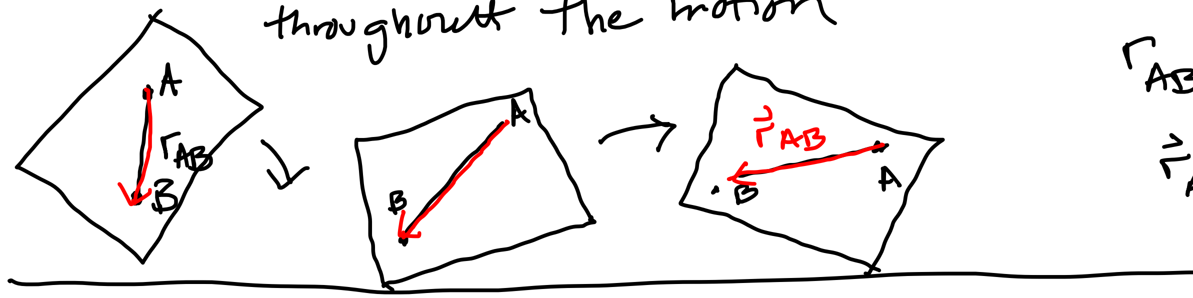
TAM 2~~1~~2 Class 15: Rigid Bodies.

ANNOUNCEMENTS: MT1

* MUST fill out conflict form by end of FRIDAY
* covers up to & including PL #4 (T/N)
* one piece of paper, handwritten notes, both sides.
* PL Practice Exam Mode
* CARE Program
  - M 3/2 8-10pm
  - W 3/4 9-11pm } 429 Grainger
* Office Hours: ~~Th~~ W / Th: extra TAs

Rigid Body $\longrightarrow$ "non-deformable"

distance r_{AB} between any two points A & B is constant throughout the motion



r_{AB} = constant

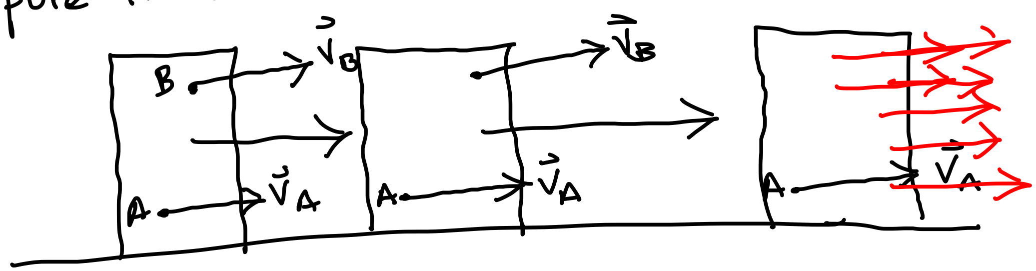
$\vec r_{AB}$ → varies

Significance:

The velocities of various points on a R.B. can be different, but they are related to each other by the angular velocity of the rigid body.

Types of Motion:

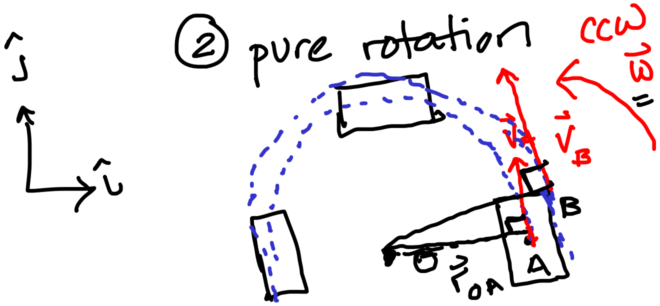
① pure translation



all points have the same velocity

② pure rotation

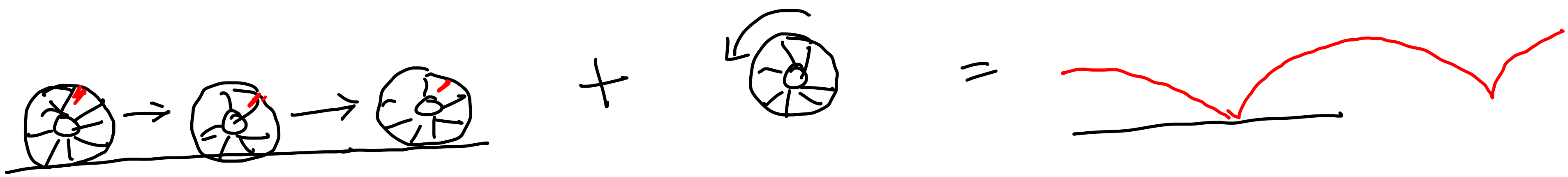
ccw $\vec\omega = \omega\hat k$



$$\vec V_A = \vec\omega \times \vec r_{OA} \Longrightarrow \text{in } \hat j \text{ direction}$$

$$\vec V_B = \vec\omega \times \vec r_{OB} \Longrightarrow \vec V_B \text{ is } \perp \text{ to } \vec r_{OB}$$

ex) riding a bicycle: movement of tire is a combination of a pure translation & pure rotation



For a R.B. given $\vec{v}_A$ and $\vec{\omega}$ of the R.B, we can always find the velocity of any other point.

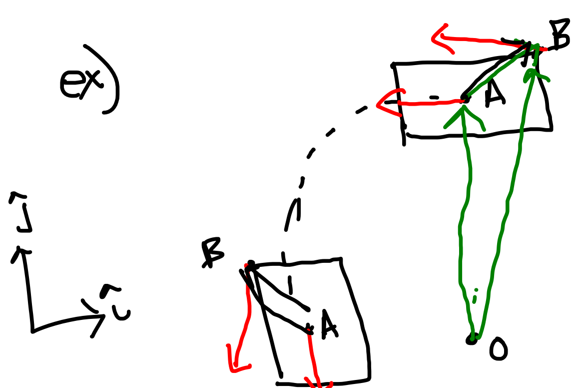
* Angular velocity is the same for all points on a R.B.; $\vec{v}$ are not the same.

PLANE MOTION OF A RIGID BODY

- fixed axis of rotation: for us, usually $\hat{k}$

  $\vec{\omega} = \omega \hat{k}$

ex)



If we know

(1) $\vec{r}_{OA}$, $\vec{v}_A$, $\vec{\omega}$

(2) orientation of point B w/r/t point A $\vec{r}_{AB}$

Then: we can always find $\vec{v}_B$

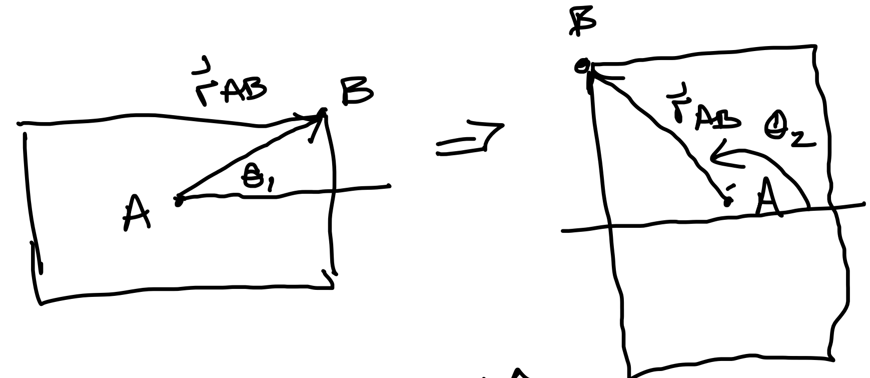
Rigid Body

$$\vec{r}_{OB} = \vec{r}_{OA} + \vec{r}_{AB}$$

$$\frac{d}{dt}\left(\vec{r}_{OB} = \vec{r}_{OA} + \vec{r}_{AB}\right)$$

$$\boxed{\vec{V}_B = \vec{V}_A + \underbrace{\frac{d}{dt}\vec{r}_{AB}}}$$

$$\vec{r}_{AB} = \underbrace{r_{AB}}_{\text{const}}\ \underbrace{\hat{u}}_{\text{varies}}$$



$$= \vec{V}_A + \frac{d}{dt}\left(r_{AB}\,\hat{u}\right)$$

$$= \vec{V}_A + r_{AB}\frac{d}{dt}\hat{u}$$

$$* \quad = \vec{V}_A + r_{AB}\,\underbrace{\dot{\theta}\left[-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right]}$$

$$\hat{u} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

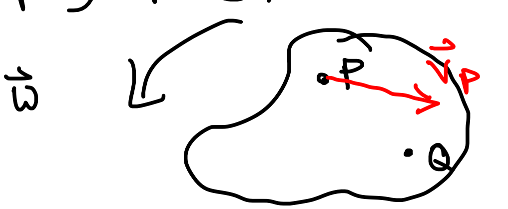
$$\frac{d\hat{u}}{dt} = -\dot{\theta}\sin\theta\,\hat{i} + \dot{\theta}\cos\theta\,\hat{j}$$

recall $\dot{\theta} = \omega$

$$\vec{r}_{AB} = r_{AB}\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$* \quad \boxed{\vec{V}_B = \underbrace{\vec{V}_A}_{\text{transl.}} + \underbrace{\vec{\omega}\times\vec{r}_{AB}}_{\text{rotation}}}$$

example) R.B. rotates w/ $\vec{\omega} = \hat{k}$ rad/sec. Given



$$\vec{r}_{PQ} = \hat{i} - 4\hat{j}\ \text{m}$$

$$\vec{V}_P = 2\hat{i} - 2\hat{j}\ \text{m}$$

Find $\vec{V}_Q$.

$\vec{V}_Q = \vec{V}_P + \vec{\omega} \times \vec{r}_{PQ}$

$= (2\hat{i} - 2\hat{j}) + (\hat{k}) \times (\hat{i} - 4\hat{j})$

$= (2\hat{i} - 2\hat{j}) + (\hat{j} + 4\hat{i})$

A) $\vec{V}_Q = 6\hat{i} - \hat{j}$

B) $\vec{V}_Q \neq 6\hat{i} - \hat{j}$