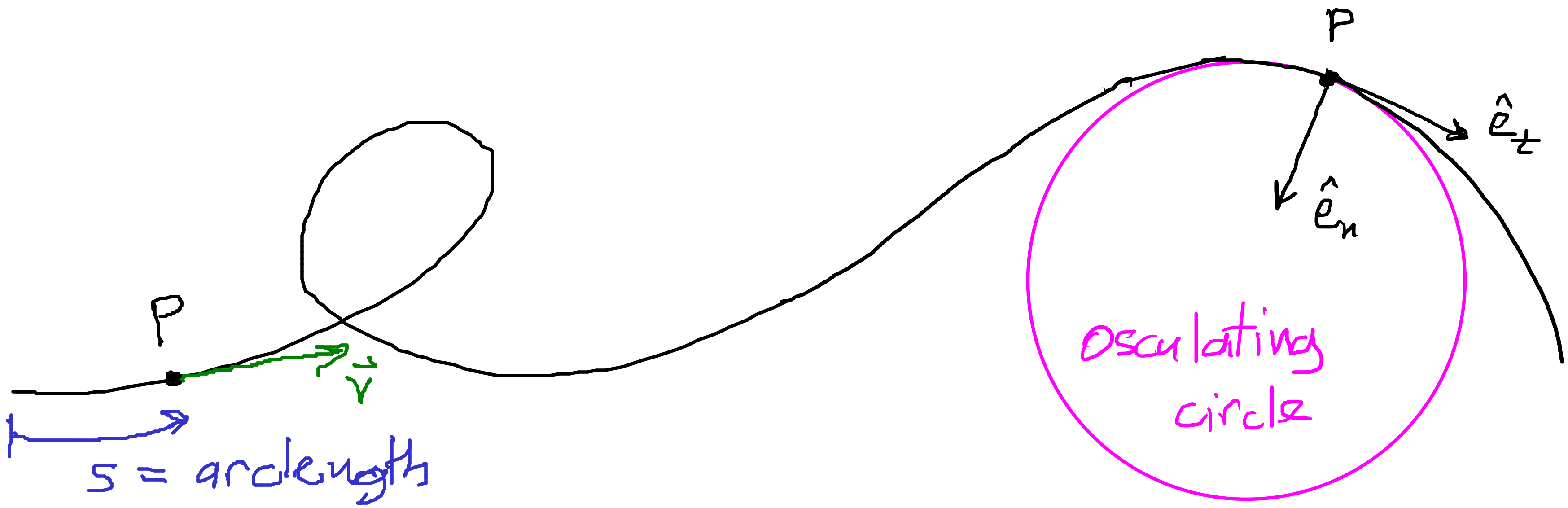
Recap

	Cartesian	Polar	Tang/Norm
position P	x, y	r, θ	s
position vec.	$\vec{r} = x\hat{i} + y\hat{j}$	$\vec{r} = r\hat{e}_r$	—
velocity	$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	$\vec{v} = \dot{s}\hat{e}_t$
acceleration	$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$	$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$	$\vec{a} = ?$

Particle P moving along a path



Velocity $\vec{v}$ is always tangent to the path

$\hat{e}_t$ = tangential basis vector — tangential in direction of motion

$\hat{e}_n$ = normal basis vector — orthogonal, inwards direction

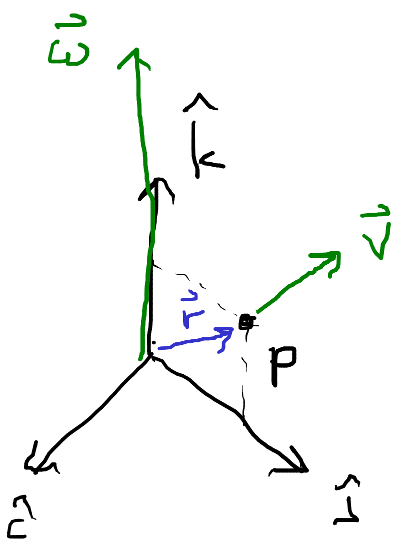
s = arclength

$\dot{s}$ = speed

$$\vec{v} = \dot{s}\,\hat{e}_t \qquad \hat{e}_t = \hat{v}$$

Today:
- angular velocity and acceleration vectors
- direction of acceleration
- acceleration in tang/norm basis

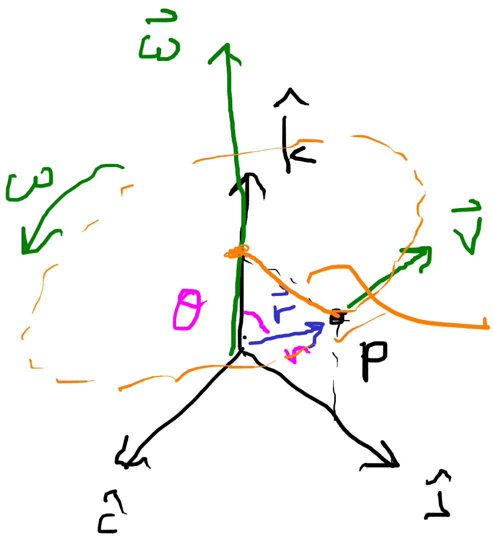
Angular velocity and acceleration for a particle



$\vec{\omega}$ = angular velocity

$$\vec{\omega} \begin{cases} \omega = \text{rate of rotation} \\ \hat{\omega} = \text{axis of rotation} \\ \quad \text{(orthogonal to plane of rotation)} \end{cases}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$



$\vec{v} = \vec{\omega} \times \vec{r}$

effective radius of rotation

Ex

$\vec{r} = 4\hat{j} + 3\hat{k}$

$\vec{\omega} = 3\hat{k}$

what is $\vec{v}$?

A. $15\,\hat{i}$ m/s

B. $-15\,\hat{i}$ m/s

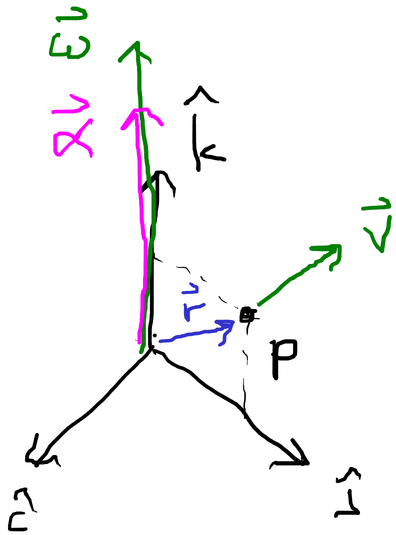
C. $12\,\hat{i}$ m/s

D. $-12\,\hat{i}$ m/s

$$\vec{v} = \vec{\omega} \times \vec{r}$$
$$= 3\hat{k} \times (4\hat{j} + 3\hat{k})$$
$$= 12\,\underbrace{\hat{k} \times \hat{j}}_{-\hat{i}} + \cancel{9\,\hat{k} \times \hat{k}}$$

$v = \omega(r\sin\theta)$

$v = \omega r \sin\theta$



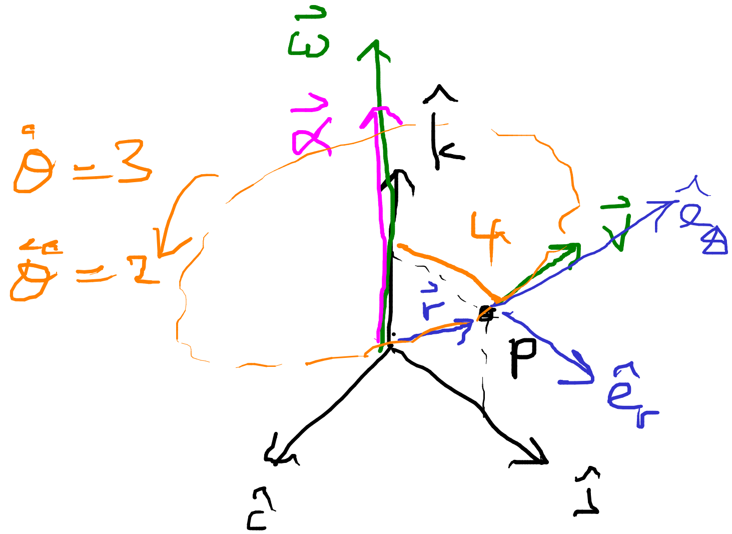
$\vec{\alpha}$ = angular acceleration vector

$\vec{\alpha} = \dot{\vec{\omega}}$

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

$$\vec{a} = \dot{\vec{v}} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \underbrace{\dot{\vec{\omega}}}_{\vec{\alpha}} \times \vec{r} + \vec{\omega} \times \underbrace{\dot{\vec{r}}}_{\vec{\omega} \times \vec{r}}$$

$$\boxed{\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})}$$



angular   centripetal

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \vec{\alpha} \times \vec{r} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})}$$

$$3\hat{k} \times (3\hat{k} \times (4\hat{j} + \cancel{3\hat{k}}))$$

$$= 3\hat{k} \times (-12\hat{i})$$

$$= -36\, \underbrace{\hat{k} \times \hat{i}}_{+\hat{j}}$$

Ex

$$\vec{r} = 4\hat{j} + 3\hat{k}$$

$$\vec{\omega} = \underbrace{3\hat{k}}_{\dot{\theta}}$$

$$\vec{\alpha} = \underbrace{2\hat{k}}_{\ddot{\theta}}$$

what is $\vec{a}$?

$r\ddot{\theta}$ $-r\dot{\theta}^2$

A. $8\hat{i} + 6\hat{j}$ m/s²

B. $-8\hat{i} - 36\hat{j}$ m/s²

C. $-8\hat{i}$ m/s²

D. $-12\hat{i} - 12\hat{j}$ m/s²

E. $-12\hat{i}$ m/s²

Particle P moving along a path

Velocity $\vec{v}$ is always tangent to the path

$\hat{e}_t$ = tangential basis vector — tangential in direction of motion

$\hat{e}_n$ = normal basis vector — orthogonal, inwards direction

s = arclength

$\dot{s}$ = speed

$$\vec{v} = \dot{s}\,\hat{e}_t$$

$$\hat{e}_t = \hat{v}$$

$$\dot{s} = v$$

$$\vec{v} = \dot{s}\hat{e}_t$$

$$\vec{a} = \dot{\vec{v}} = \ddot{s}\,\hat{e}_t + \dot{s}\,\dot{\hat{e}}_t$$

rate of change of speed

$$\frac{d}{dt}\hat{e}_t = \frac{d\hat{e}_t}{ds}\underbrace{\frac{ds}{dt}}_{\dot{s}}$$

$$\vec{a} = \ddot{s}\,\hat{e}_t + \dot{s}^2\frac{d\hat{e}_t}{ds}$$

in the direction $\hat{e}_n$

say $\kappa\hat{e}_n$

kappa

$$\boxed{\vec{a} = \ddot{s}\,\hat{e}_t + \kappa\dot{s}^2\,\hat{e}_n}$$

$$\boxed{\kappa = \text{curvature of the path}}$$