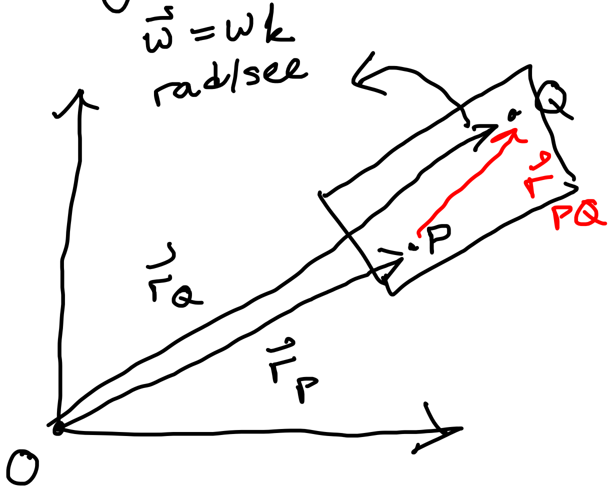
TAM 212 Class 21: Angular Acceleration of RBs

angular accelerations of R.B.'s



① $\vec{r}_Q = \vec{r}_P + \vec{r}_{PQ}$

② take derivative:

$$\underline{\underline{\vec{v}_Q}} = \vec{v}_P + \dot{\vec{r}}_{PQ} \qquad \boxed{\underline{\dot{\vec{r}}_{PQ}} = \vec{\omega} \times \vec{r}_{PQ}}$$

$$= \underline{\underline{\vec{v}_P + \vec{\omega} \times \vec{r}_{PQ}}}$$

③ take derivative

$\dot{\vec{\omega}} = \vec{\alpha} = \alpha\hat{k} \equiv$ angular acceleration

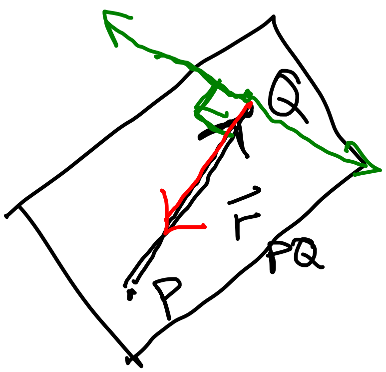
$$\vec{\alpha} = \frac{d}{dt}(\vec{\omega}) = \frac{d}{dt}(\omega\hat{k}) = \alpha\hat{k}$$

$$\frac{d}{dt}\left(\underline{\vec{V}_Q} = \underline{\vec{V}_P} + \vec{\omega} \times \vec{r}_{PQ} \right)$$

$$\vec{a}_Q = \vec{a}_P + \frac{d}{dt}\left(\vec{\omega} \times \vec{r}_{PQ} \right)$$

$$= \vec{a}_P + \underline{\dot{\vec{\omega}}} \times \vec{r}_{PQ} + \vec{\omega} \times \boxed{\dot{\vec{r}}_{PQ}}$$

$$\boxed{\vec{a}_Q = \vec{a}_P + \vec{\alpha} \times \vec{r}_{PQ} + \boxed{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{PQ})}}$$



perpendicular to $\vec{r}_{PQ}$

(velocity changes magnitude)

antiparallel to $\vec{r}_{PQ}$

"centrepetal acceleration"

(velocity changes direction)

$$T/N: \quad \vec{a} = \ddot{s}\,\hat{e}_t + \frac{(\dot{s})^2}{\rho}\,\hat{e}_n$$

consider $\vec{\omega} \times (\vec{\omega} \times \vec{r}_{PQ}) \underset{\text{WARNING}}{\neq} \underbrace{(\vec{\omega} \times \vec{\omega})}_{0} \times \vec{r}_{PQ}$

$$\vec{\omega} = \omega \hat{k}$$

$$\vec{r} = r_x \hat{i} + r_y \hat{j}$$

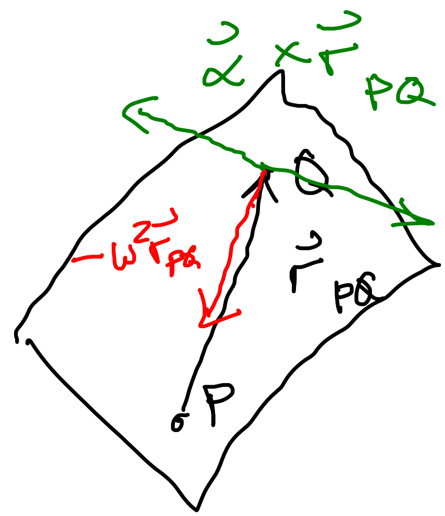
$$\vec{\omega} \times \vec{r} = (\omega \hat{k}) \times (r_x \hat{i} + r_y \hat{j})$$

$$= (\omega r_x \hat{j} - \omega r_y \hat{i})$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = (\omega \hat{k}) \times (\omega r_x \hat{j} - \omega r_y \hat{i})$$

$$= (-\omega^2 r_x \hat{i} - \omega^2 r_y \hat{j})$$

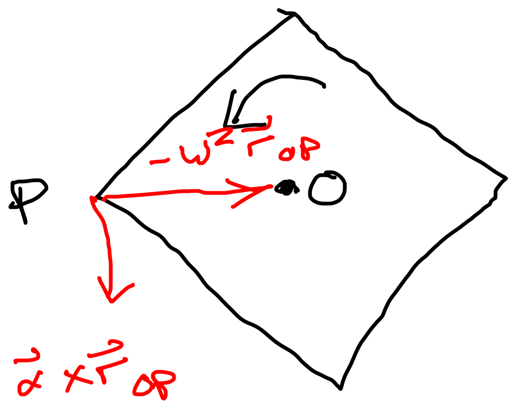
$$= -\omega^2 \vec{r}$$



angular acceleration!

$$\vec{a}_Q = \vec{a}_P + \vec{\alpha} \times \vec{r}_{PQ} - \omega^2 \vec{r}_{PQ}$$

iclickers:



$$\vec{\omega} = 10\ \hat{k} \ \text{rad/sec}$$

$$\vec{\alpha} = 10\ \hat{k} \ \text{rad/sec}^2$$

Which direction is closest to $\vec{a}_P$

A) $\uparrow$ B) $\downarrow$ C) $\rightarrow$ D) $\leftarrow$

$$\vec{a}_P = \cancel{\vec{a}_O} + \vec{\alpha} \times \vec{r}_{OP} - \omega^2 \vec{r}_{OP}$$