TAM 212 Class 13 T/N Wrap-Up

review: T/N

$$\boxed{\begin{aligned}\vec{v} &= \dot{s}\,\hat{e}_t \\ \vec{a} &= \cancel{\ddot{s}\,\hat{e}_t} + \kappa(\dot{s})^2\,\hat{e}_n\end{aligned}}$$

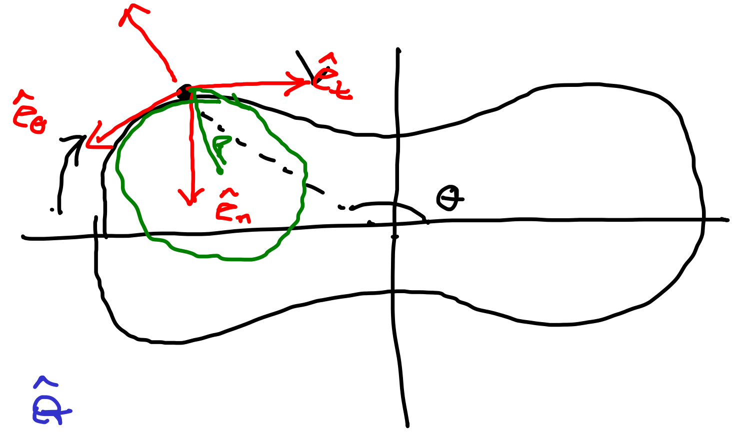
$= -4\hat{e}_r - 4\hat{e}_\theta$

$= -12\hat{e}_r + 12\hat{e}_\theta$

Finish Problem from last time:

Bicycle on a track given by $r(\theta) = 2 + \cos(2\theta)$

At an instant: $\theta = 3\pi/4$ rad $\quad \dot{\theta} = -2$ rad/sec $\quad \ddot{\theta} = -2$ rad/sec^2



a) sketch the track

b) find $\vec{v}, \vec{a}$ in the polar basis

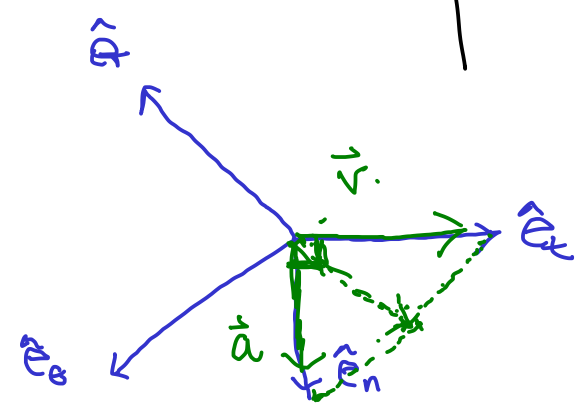
* c) bicycle speeding up or slowing down?

d) radius of curvature at this instant?

last time: $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = -4\,\hat{e}_r - 4\hat{e}_\theta$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$= -12\hat{e}_r + 12\,\hat{e}_\theta$



the bicycle is:

A) speeding up

B) slowing down

C) neither

D) both

$\vec{a} \cdot \vec{v} = 0$

if $\vec{a} \cdot \vec{v} < 0 \Rightarrow$ slowing down

$\vec{a} \cdot \vec{v} > 0 \Rightarrow$ speeding up

$$\vec{a} = \underbrace{K(\dot{s})^2}\, \underbrace{\hat{e}_n} = -12\hat{e}_r + 12\hat{e}_\theta$$

$$\hat{e}_n = \frac{1}{\sqrt{2}}\left(-\hat{e}_r + \hat{e}_\theta\right)$$

$\Rightarrow$ equate directions

$\Rightarrow$ equate magnitude

$$K(\dot{s})^2 = 12\sqrt{2}$$

$$K(4\sqrt{2})^2 = 12\sqrt{2}$$

$$K = \frac{1}{\rho} = \frac{3\sqrt{2}}{8} \text{ m}$$

3D T/N Coords

(velocity is STILL tangent to path)

3 basis vectors:

$\hat{e}_t$ = tangent to path = $\frac{\vec{v}}{v} = \frac{\vec{v}}{\dot{s}}$

$\hat{e}_n$ = points to inside of curve, $\perp$ to $\hat{e}_t$

$\hat{e}_b$ = binormal basis vector: $\hat{e}_t \times \hat{e}_n$

STILL

$\vec{v} = \dot{s}\hat{e}_t$

$$\boxed{\vec{a} = \ddot{s}\hat{e}_t + \kappa(\dot{s})^2\hat{e}_n}$$

eg) $\vec{v} = 74\hat{e}_\theta + 8\hat{k}$ m/s

$$\boxed{\vec{a} = -148\hat{e}_r + 37\hat{e}_\theta - 20\hat{k} \text{ m/s}^2}$$

Find ρ, radius of curvature, at this instant?

$\ast\ \hat{e}_t = \frac{\vec{v}}{v} \Rightarrow$ tangent direction

$\hat{e}_n$??

$$\vec{a} - \underbrace{(\vec{a}\cdot\hat{e}_t)\hat{e}_t}_{\ddot{s}\hat{e}_t} = \underbrace{\kappa(\dot{s})^2\hat{e}_n}$$

magnitude is $\kappa(\dot{s})^2$

direction is $\hat{e}_n$

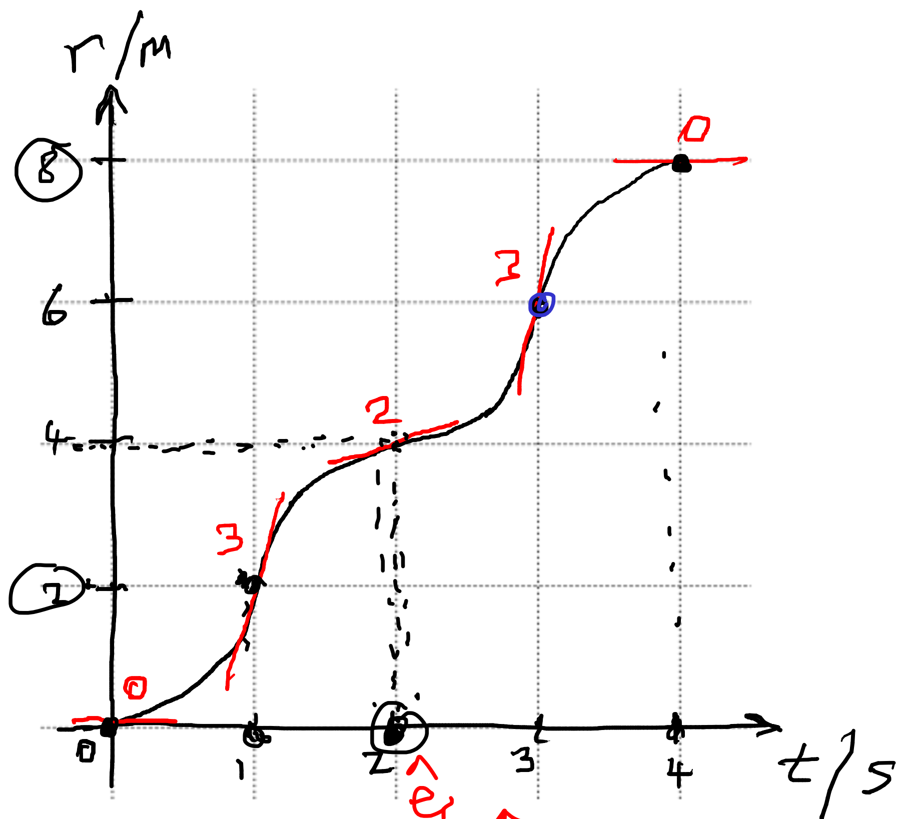
why does $\vec{a}\cdot\hat{e}_t = \ddot{s}$

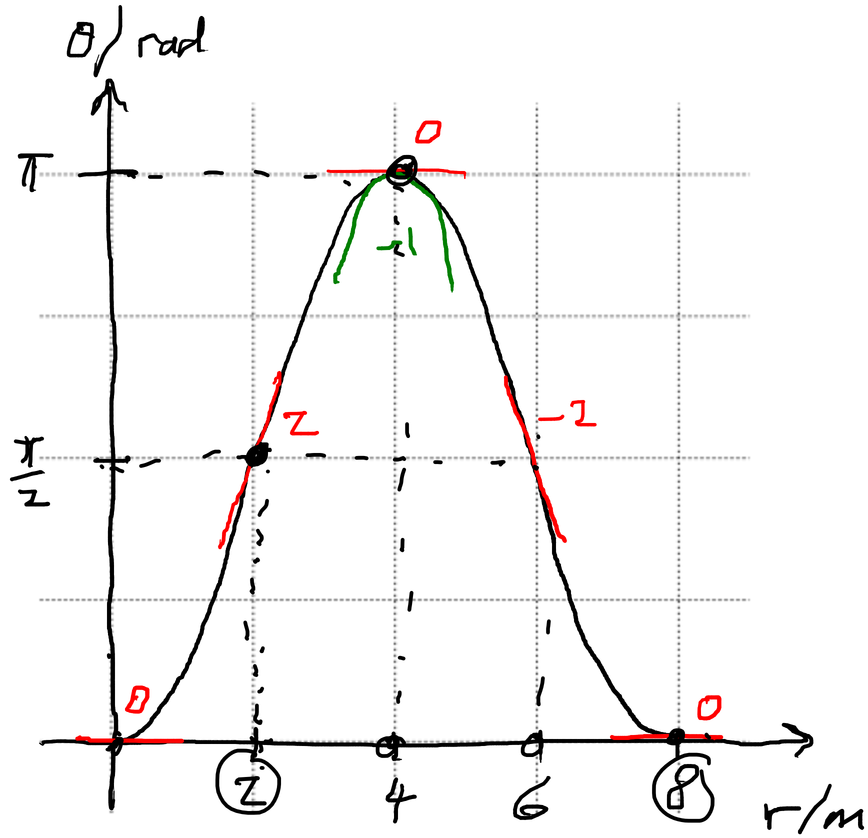
$\downarrow$

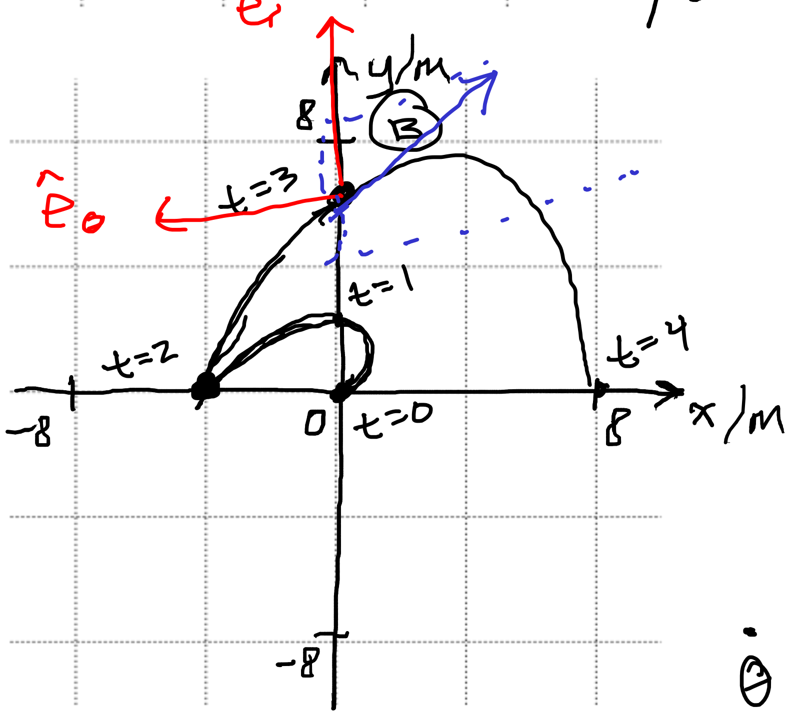
$(\ddot{s}\hat{e}_t + \kappa(\dot{s})^2\hat{e}_n)\cdot\hat{e}_t$

$\ddot{s}(\hat{e}_t\cdot\hat{e}_t) + \cancel{\kappa(\dot{s})^2(\hat{e}_n\cdot\hat{e}_t)}$

$\ddot{s}$







question: find $\dot{\theta} = \frac{d\theta}{dt}$ at t=2sec

$$\frac{d\theta}{dt} = \frac{d\theta}{dr}\,\frac{dr}{dt}$$

$$= (2)\,(2)$$

$$\dot{\theta} = 4 \text{ rad/sec}$$

$\dot{\theta}$ at t=1 sec? $\frac{d\theta}{dt} = \left(\frac{d\theta}{dr}\right)\left(\frac{dr}{dt}\right) = (2)(3) = 6$

$$\vec{v} = \underline{\underline{\dot{r}}}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

Find $\vec{v}$ at pt B, $t = 3$ sec?

$\dot{r} = 3$ at $t = 3$ sec

$r = 6$

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{d\theta}{dr}\left(\frac{dr}{dt}\right) = (-2)(3) = -6$$

$$\vec{v} = 3\hat{e}_r - 36\,\hat{e}_\theta$$