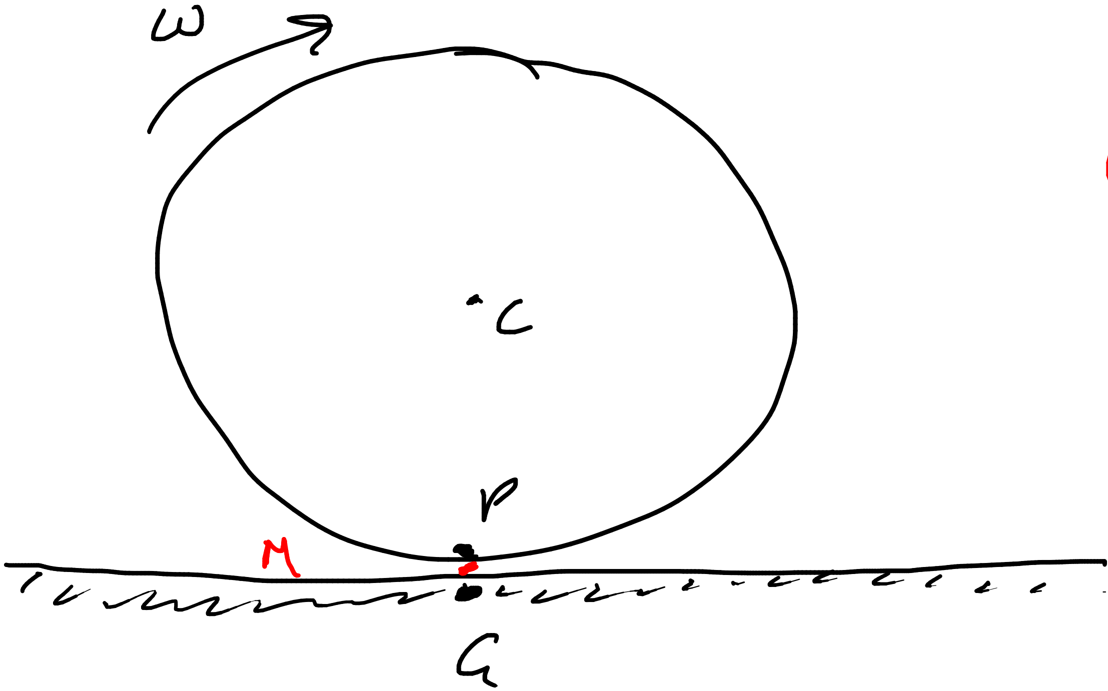
Rolling without slipping



P = point fixed to wheel

G = point fixed to ground

M = point that is always the contact location (not attached to an atom)

at the instant shown:

A. $v_P = v_G = v_M$

B. $v_P = v_G \neq v_M$

C. $v_G = v_M \neq v_P$

D. $v_P = v_M \neq v_G$

E. all different

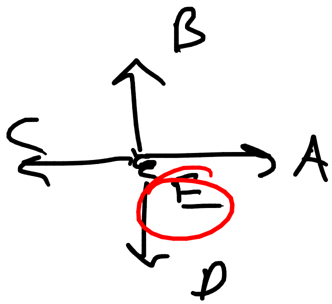
it is true:

$$\vec{r}_P = \vec{r}_G = \vec{r}_M$$

at this instant

at this instant,

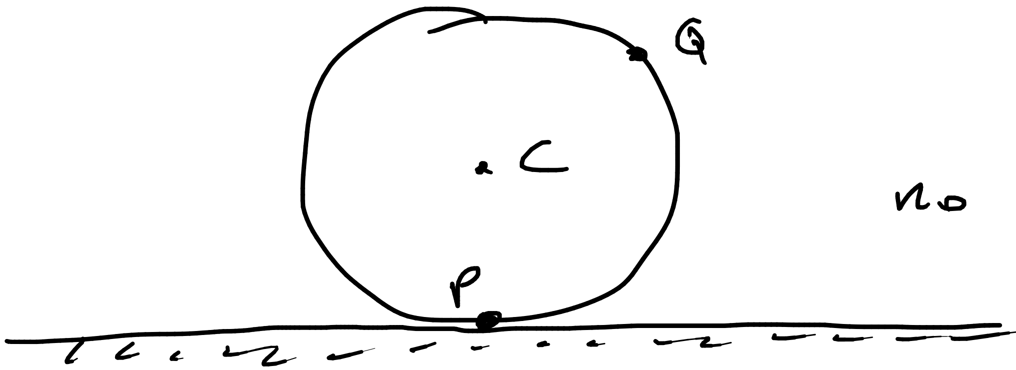
$\vec{v}_G$



$\vec{v}_P$ also zero

$\vec{v}_M = \dot{\vec{r}}_M$ A

$$\vec{\omega} = \omega_z \hat{k}$$

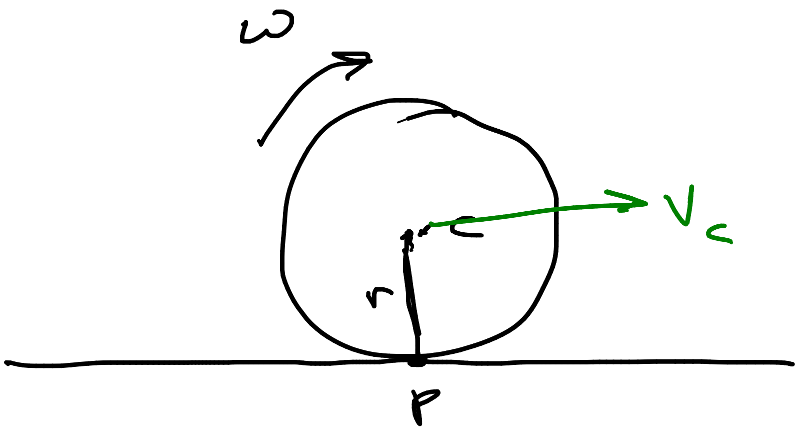


no slip

$$\vec{v}_P = 0$$

$$\vec{v}_C = \cancelto{0}{\vec{v}_P} + \vec{\omega} \times \vec{r}_{PC}$$

$$\vec{v}_Q = \vec{v}_C + \vec{\omega} \times \vec{r}_{CQ}$$



$$\boxed{v_C = \omega r}$$

$$\vec{v}_C = \omega r \hat{i}$$

$$\vec{a}_C = \dot{\omega} r \hat{i}$$

$$\boxed{a_C = \alpha r}$$

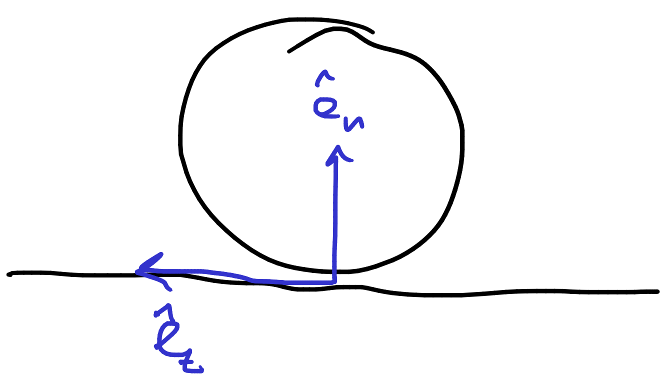
Magnitudes

or

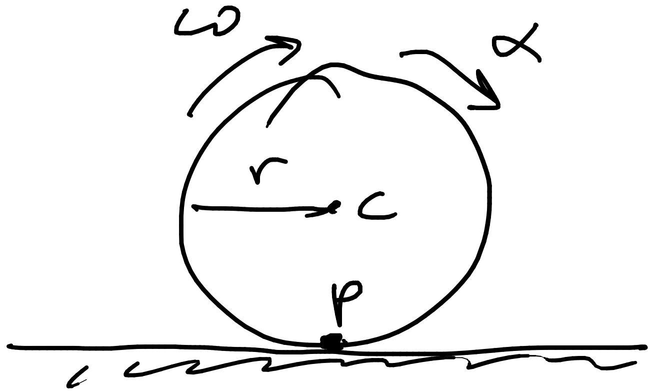
$$\boxed{\begin{array}{ccc} \vec{\omega} = -\omega\hat{k} & \Longrightarrow & \vec{V}_c = \omega r\hat{i} \\ \vec{\alpha} = -\alpha\hat{k} & \Longrightarrow & \vec{a}_c = \alpha r\hat{i} \end{array}}$$

or

$$\boxed{\begin{array}{ccc} \vec{\omega} = \omega\hat{k} & \Longrightarrow & \vec{V}_c = \omega r\hat{e}_t \\ \vec{\alpha} = \alpha\hat{k} & \Longrightarrow & \vec{a}_c = \alpha r\hat{e}_t \end{array}}$$



Ex



$\vec{v}_p = 0$

$\vec{a}_p$?

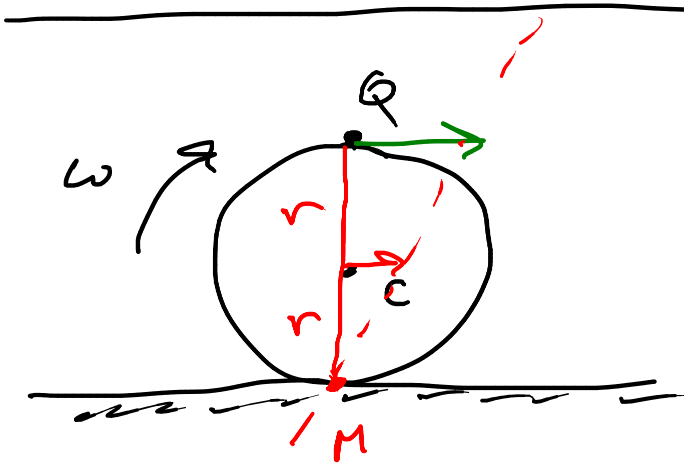
$\vec{\omega} = -\omega\hat{k}$

$\vec{\alpha} = -\alpha\hat{k}$

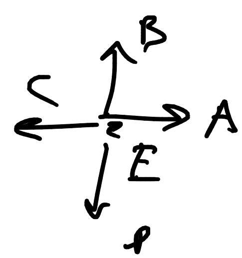
$$\vec{a}_p = \vec{a}_c + \vec{\alpha}\times\vec{r}_{cp} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{cp})$$

$$= \cancel{\alpha r\hat{i}} + \underbrace{(-\alpha\hat{k})\times(-r\hat{j})}_{\cancel{-\alpha r\hat{i}}} - \omega^2(-r\hat{j})$$

$$= \omega^2 r\hat{j}$$



$\hat{v}_Q$



V_Q

A. $< V_c$

B. $= V_c$

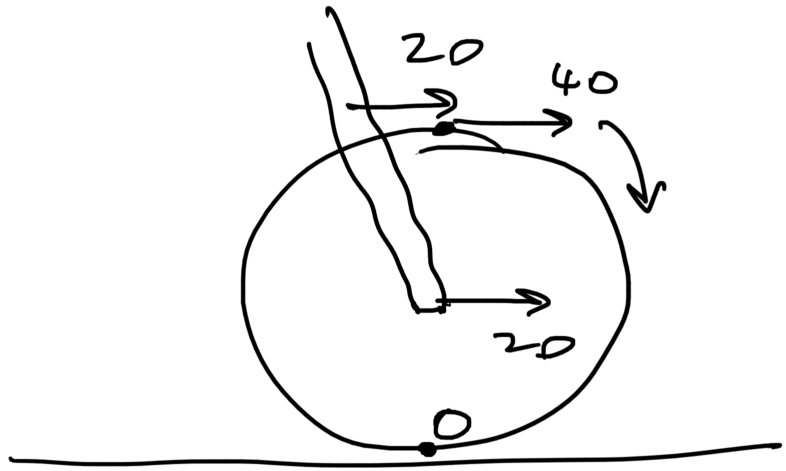
C. $V_c < V_Q < 2V_c$

D. $= 2V_c$

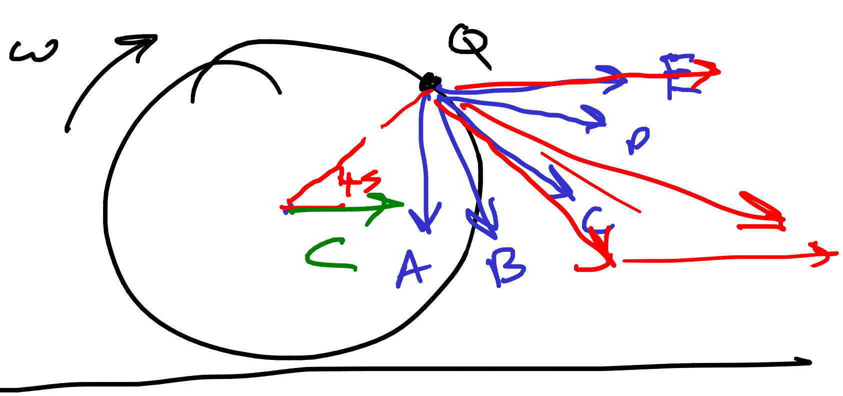
E. $> 2V_c$

$V_c = \omega r$

$\vec{v}_Q = \vec{v}_c + \vec{\omega}\times\vec{r}_{cQ}$

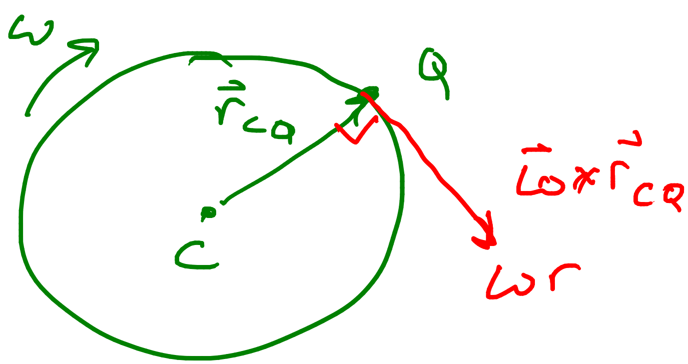


$\hat{V}_Q$

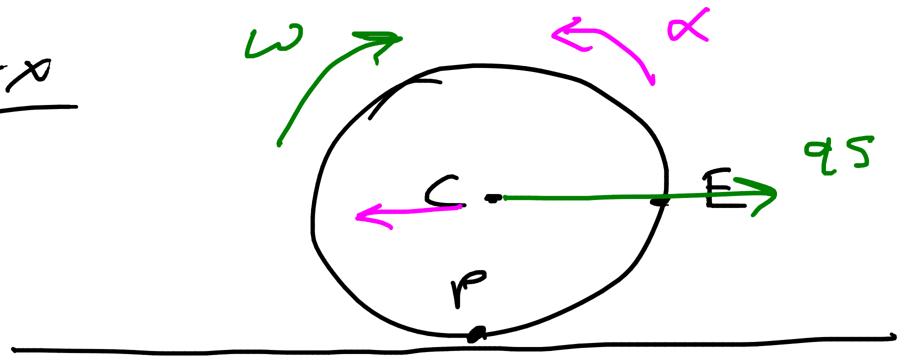


$$\vec{V}_Q = \vec{V}_C + \underline{\vec{\omega} \times \vec{r}_{CQ}}$$

ωr



Ex



no slip.

$r = 0.3$ m

$\vec{\omega} = -2\hat{k}$ rad/s

$\vec{\alpha} = 3\hat{k}$ rad/s^2

$\vec{v}_E$, $\vec{a}_E$?

$\vec{v}_P = 0$

$\left.\begin{array}{l} v_c = \omega r \\ a_c = \alpha r \end{array}\right\}$ mag

$\left.\begin{array}{l} v_c = 0.6 \\ a_c = 0.9 \end{array}\right\}$ pos because mag.

$\left.\begin{array}{l} \vec{v}_c = +0.6\,\hat{i} \\ \vec{a}_c = -0.9\,\hat{i} \end{array}\right\}$