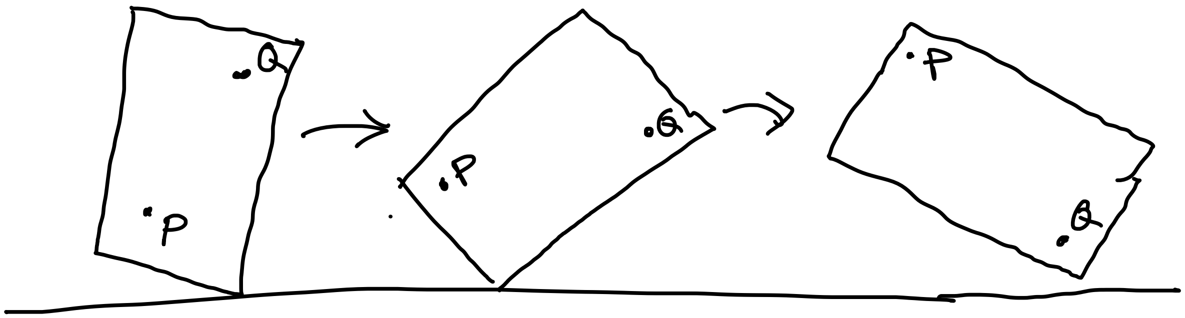
TAM 212 Class 16: Rigid Bodies Con't

* remember conflict sign up by end of Friday (~~tomorrow~~ day after)

* partial credit possible on multiple choice

A	B	C	D	E	
			•		1
		•			0
			•	•	1/2
		•	•	•	1/3
•	•	•			0
	•	•	•	•	~~1/4~~ 0

Class 16 Rigid Bodies



* all points on the R.B. HAVE the same angular velocity $\vec{\omega}$
* all points on the R.B do not necessarily have the same $\vec{v}$

If $\vec{\omega}$ is known, and the velocity of any single point P, $\vec{v}_P$ is known;
THEN: we can find the velocity of any other point Q on the R.B.

$$\boxed{\vec{V}_Q = \underbrace{\vec{V}_P}_{\substack{\text{pure transl.} \\ \text{w/ } \vec{v}=\vec{v}_P}} + \underbrace{\vec{\omega}\times\vec{r}_{PQ}}_{\substack{\text{pure rotation of} \\ \text{point Q about} \\ \text{point P.}}}}$$

* usually (for us) $\vec{\omega} = \omega\hat{k}$
  $\vec{v}_P, \vec{v}_Q, \ldots$ = lie in the xy plane

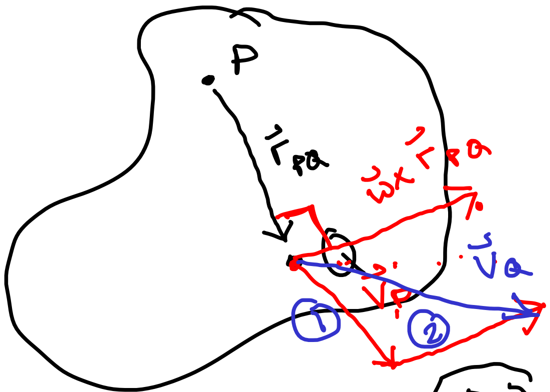
* also notice $\vec{\omega}\times\vec{r}_{PQ}$ is $\perp$ to vector $\vec{r}_{PQ}$

ex) R.B. rotates w/ $\vec{\omega} = \hat{k}$ rad/sec

$\vec{r}_{PQ} = \hat{i} - 4\hat{j}$ m

$\vec{v}_P = \underline{\underline{2\hat{i} - 2\hat{j}}}$ m/s

$\vec{r}_{PQ} \Rightarrow$ from P to Q



Find $\vec{v}_Q = \vec{v}_P + \vec{\omega}\times\vec{r}_{PQ}$

$$= (2\hat{i} - 2\hat{j}) + (\hat{k}) \times (\hat{i} - 4\hat{j})$$

$$= (2\hat{i} - 2\hat{j}) + \underbrace{(\hat{j} + 4\hat{i})}$$

$$= 6\hat{i} - \hat{j}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & -4 & 0 \end{vmatrix}$$

i j k i j k

+

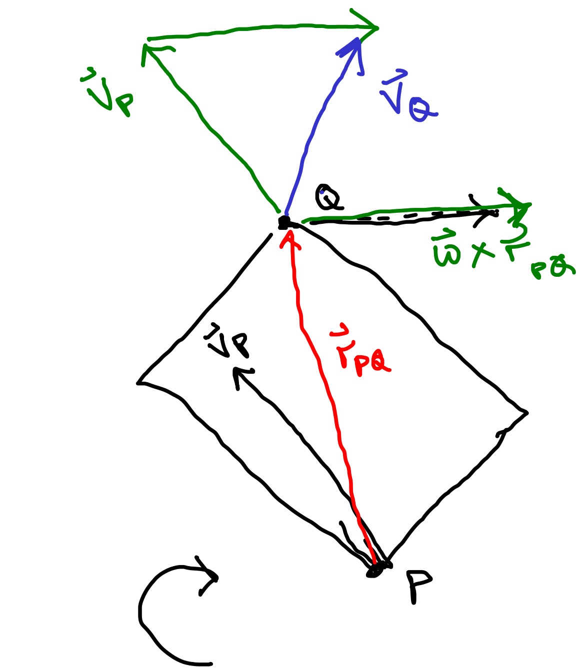
−

$\hat{k}\times\hat{i} = \hat{j}$

$\hat{k}\times\hat{j} = -\hat{i}$

ex) R.B. rotates CW. We are told $\|\omega r_{PQ}\| = \|v_P\|$

Find the direction of $\vec{v}_Q$



A) ↗

B) ↖

C) ↙

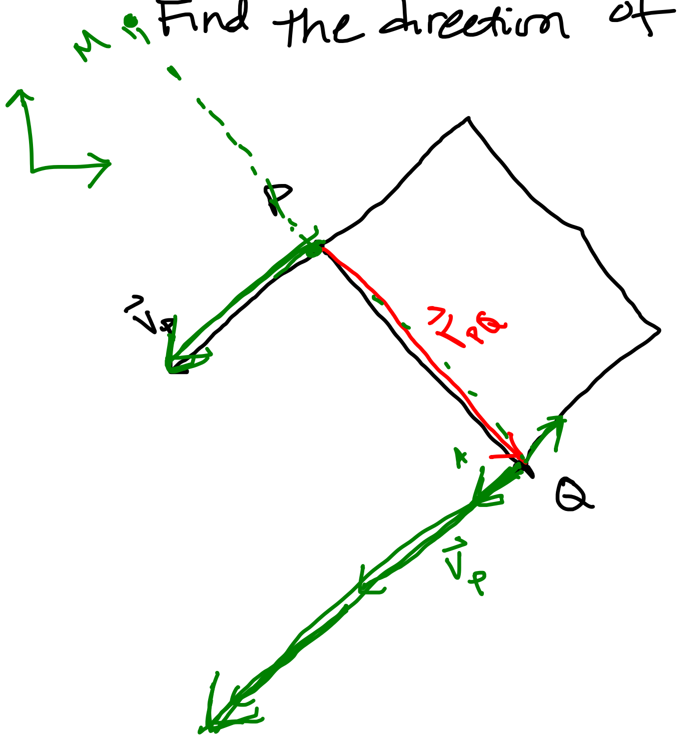
D) ↘

$$\vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ}$$

CW: $\vec{\omega} = \omega \hat{k}$ $\quad \omega < 0$

ex) R.B. moving as shown with a CW rotation. The angular velocity ω, distance r_{PQ}, and the speed v_P satisfy $\omega r_{PQ} = v_P$.

Find the direction of $\vec{v}_Q$



A) ↖

B) ↗

C) ↙

D) ↘

$$\vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ}$$

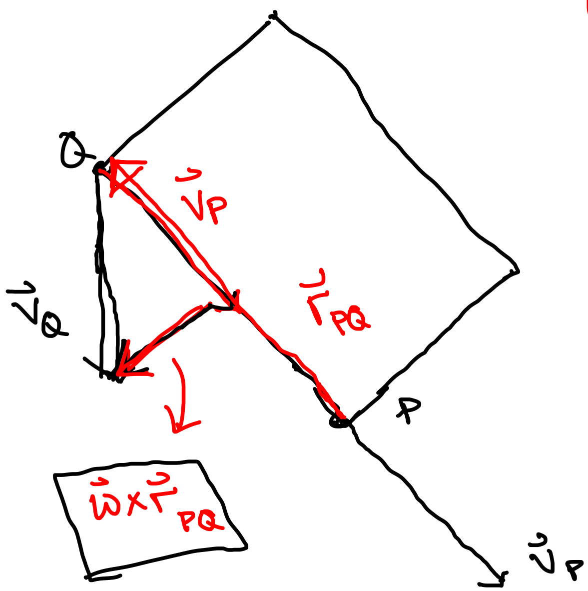
$$\vec{\omega} \times \vec{r}_{PQ}$$

$$(-\omega \hat{k}) \times (+\hat{i} - \hat{j})$$

$$(-\omega)(\hat{j}) + (+\omega)(-\hat{i})$$

$$\underline{-\omega \hat{i} - \omega \hat{j}}$$

ex) RB moves in 2D as shown below. What is the direction of the angular velocity?



A) CCW

B) CW

$$\underline{\underline{\vec{V}_Q}} = \underline{\underline{\vec{V}_P}} + \underline{\underline{\vec{\omega} \times \vec{r}_{PQ}}}$$

Say $\vec{\omega} = \omega \hat{k}$

What is the sign of ω so that $\vec{\omega} \times \vec{r}_{PQ}$ is "down-left" $\swarrow$?

$\omega > 0$, CCW

CONSTRAINED MOTION