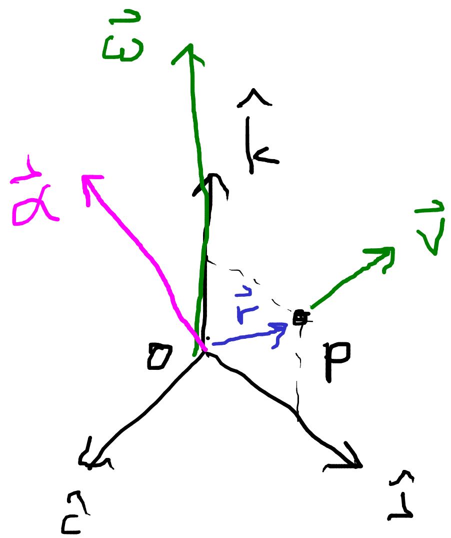
Recap



$\vec{\omega}$ = angular velocity vector

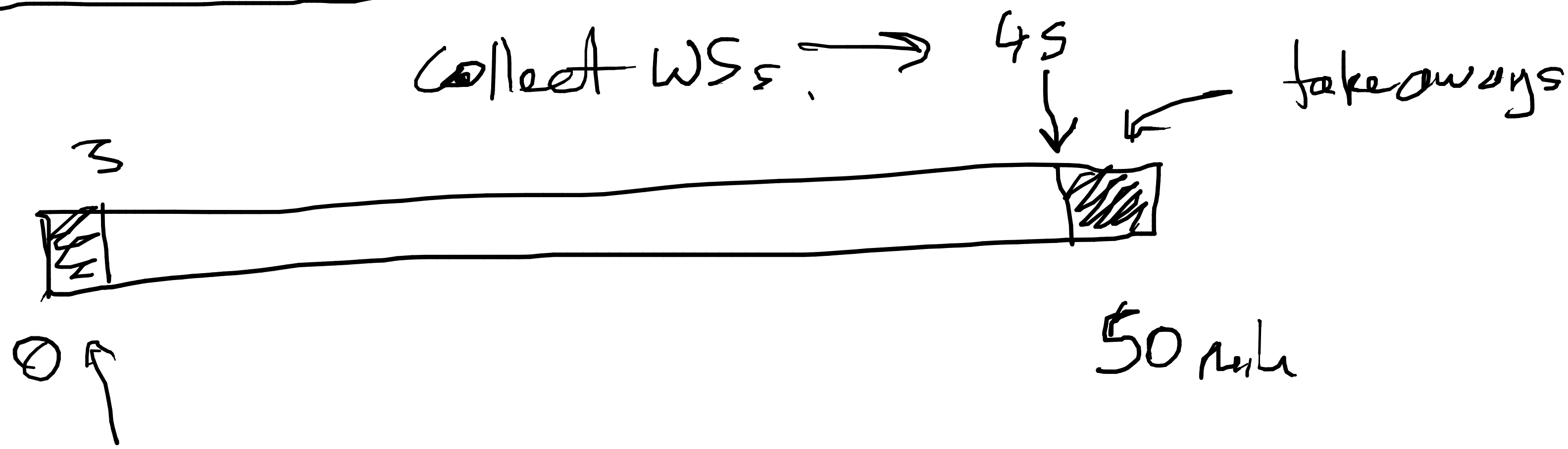
$$\vec{\omega} \begin{cases} \omega = \text{rate of rotation} \\ \hat{\omega} = \text{axis of rotation} \\ \quad \text{(orthogonal to plane of rotation)} \end{cases}$$

$\vec{\alpha}$ = angular acceleration vector

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

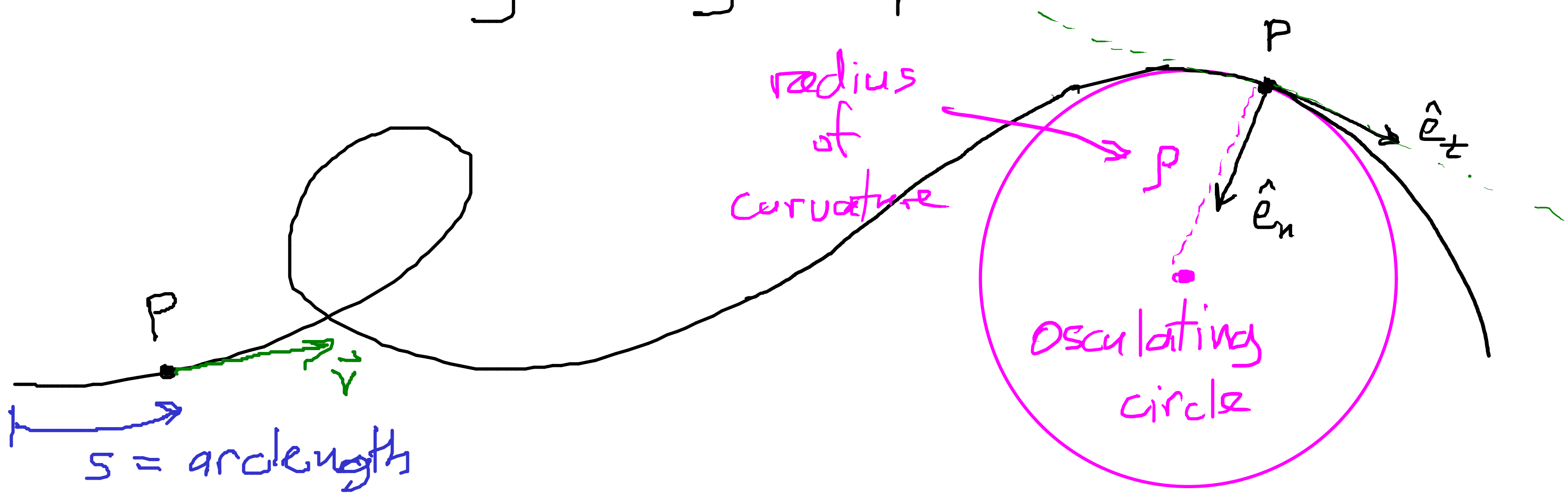
Today:
- HW extended to Sat 11:59 pm
- computer charger left in Grainger
- CAs joining class next week
- discussion section format
- acceleration in tangential/normal basis
- tangential/normal basis in 3D
- roller coasters (video at t = 2:15)



grading based on participation

	Cartesian	Polar	Tang/Norm
position P	x, y	r, θ	s
position vec.	$\vec{r} = x\hat{\imath} + y\hat{\jmath}$	$\vec{r} = r\hat{e}_r$	—
velocity	$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	$\vec{v} = \dot{s}\hat{e}_t$
acceleration	$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$	$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$	$\vec{a} = ?$

Particle P moving along a path



Velocity $\vec{v}$ is always tangent to the path

$\hat{e}_t$ = tangential basis vector — tangential in direction of motion

$\hat{e}_n$ = normal basis vector — orthogonal, inwards direction

s = arclength

$\dot{s}$ = speed

$\hat{e}_t = \hat{v}$

$\dot{s} = v$

speed

$\vec{v} = \dot{s}\hat{e}_t$

curvature of path

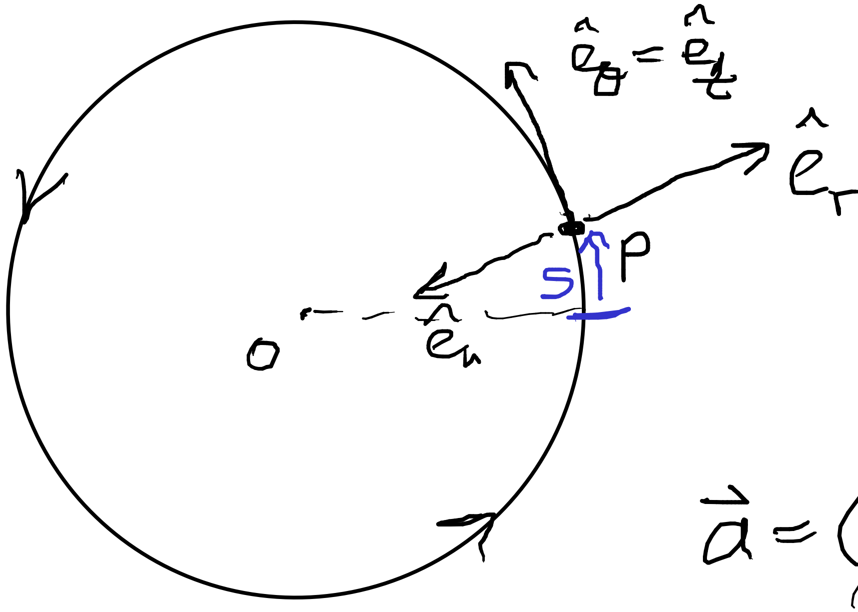
$\vec{a} = \ddot{s}\hat{e}_t + \dot{s}^2 \kappa \hat{e}_n$

$\kappa = \frac{1}{\rho}$

rate of change of speed

What is K?

circular motion:



$\hat{e}_t = e_\theta$

$\hat{e}_n = -\hat{e}_r$

$r = \text{const}$

$\dot{r} = \ddot{r} = 0$

$$\vec{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + \cancel{2\dot{r}\dot{\theta}})\hat{e}_\theta$$

$$\vec{a} = -r\dot{\theta}^2(-\hat{e}_n) + r\ddot{\theta}\hat{e}_t$$

$$= \underline{r\ddot{\theta}}\,\hat{e}_t + \underline{r\dot{\theta}^2}\,\hat{e}_n$$

$$\vec{a} = \underline{\ddot{s}\,\hat{e}_t} + \underline{\dot{s}^2 K}\,\hat{e}_n$$

relate s to r, θ:

$$s = r\theta$$

$$\dot{s} = \cancel{\dot{r}\theta} + r\dot{\theta}$$

$$\ddot{s} = r\ddot{\theta}$$

$$r\ddot{\theta} = \ddot{s}$$

$$r\dot{\theta}^2 = r\left(\frac{\dot{s}}{r}\right)^2 = \frac{\dot{s}^2}{r} = \dot{s}^2 \kappa$$

$$\boxed{\kappa = \frac{1}{r}}$$

Curvature

radius of curvature

$$\vec{v} = \dot{s}\,\hat{e}_t$$

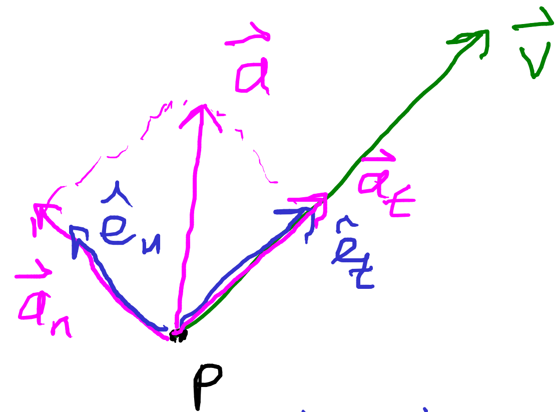
$$\vec{a} = \underbrace{\ddot{s}\,\hat{e}_t}_{\vec{a}_t} + \underbrace{\frac{\dot{s}^2}{\rho}\,\hat{e}_n}_{\vec{a}_n}$$

If we know $\vec{v}, \vec{a}$

How do we find $\dot{s}, \ddot{s}, \rho, \hat{e}_t, \hat{e}_n$

$$\dot{s} = v = \|\vec{v}\|$$

$$\hat{e}_t = \hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$$



$$\vec{a} = \vec{a}_t + \vec{a}_n$$

remainder ← complementary proj

component in $\hat{e}_t$ ← projection in $\vec{v}$

$$\vec{a}_t = \mathrm{proj}(\vec{a}, \vec{v})$$ ← causes speeding up or slowing down

$$\vec{a}_n = \mathrm{comp}(\vec{a}, \vec{v})$$ ← causes curvature

$$\hat{e}_n = \hat{a}_n = \frac{\vec{a}_n}{\|a_n\|} = \frac{\mathrm{comp}(\vec{a}, \vec{v})}{\|\mathrm{comp}(\vec{a}, \vec{v})\|}$$

$$\ddot{s} = a_t = \|\vec{a}_t\| = \|\operatorname{proj}(\vec{a}, \vec{v})\|$$

$$\frac{\dot{s}^2}{\rho} = a_n = \|\vec{a}_n\| = \|\operatorname{comp}(\vec{a}, \vec{v})\|$$

$\Rightarrow$ Solve for ρ

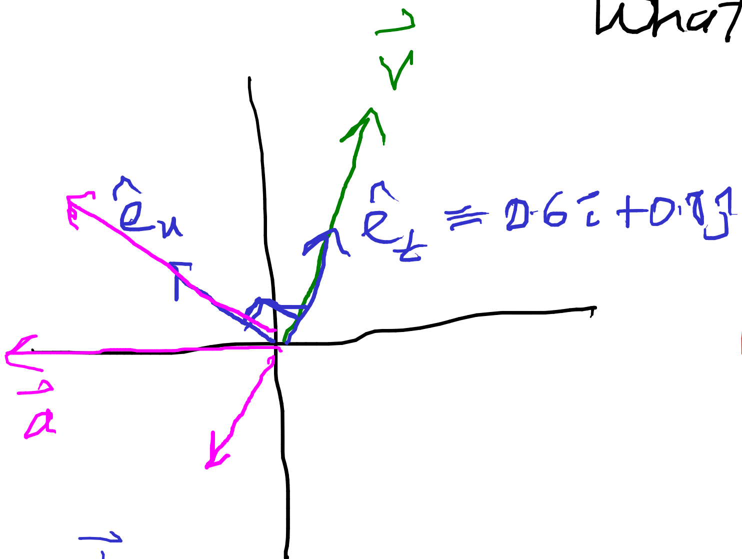
$$\rho = \frac{v^2}{a_n}$$

Ex

$$\vec{v} = 3\hat{i} + 4\hat{j} \ \ m/s$$

$$\vec{a} = -8\hat{i} \ \ m/s^2$$

what is $\hat{e}_n$?

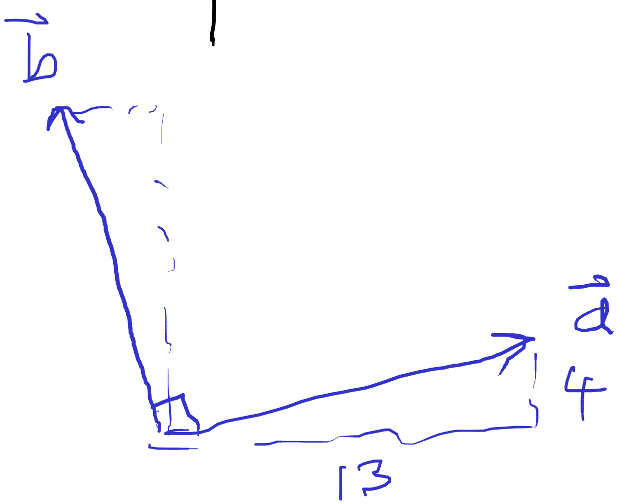


A. $-0.8\hat{i} - 0.6\hat{j}$

B. $-0.8\hat{i} + 0.6\hat{j}$

C. $-0.6\hat{i} + 0.8\hat{j}$

D. $-0.6\hat{i} - 0.8\hat{j}$



$$\vec{a} = 13\hat{i} + 4\hat{j}$$

$$\vec{b} \perp \vec{a}$$

$$\vec{b} = -4\hat{i} + 13\hat{j}$$