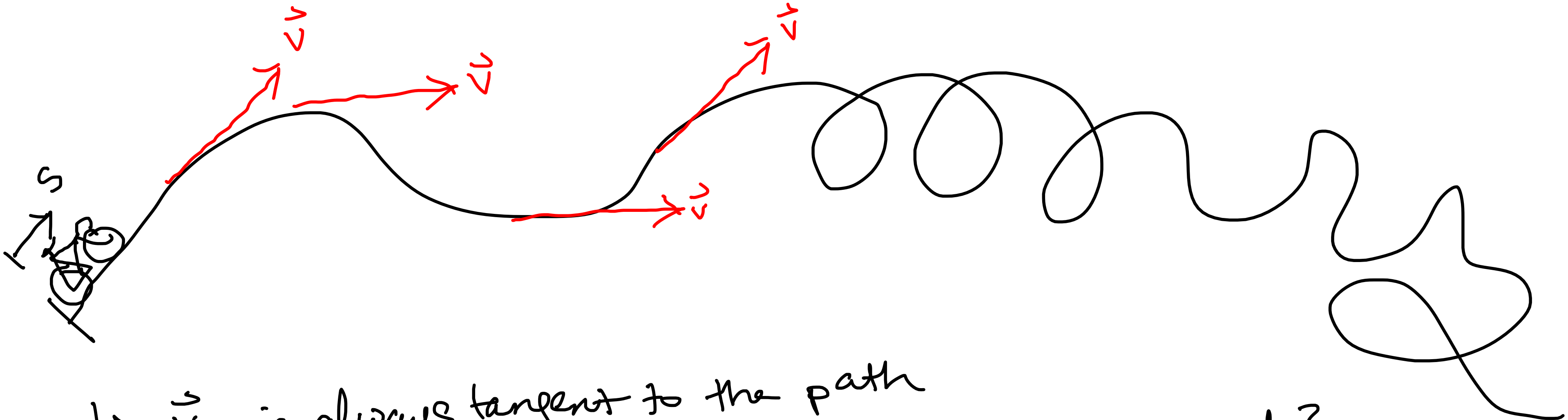
TAM 212: Class 12 T/N

Review

	cartesian	polar	T/N
pos.	$\vec{r} = x\hat{\imath} + y\hat{\jmath}$	$\vec{r} = r\hat{e}_r$	— —
vel.	$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	$\vec{v} = \dot{s}\underline{\underline{\hat{e}_t}}$
accel.	$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$	$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$	$\vec{a} = \underbrace{\ddot{s}\underline{\underline{\hat{e}_t}}}_{\text{changes in speed}} + \underbrace{\kappa(\dot{s})^2\underline{\underline{\hat{e}_n}}}_{\text{changes to direction of } \vec{v}}$



↳ $\vec{v}$ is always tangent to the path

↳ TRUE (A) OR FALSE (B)

① $\dot{s}$ is the speed? TRUE

② $\ddot{s}$ is the acceleration? FALSE

③ $\ddot{s}$ is the magnitude of the accel? F

$$\vec{a} = \boxed{\ddot{s}\,\hat{e}_t} + \boxed{K(\dot{s})^2\,\hat{e}_n}$$

magn of $\vec{a}$ $= \|\vec{a}\| = \left[(\ddot{s})^2 + K^2(\dot{s})^4\right]^{1/2}$

ex) A ~~car~~ bicycle is moving on a track defined by $r(\theta) = 2 + \cos(2\theta)$. At an instant, we measure

$$\theta = \frac{3\pi}{4} \text{ rad} \;;\; \dot{\theta} = -2 \text{ rad/sec} \;;\; \ddot{\theta} = -2 \text{ rad/sec}^2$$

a) sketch the track

b) Find $\vec{v}$, $\vec{a}$ in the polar basis

c) is the car speeding up or slowing down?

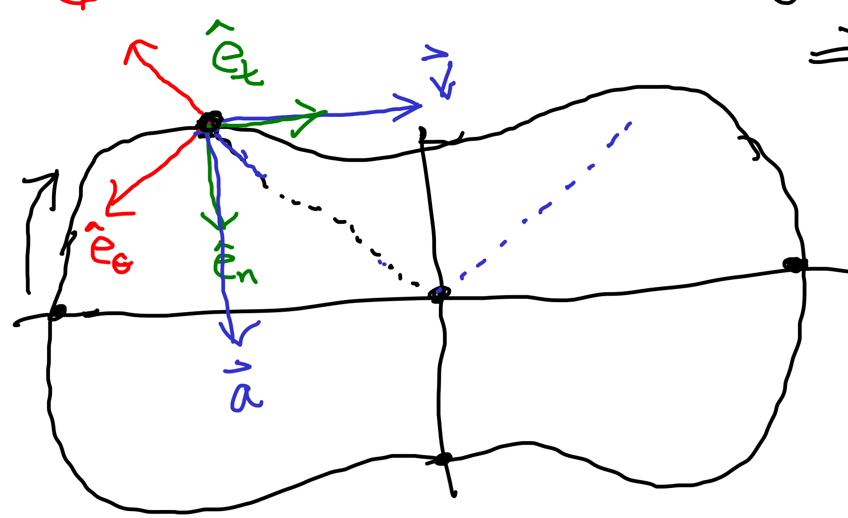
d) what is the radius of curvature of the track?

$\dot{\theta} < 0$

$\Rightarrow$ CW

a)

θ	r
0	3
$\pi/4$	2
$\pi/2$	1
$3\pi/4$	2
π	3



b) $\vec{r} = r\hat{e}_r$ $\qquad\qquad r = \underline{2+\cos(2\theta)}$

$\vec{v} = \underline{\underline{\dot{r}}}\,\hat{e}_r + \underline{\underline{r\dot{\theta}}}\,\hat{e}_\theta$

Need $\dot{r} = \dfrac{dr}{dt} = \dfrac{d}{dt}(2+\cos 2\theta)$

$= \left(\dfrac{dr}{d\theta}\right)\left(\dfrac{d\theta}{dt}\right)$

$= (-2\sin 2\theta)(\dot{\theta})$

$= (-2)(-1)(-2)$

$= -4$ m/s

Need $r\dot{\theta} = (2+\cos 2\theta)(-2)$

$= (2+\cos\frac{3\pi}{2})(-2)$

$= -4$ m/s

$$\boxed{\vec{v} = -4\hat{e}_r - 4\hat{e}_\theta}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta \qquad r = 2 + \cos(2\theta)$$

$$\ddot{r} = \frac{d}{dt}(\dot{r}) = \frac{d}{dt}\left(\underbrace{\frac{dr}{d\theta}}_{a}\;\underbrace{\frac{d\theta}{dt}}_{b}\right)$$

$$= \underbrace{\left[\frac{d}{dt}\left(\frac{dr}{d\theta}\right)\right]}\left(\frac{d\theta}{dt}\right) + \left(\frac{dr}{d\theta}\right)\left[\frac{d}{dt}\left(\frac{d\theta}{dt}\right)\right]$$

$$= \left[\underbrace{\frac{d}{d\theta}\left(\frac{dr}{d\theta}\right)}\frac{d\theta}{dt}\right]\dot{\theta} + \left(\frac{dr}{d\theta}\right)\ddot{\theta}$$

$$= \left[\left(\frac{d^2 r}{d\theta^2}\right)\dot{\theta}\right]\dot{\theta} + \left(\frac{dr}{d\theta}\right)\ddot{\theta}$$

$$= \frac{d}{d\theta}\left(-2\sin 2\theta\right)(\dot{\theta})^2 + (-2\sin 2\theta)\,\ddot{\theta}$$

$$= (-2)(2)\cos(2\theta)\,(\dot{\theta})^2 - 2\sin(2\theta)\,(\ddot{\theta})$$

$$= (-4)(0)(-2)^2 - 2(-1)(-2)$$

$$= -4 \text{ m/s}^2$$

$$\vec{a} = \left(-4 - 2(-2)^2\right)\hat{e}_r + \left(2(-2) + 2(-4)(-2)\right)\hat{e}_\theta = \boxed{-12\,\hat{e}_r + 12\,\hat{e}_\theta}$$

c) since $\vec{a} \perp \vec{v}$, bicycle is not speeding up or slowing down