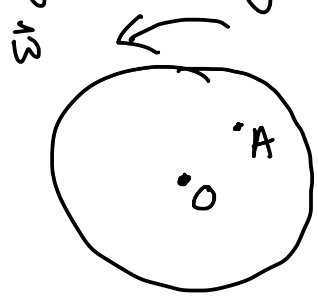
TAM 212: Rigid Bodies / Constrained R.B.'s

Rigid Body: ∀

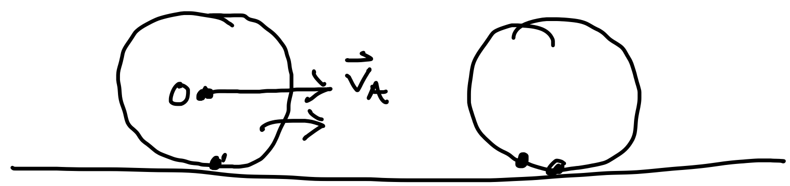


$$\boxed{\vec{v}_A = \vec{\omega} \times \vec{r}_{OA}}$$

pure rotation

pure translation:
$\vec{v}_A$ all same

translation + rotation



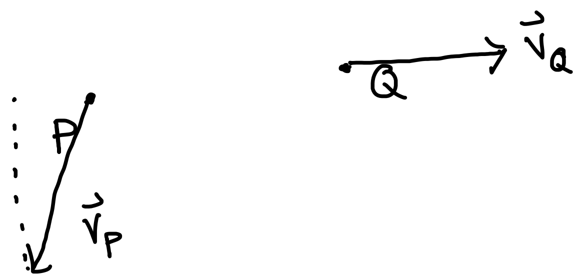
$$\boxed{\vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ}}$$

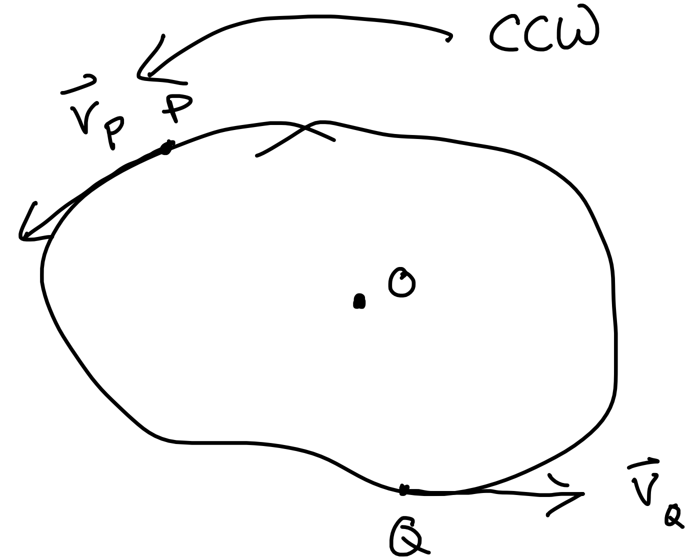
Ex) A rigid body has $\vec{\omega} = \underline{\underline{\omega}}\,\hat{k} = \underline{\underline{6}}\,\hat{k}$ rad/sec. Two points have velocity

$$\vec{v}_P = -\hat{i} - 3\hat{j} \quad \text{m/s}$$

$$\vec{v}_Q = 3\hat{i} \quad \text{m/s}$$

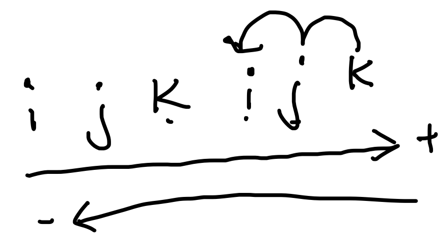
Find: $\underline{\underline{r_{PQ}}}$





Ex) $\boxed{\vec{V}_Q = \vec{V}_P + \vec{\omega} \times \vec{r}_{PQ}}$

unknown $\quad \vec{r}_{PQ} = x\,\hat{i} + y\,\hat{j}$



$$0\hat{j} + (3\hat{i}) = (-\hat{i} - 3\hat{j}) + (+6\hat{k}) \times (x\hat{i} + y\hat{j})$$

$$= (-\hat{i} - 3\hat{j}) + (6x\;\hat{k} \times \hat{i}) + (6y\;\hat{k} \times \hat{j})$$

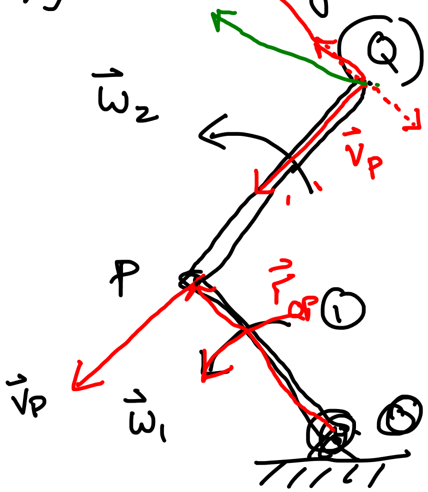
$$= (-\hat{i} - 3\hat{j}) + (6x\,\hat{j}) + (-6y\,\hat{i})$$

$$= (-1 - 6y)\,\hat{i} + (-3 + 6x)\,\hat{j}$$

$\hat{i}: \quad 3 = -1 - 6y \qquad\qquad y = -2/3 \qquad\qquad \vec{r} = \frac{1}{2}\hat{i} - \frac{2}{3}\hat{j}$

$\hat{j}: \quad 0 = -3 + 6x \qquad\qquad x = 1/2$

Ex) Two rigid bodies of equal length, connected as shown:



$\vec{\omega}_1 = \omega_1 \hat{k}$ $\quad \omega_1 > 0$

$\vec{\omega}_2 = \omega_2 \hat{k}$ $\quad \boxed{\omega_2 = 2\omega_1}$

What is the direction of $\vec{v}_Q$?

A) ↖ B) ↘ C) ↗ D) ↙

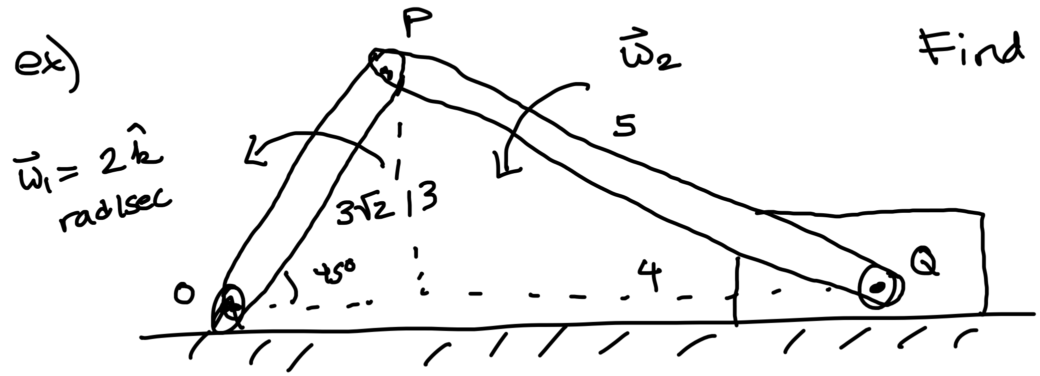
$\vec{v}_0 = 0$

$\vec{v}_P = \cancel{\vec{v}_0} + \vec{\omega}_1 \times \vec{r}_{OP}$

$\vec{v}_Q = \vec{v}_P + \vec{\omega}_2 \times \vec{r}_{PQ}$

CONSTRAINED MOTION OF A RIGID BODY

ex)



Find $\vec{\omega}_2$

A) CW

B) CCW

Constrained: $\vec{V}_Q = V_x\hat{i} + V_y\hat{j} = V_x\hat{i}$ $\boxed{V_y = 0}$

approach: use $\vec{V}_Q = \vec{V}_P + \vec{\omega} \times \vec{r}_{PQ}$

- start at a place where $\vec{v}$ is known, work along the linkage to obtain the velocities of other points,
- apply common sense constraints to solve for unknowns.

$\vec{V}_O = 0$ $\qquad \vec{V}_P = \cancel{\vec{V}_O} + \vec{\omega}_1 \times \vec{r}_{OP} = (2\hat{k}) \times (3\hat{i} + 3\hat{j}) = -6\hat{i} + 6\hat{j}$

$$\vec{V}_Q = \vec{V}_P + \vec{\omega}_2 \times \vec{r}_{PQ}$$

$$= (-6\hat{i} + 6\hat{j}) + (\omega_2 \hat{k}) \times (4\hat{i} - 3\hat{j})$$

$$= (-6\hat{i} + 6\hat{j}) + (4\omega_2 \hat{j} + 3\omega_2 \hat{i})$$

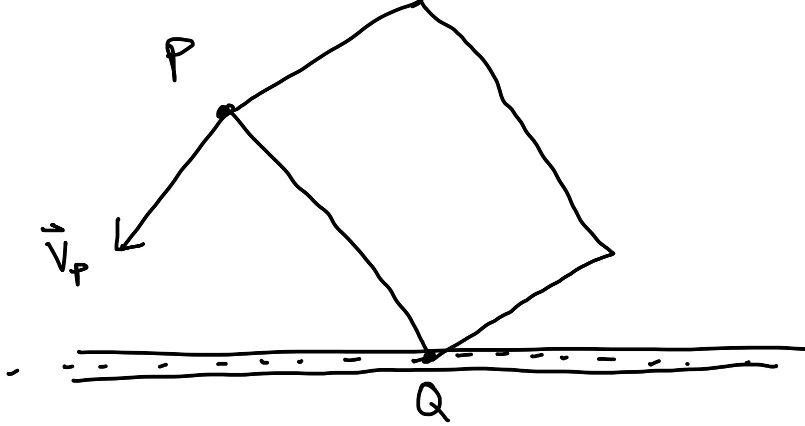
$$= (3\omega_2 - 6)\hat{i} + (4\omega_2 + 6)\hat{j}$$

use constraint: y comp of $V_Q = 0$

$$4\omega_2 + 6 = 0$$

$\omega_2 = -1.5$ rad/sec CW

$\vec{\omega}_2 = -1.5\,\hat{k}$ rad/sec ex)



$\vec{\omega} = \omega \hat{k}$

$\vec{V}_Q = V_x \hat{\imath} + \underline{\underline{V_y}} \hat{\jmath} = V_x \hat{\imath}$

given: $\vec{r}_{PQ} = 3\hat{\imath} - 2\hat{\jmath}$ m

$\vec{V}_P = -\hat{\imath} - 3\hat{\jmath}$ m/s

find $\vec{\omega} = \omega \hat{k}$ rad/sec

$$\begin{aligned}
\vec{V}_Q &= \vec{V}_P + \vec{\omega} \times \vec{r}_{PQ} \\
&= (-\hat{\imath} - 3\hat{\jmath}) + (\omega \hat{k}) \times (3\hat{\imath} - 2\hat{\jmath}) \\
&= (-\hat{\imath} - 3\hat{\jmath}) + (3\omega \hat{\jmath}) + (+2\omega \hat{\imath}) \\
&= (2\omega - 1)\hat{\imath} + (3\omega - 3)\hat{\jmath}
\end{aligned}$$

constraint $3\omega - 3 = 0$

$\omega = 1$ rad/sec

$\vec{\omega} = \hat{k}$ rad/sec.