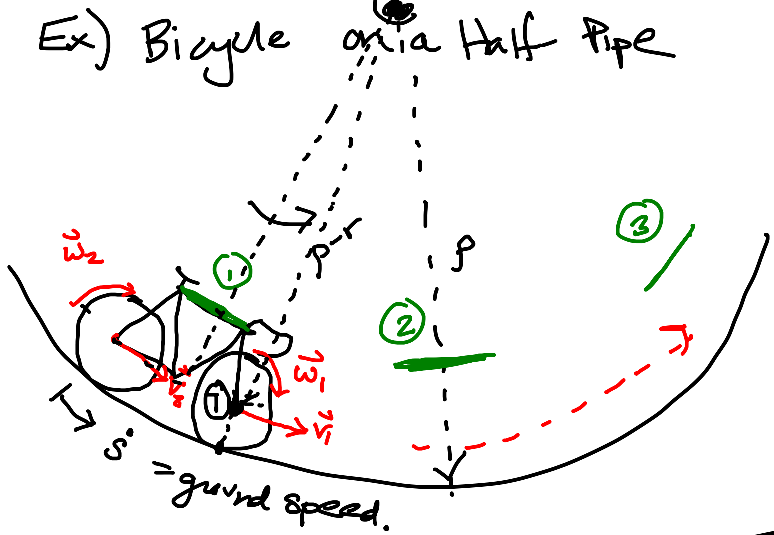
$\ddot s$ = ground acceleration

$$\vec v_C = \left(\frac{\rho - r}{\rho}\right)\dot s\,\hat e_t = \underline{r\omega\,\hat e_t}$$

$$\vec a_C = r\alpha\,\hat e_t + \left(\frac{r}{\rho - r}\right) r\omega^2\,\hat e_n$$

$$\dot s = \left(\frac{\rho}{\rho - r}\right) r\omega \qquad \ddot s = \left(\frac{\rho}{\rho - r}\right) r\alpha$$

Ex) Bicycle on a Half Pipe

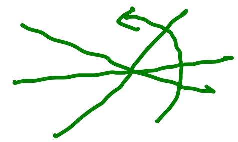


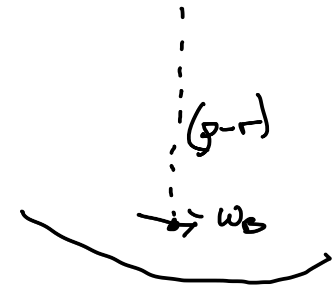
How many distinct R.B.'s? 3

Wheel 1, wheel 2, frame

→ $\vec{\omega}_1, \vec{\omega}_2$ are CW, call it $\vec{\omega} = -\omega \hat{k}$

$\vec{\omega}_B$ is CCW; $\vec{\omega}_B = \omega_B \hat{k}$



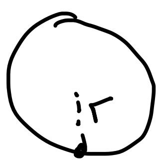


Find ω_B in terms of ω, r, ρ

Find ω_B in terms of $\dot{s}$

$$\underbrace{\vec{v}_1 = r\omega_1 \hat{e}_t}_{\text{using I.C. of wheel is contact point}}$$

$$\underbrace{\vec{v}_1 = (\rho - r)\omega_B \hat{e}_t}_{\text{using I.C. of frame at center of half pipe}}$$



SET EQUAL: $r\omega_1 \cancel{\hat{e}_t} = (\rho - r)\omega_B \cancel{\hat{e}_t}$

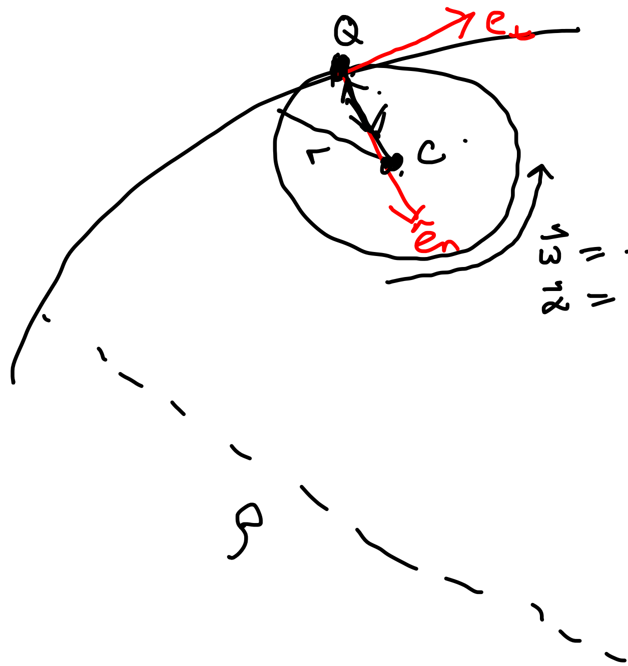
$$\boxed{\omega_B = \left(\frac{r}{\rho - r}\right)\omega_1}$$

then $\dot{s} = \left(\frac{\rho}{\rho - r}\right) r\omega_1$ for the wheel so

$\omega_1 = \left(\frac{\rho - r}{\rho}\right)\frac{1}{r}\dot{s}$ $\Longrightarrow$ $\omega_B = \left(\frac{r}{\rho - r}\right)\left(\frac{\rho - r}{\rho}\right)\frac{1}{r}\dot{s}$

$$\boxed{\omega_B = \frac{1}{\rho}\dot{s}}$$

Example)



$\vec{\omega} = -\omega\hat{k}$

$\vec{\alpha} = -\alpha\hat{k}$

Given:

$r = 2\ m$

$\omega = 3\ rad/sec$ CCW

$\alpha = 2\ rad/sec^2$ CW

$|\vec{a}_Q| = 27\ m/sec^2$

Note: $\hat{e}_b \times \hat{e}_n = -\hat{e}_t$

Find ρ

$$\vec{a}_Q = \vec{a}_c + \vec{\alpha} \times \vec{r}_{CQ} - \omega^2 \vec{r}_{CQ}$$

$$= \left[r\alpha\hat{e}_t + \left(\frac{r}{\rho - r}\right) r\omega^2 \hat{e}_n\right] + \left[(-\alpha\hat{e}_b) \times (-r\hat{e}_n) - \omega^2(-r\hat{e}_n)\right]$$

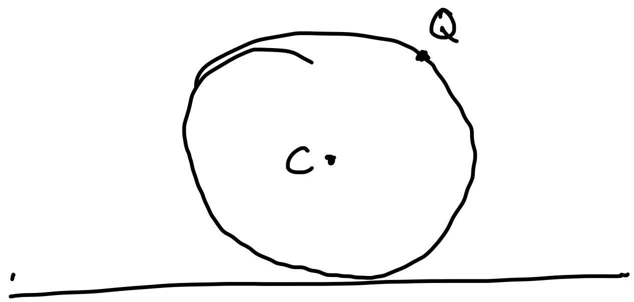
$$= \left[\cancel{r\alpha}\hat{e}_t + \left(\frac{r}{\rho - r}\right) r\omega^2 \hat{e}_n\right] + \left[\cancel{(-\alpha)(-r)}(-\hat{e}_t) + r\omega^2\hat{e}_n\right]$$

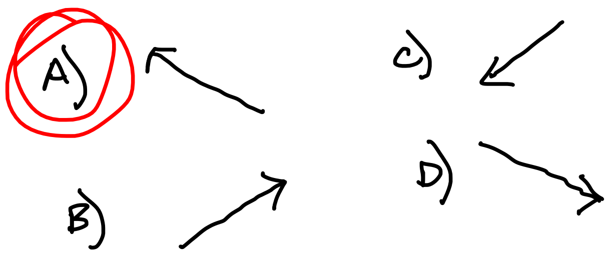
$$\vec{a}_Q = \underbrace{\left[\frac{r^2\omega^2}{\rho - r} + r\omega^2\right]}\hat{e}_n$$

$$|\vec{a}_Q| = 27 = \frac{r^2\omega^2}{\rho - r} + r\omega^2$$

solve for $\rho \Rightarrow \rho = 6\ m$

Example)





Given: R.B. rolls w/o slip.

$\vec{\alpha} = \alpha \hat{k}$ and $\vec{\omega} = \omega \hat{k}$

$\boxed{\alpha = 2\omega^2}$ $\qquad \omega^2 = \frac{\alpha}{2}$

Find direction of $\vec{a}_Q$

$\vec{v}_c = -r\omega \hat{i}$

$\vec{a}_c = -r\alpha \hat{i}$

$$\vec{a}_Q = \vec{a}_c + \vec{\alpha} \times \vec{r}_{CQ} - \omega^2 \vec{r}_{CQ}$$

$$= (-r\alpha \hat{i}) + (\alpha \hat{k} \times r \hat{r}_{CQ}) - \omega^2 (r \hat{r}_{CQ})$$

$$= (-r\alpha \hat{i}) + (\alpha r)(\hat{k} \times \hat{r}_{CQ}) - \omega^2 r \, \hat{r}_{CQ}$$

$$= \underbrace{(-r\alpha \hat{i})}_{①} + (\alpha r)(\hat{r}_{CQ}^{\perp}) - \omega^2 r \, \hat{r}_{CQ}$$

③ $-\frac{\alpha}{2} \hat{r}_{CQ}$