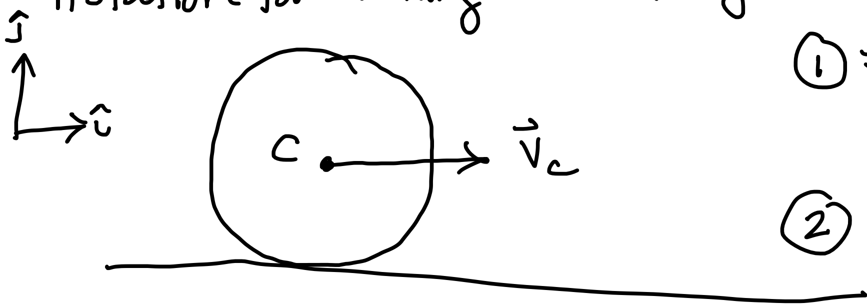
Class 28 : Rolling on Curved Surfaces

notation for rolling w/o slipping



① $\Rightarrow$ Wheel must be rotating clockwise so angular velocity vector $\vec{\omega}$ has neg. sign

② $\Rightarrow$ wheel moves to the right, $\vec{v}_c$ has positive sign.

SUGGESTED CONVENTION:

Ⓐ $\left[\begin{array}{l} \vec{\omega} = -\omega \hat{k} \\ \vec{v}_c = r\omega \hat{i} \end{array}\right\}$ if $\omega > 0$ then

① $\vec{\omega} = -\omega \hat{k}$ is negative. ✓

② $\vec{v}_c = r\omega \hat{i}$ is positive ✓

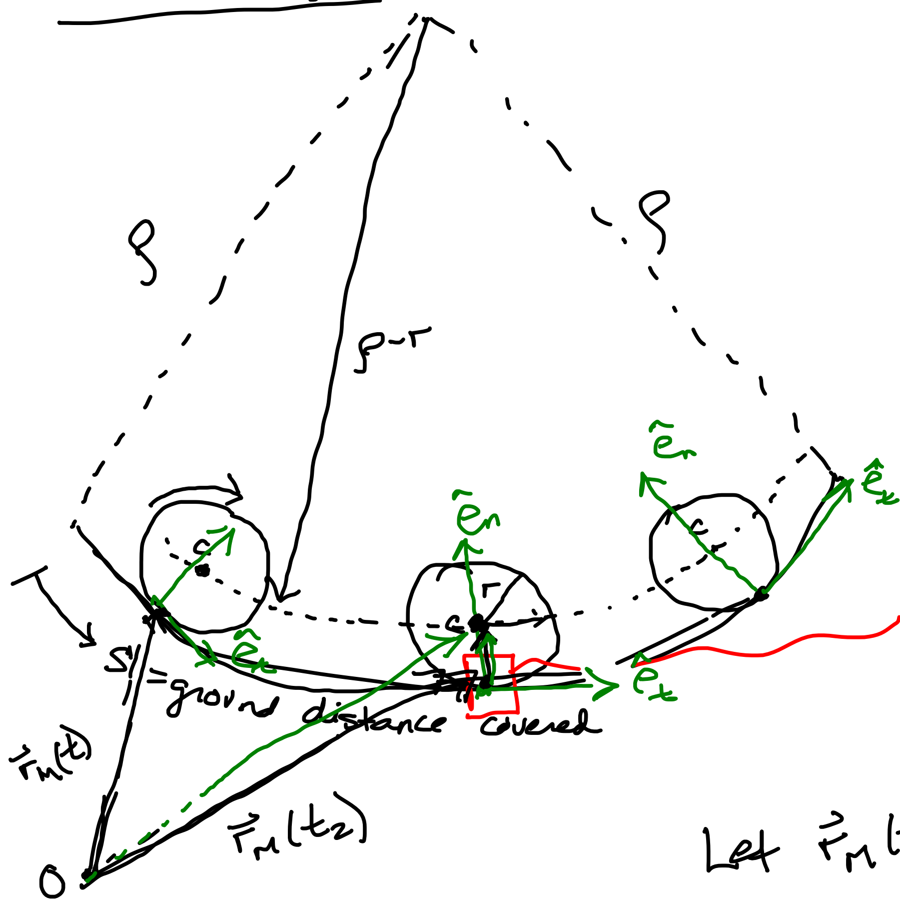
YOU CAN ALSO CHOOSE:

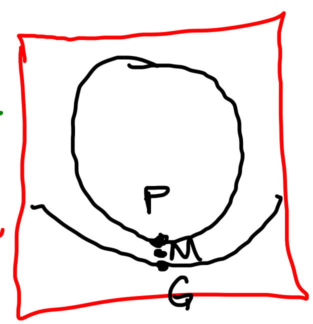
Ⓑ $\left[\begin{array}{l} \vec{\omega} = \omega \hat{k} \\ \vec{v}_c = -r\omega \hat{i} \end{array}\right\}$ if $\omega < 0$ then

① $\vec{\omega} = \omega \hat{k}$ is negative ✓

② $\vec{v}_c = -r\omega \hat{i}$ is positive ✓

Curved Surface





ρ = radius of curv. of ground

r = radius of wheel

P = point on wheel in contact w/ ground

G = point on ground in contact w/ wheel

M = point denoting the I.C. of rotation as funct. of time

Let $\vec r_M(t)$ denote the position vector of the I.C.

$$\vec V_M = \frac{d\vec r_M}{dt} \neq 0 \quad \text{BUT} \quad \vec V_P = 0, \quad \vec V_G = 0$$

We will use T/N coordinates for these problems. AND

$$\boxed{\vec V_M(t) = \dot s\,\hat e_t} \qquad \vec\omega = -\omega\,\hat e_b \qquad \vec\alpha = -\alpha\,\hat e_b$$

FIND: (1) $\boxed{\vec v_c}$, $\boxed{\vec a_c}$ in terms of r, ρ, ω, α (2) ω, α in terms $\dot s, \ddot s, r, \rho$

For $\vec{v}_c$, use two approaches & set equal

① $\vec{v}_c = \cancel{\vec{v}_p} + \vec{\omega} \times \vec{r}_{pc}$

$= (-\omega\hat{e}_b \times r\hat{e}_n)$

$= (-\omega r)\,\hat{e}_b \times \hat{e}_n$

$$\boxed{\vec{v}_c = \omega r\,\hat{e}_t}$$

② $\vec{r}_c = \vec{r}_M + r\hat{e}_n$

take $\frac{d}{dt}$ of both sides:

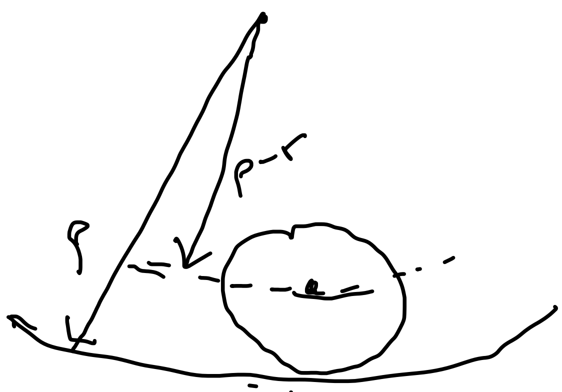
$\vec{v}_c = \vec{v}_M + \frac{d}{dt}(r\hat{e}_n)$

$= \vec{v}_M + \cancel{\left(\frac{dr}{dt}\right)}\hat{e}_n + r\frac{d\hat{e}_n}{dt}$

$= \vec{v}_m + r\left(\frac{d\hat{e}_n}{dt}\right)$

recall: curvature

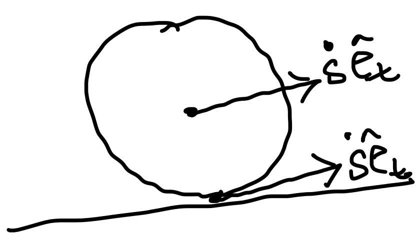
$\frac{d\hat{e}_n}{dt} = \frac{-\dot{s}}{\rho}\hat{e}_t$



$\vec{v}_c = \vec{v}_M + r\left(\frac{-\dot{s}}{\rho}\hat{e}_t\right)$

$= \dot{s}\hat{e}_t - \frac{r\dot{s}}{\rho}\hat{e}_t$

$$\boxed{\vec{v}_c = \boxed{\left(\frac{\rho - r}{\rho}\right)\dot{s}}\,\hat{e}_t}$$



Notice:

$\lim_{\rho\to\infty} \vec{v}_c = \lim_{\rho\to\infty} \left(\frac{\rho - r}{\rho}\right)\dot{s}\hat{e}_t$

$= \dot{s}\hat{e}_t \lim_{\rho\to\infty}\left(\frac{\rho - r}{\rho}\right)$

$= \dot{s}\hat{e}_t$

Setting equal:

$$r\omega \cancel{\hat{e}_t} = \left(\frac{\rho - r}{\rho}\right)\dot{s}\, \cancel{\hat{e}_t}$$

$$\rightarrow \boxed{\dot{s} = \left(\frac{\rho}{\rho - r}\right) r\omega}$$

ground speed in terms of ρ, r, ω

Now onto the accelerations:

$$\vec{a}_c = \frac{d\vec{v}_c}{dt} = \frac{d}{dt}\left(r\omega \hat{e}_t\right)$$

$$= \underbrace{\frac{d}{dt}(r\omega)}\hat{e}_t + r\omega \boxed{\frac{d}{dt}\hat{e}_t}$$

$$= r\alpha \hat{e}_t + r\omega\left(\frac{\dot{s}}{\rho}\hat{e}_n\right)$$

$$= r\alpha \hat{e}_t + \frac{r\omega}{\cancel{\rho}}\left(\frac{\cancel{\rho}}{\rho - r}\right) r\omega \hat{e}_n$$

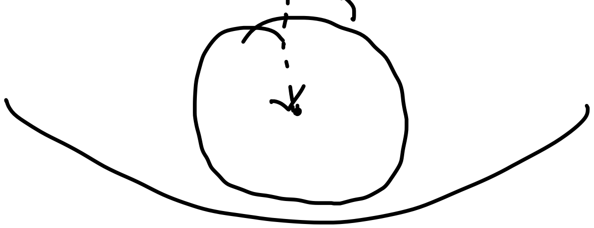
$$\boxed{\vec{a}_c = r\alpha \hat{e}_t + \left(\frac{r}{\rho - r}\right) r\omega^2 \hat{e}_n}$$

Again notice that as $\rho \rightarrow \infty$ we recover $\vec{a}_c = r\alpha \hat{e}_t$ which is the flat surface result.

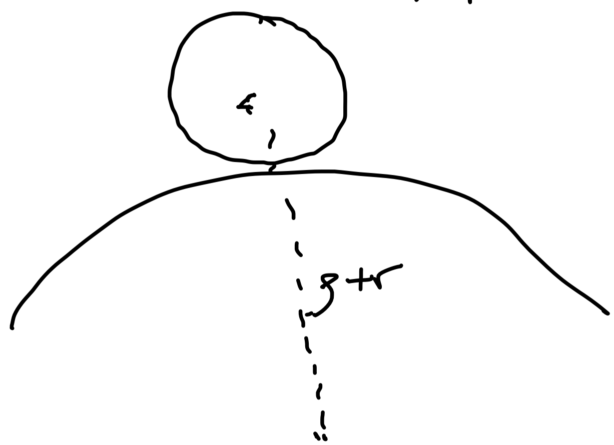
Finally for $\ddot{s}$, just differentiate!

$$\boxed{\begin{aligned} \dot{s} &= \left(\frac{\rho}{\rho - r}\right) r\omega \\ \ddot{s} &= \left(\frac{\rho}{\rho - r}\right) r\alpha \end{aligned}}$$

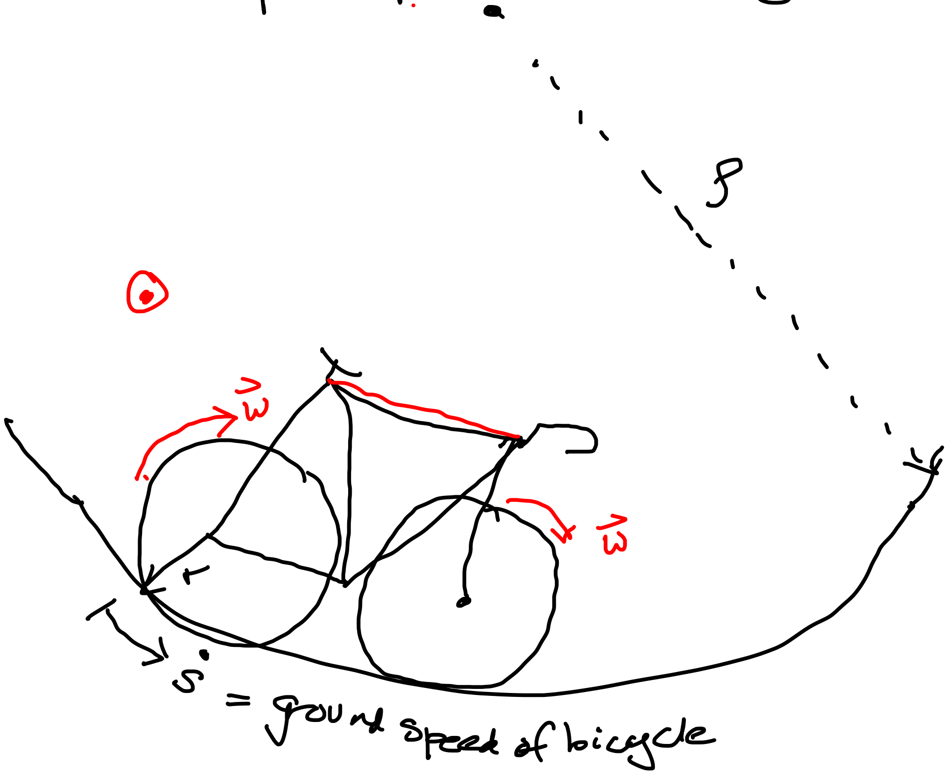
We covered rolling on inside $\Longrightarrow$



for rolling on outside: look at class notes online



Example Problem: Bicycle on a Half Pipe



How many distinct R.B.'s are on the bicycle?

A) 1

B) 2

C) 3

D) 4

2 wheels, one frame

CW

CCW