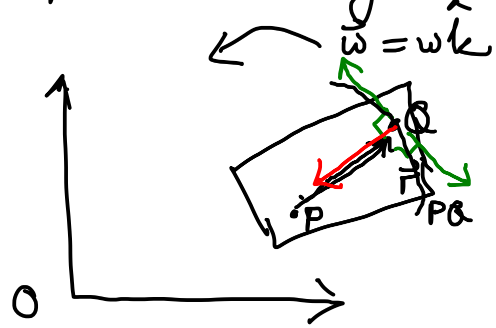
TAM 212 Class 22:

RB Acceleration

Gear Systems

R.B. Angular Acceleration



$$\vec{a}_Q = \vec{a}_P + (\vec{\alpha} \times \vec{r}_{PQ}) + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{PQ})$$

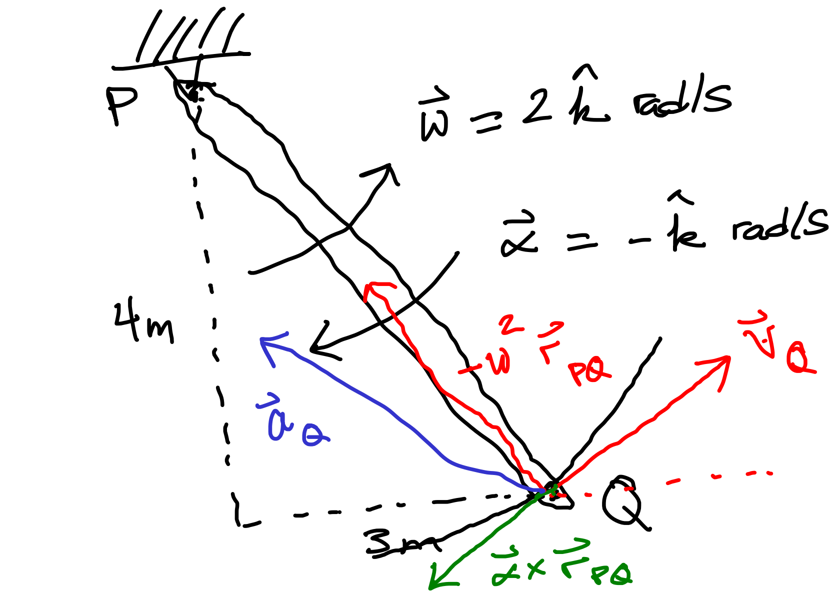
if $\vec{\omega} = \omega\hat{k}$ and $\vec{r}_{PQ}$ is in xy plane:

$$= \vec{a}_P + (\vec{\alpha} \times \vec{r}_{PQ}) - \omega^2 \vec{r}_{PQ}$$

① tangential

② inwards, centrepetal

ex) swinging pendulum



Find $\vec{v}_Q$, $\vec{a}_Q$ at this instant.

$$\vec{v}_Q = \cancel{\vec{v}_P} + (\vec{\omega} \times \vec{r}_{PQ})$$

$$= (2\hat{k}) \times (3\hat{i} - 4\hat{j})$$

$$= 6\hat{j} + \underline{8}\hat{i} \quad m/s$$

$$\vec{a}_Q = \cancel{\vec{a}_P} + (\vec{\alpha} \times \vec{r}_{PQ}) \underline{- \omega^2 \vec{r}_{PQ}}$$

$$= \underbrace{(-\hat{k}) \times (3\hat{i} - 4\hat{j})} \underbrace{-(2^2)(3\hat{i} - 4\hat{j})}$$

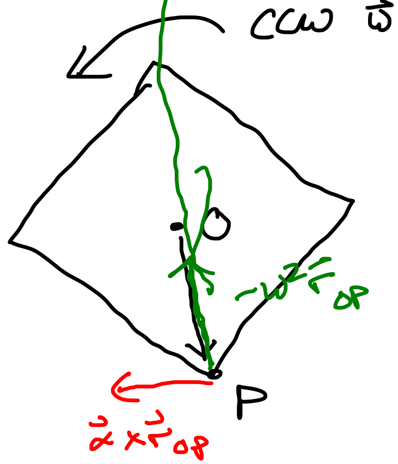
$$= \underbrace{(-3\hat{j} - 4\hat{i})}_{\substack{\vec{\alpha} \times \vec{r}_{PQ} \\ \text{tangent}}} + \underbrace{(-12\hat{i} + 16\hat{j})}_{-4(3\hat{i} - 4\hat{j})}$$

$$= -16\hat{i} + 13\hat{j}$$

ex) R.B. rotates about O. At this instant $\vec{\omega} = 10\hat{k}$ rad/s

$\vec{\alpha} = -10\hat{k}$ rad/s^2

CCW $\vec{\omega}$



Which direction is closest to the acceleration of point P?

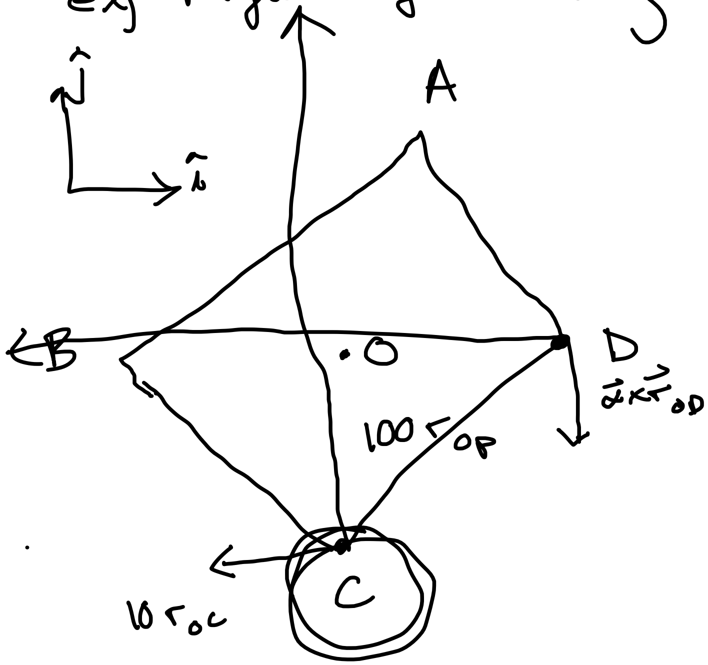
A) $\longrightarrow$ B) $\longleftarrow$ C) $\downarrow$ D) $\uparrow$

$$\vec{a}_P = \cancel{\vec{a}_O} + (\vec{\alpha}\times\vec{r}_{OP}) \underbrace{- \omega^2\vec{r}_{OP}}$$

$\leftarrow$ mag: $10 r_{OP}$

$\uparrow$ mag: $100 r_{OP}$

ex) Rigid body rotating about pt O.

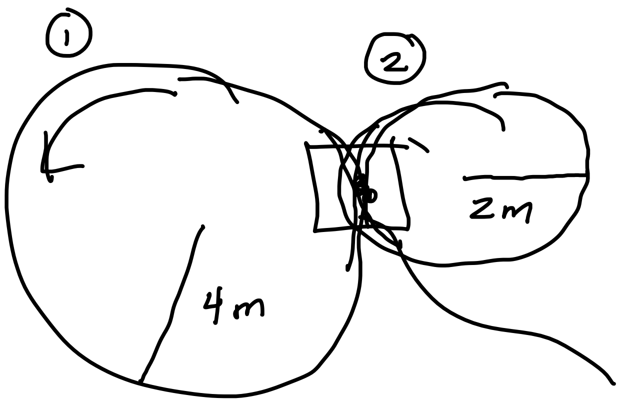


$\vec{\omega} = 10\hat{k}$ rad/s

$\vec{\alpha} = \underline{\underline{-10\hat{k}}}$ rad/s^2

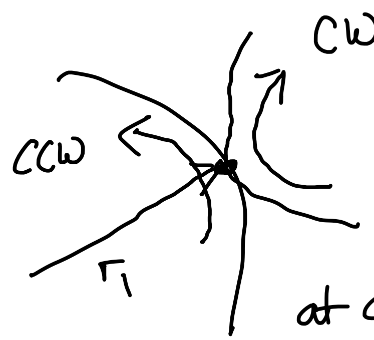
Which pt. has the largest, positive acceleration along $\hat{j}$?

Gears Example



$\vec{w}_1 = \hat{k}$ rad/sec

$\vec{w}_2$?



$$\boxed{\vec{w}_2 = -2\hat{k} \text{ rad/s}}$$

at contact point, $\vec{v}$ is the same

$$\|v\| = \|r_1 \omega_1\| = \|r_2 \omega_2\|$$

$$4(1) = 2(\|\omega_2\|)$$

$$\|\omega_2\| = 2$$