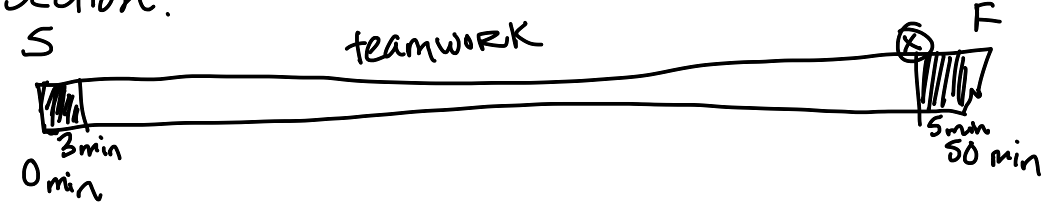
Discussion Section:

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Class 11 : T/N COORDS

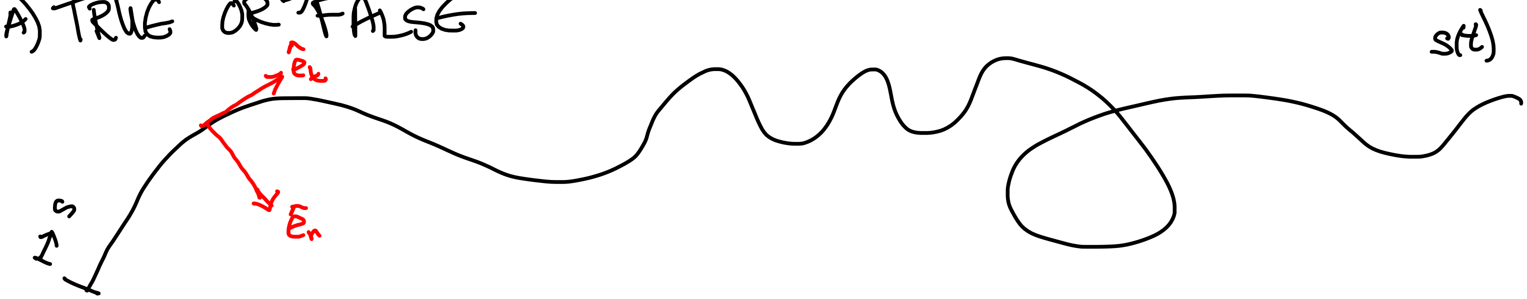
MADLIBS: T/N COORDS are not the same as POLAR COORDS

| | cartesians | polar | T/N |
|---|---|---|---|
| pos. | | | — — |
| vel. | | | $\vec{v} = \dot{s}\hat{e}_t$ |
| accel | | | $\vec{a} = \ddot{s}\hat{e}_t + \kappa(\dot{s})^2\hat{e}_n$ |

$\ddot{s}\hat{e}_t$: changes in magn. of $\vec{v}$

$\kappa(\dot{s})^2\hat{e}_n$: changes in direction of $\vec{v}$

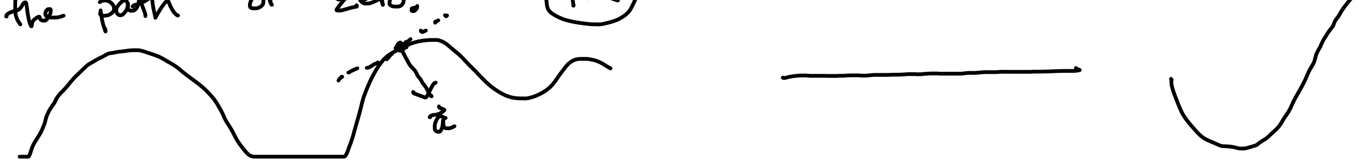
A) TRUE OR B) FALSE

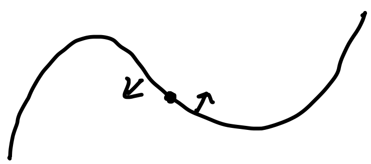



1) velocity of a moving particle is ALWAYS tangent to the path : TR

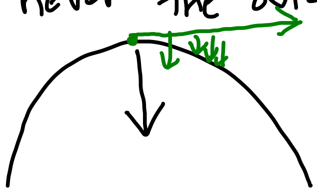
2) accel. of a moving particle is ALWAYS normal to the path: FA

3) accel. of a particle moving w/ constant speed is ALWAYS normal to the path or zero. TR

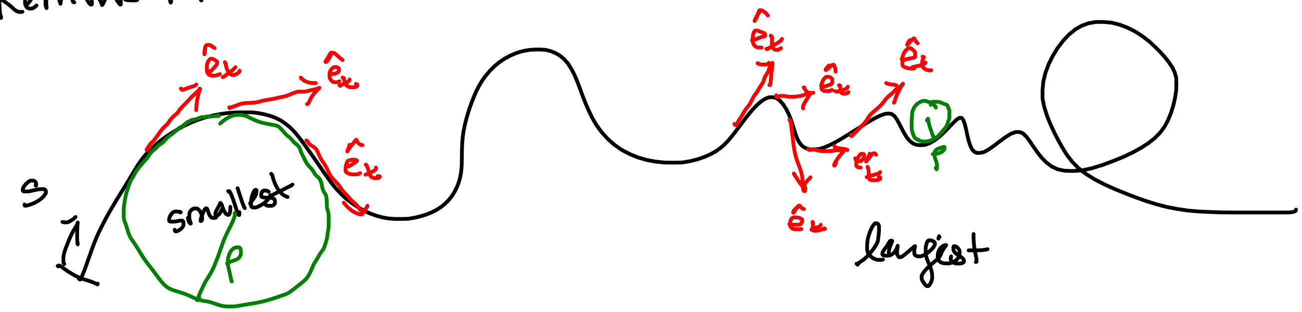




4) In classical dynamics the acceleration can point to the inside but never the outside of the curve. $\boxed{\text{TRUE}}$



Reminder:




## CURVATURE VECTOR:

$$\frac{d\hat{e}_t}{ds} = 0$$

$$\frac{d\hat{e}_t}{ds} = K\,\hat{e}_n$$

$K$ → magnitude: $K = \frac{1}{\rho}$

$\hat{e}_n$ → direction

$\vec{v} = \dot{s}\,\hat{e}_t$

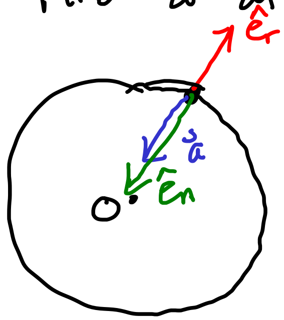
$$\boxed{\vec{a} = \ddot{s}\,\hat{e}_t + K(\dot{s})^2\,\hat{e}_n}$$

$\ddot{s}\,\hat{e}_t$ → parallel to $\vec{v}$

$K(\dot{s})^2\,\hat{e}_n$ → perp. to vel.

$$K(\dot{s})^2 \iff \frac{1}{r}v^2 = \frac{v^2}{r}$$

ex) particle moves on a circular path, constant ang. velocity $\omega$.
Find $\vec{a}$ at the moment shown in polar and T/N coords.




**POLAR**

$$\vec{r} = r\hat{e}_r$$

$$\vec{v} = \cancel{\dot{r}\hat{e}_r} + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\dot{\hat{e}}_\theta$$

$$= \cancel{r\ddot{\theta}\hat{e}_\theta} - r\dot{\theta}^2\hat{e}_r$$

$$= -r\dot{\theta}^2\hat{e}_r$$

$$= -r\omega^2\hat{e}_r$$

**T/N COORDS**

$$\vec{a} = \cancel{\ddot{s}\hat{e}_t + \frac{1}{r}\dot{s}^2}$$

$$\vec{a} = \cancel{\ddot{s}\hat{e}_t} + \frac{1}{r}\dot{s}^2\hat{e}_n$$

$$= \frac{\dot{s}^2}{r}\hat{e}_n$$

$$= \frac{v^2}{r}\hat{e}_n$$

$$= \frac{(\omega r)^2}{r}\hat{e}_n$$

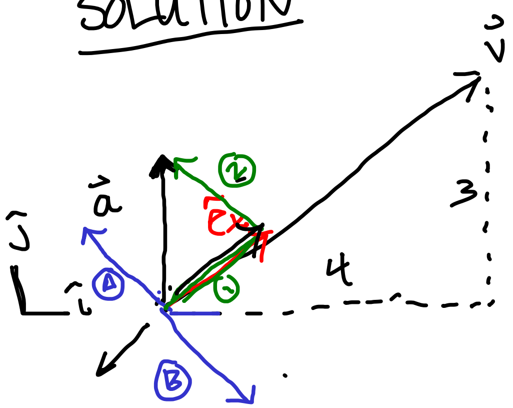
$$= r\omega^2\hat{e}_n$$

ex) A satellite tracks a bicycle moving w/ velocity $\vec{v} = 4\hat{\imath} + 3\hat{\jmath}$ m/s and $\vec{a} = 2\hat{\jmath}$ m/s$^2$.

Find: * $\hat{e}_t$, $\hat{e}_n$, radius of curvature of path ($\rho$)

* sketch bicycle's trajectory at the current instant
* is the cyclist speeding up or slowing down? SPEEDING UP $\vec{a} \cdot \vec{v} > 0$

SOLUTION




$\hat{e}_t$ : $\vec{v} = \dot{s}\,\hat{e}_t = v\,\hat{e}_t$

$$\hat{e}_t = \cancel{\frac{\vec{v}}{v}} = \frac{1}{5}(4\hat{\imath} + 3\hat{\jmath})$$

$\hat{e}_n$ : $\perp$ to $\hat{e}_t$

either: $\boxed{\frac{1}{5}(-3\hat{\imath} + 4\hat{\jmath}) \quad \text{A}}$ $\hat{e}_n$

OR $\frac{1}{5}(3\hat{\imath} - 4\hat{\jmath})$ B

$$\vec{a} = \underbrace{\ddot{s}\,\hat{e}_t}_{1} + \underbrace{\kappa(\dot{s})^2}_{2}\,\hat{e}_n$$

$$= \ddot{s}\,\hat{e}_t + \frac{1}{\rho}(\dot{s})^2\,\hat{e}_n$$

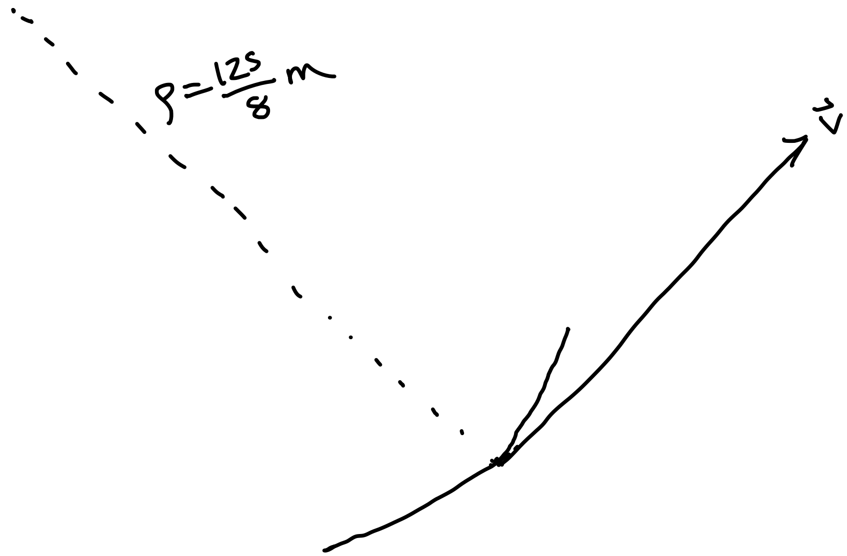
$$= a_t\,\hat{e}_t + a_n\,\hat{e}_n$$

$$\vec{a} \cdot \hat{e}_n = a_n = \frac{1}{\rho}(\dot{s})^2$$

$$(2\hat{\jmath}) \cdot \frac{1}{5}(-3\hat{\imath} + 4\hat{\jmath}) = \frac{1}{\rho}(5)^2$$

$$8/5 = \frac{1}{\rho}(25)$$

$$\rho = \frac{125}{8} \text{ m}$$



$\rho = \frac{125}{8}$ m
$\vec{v}$