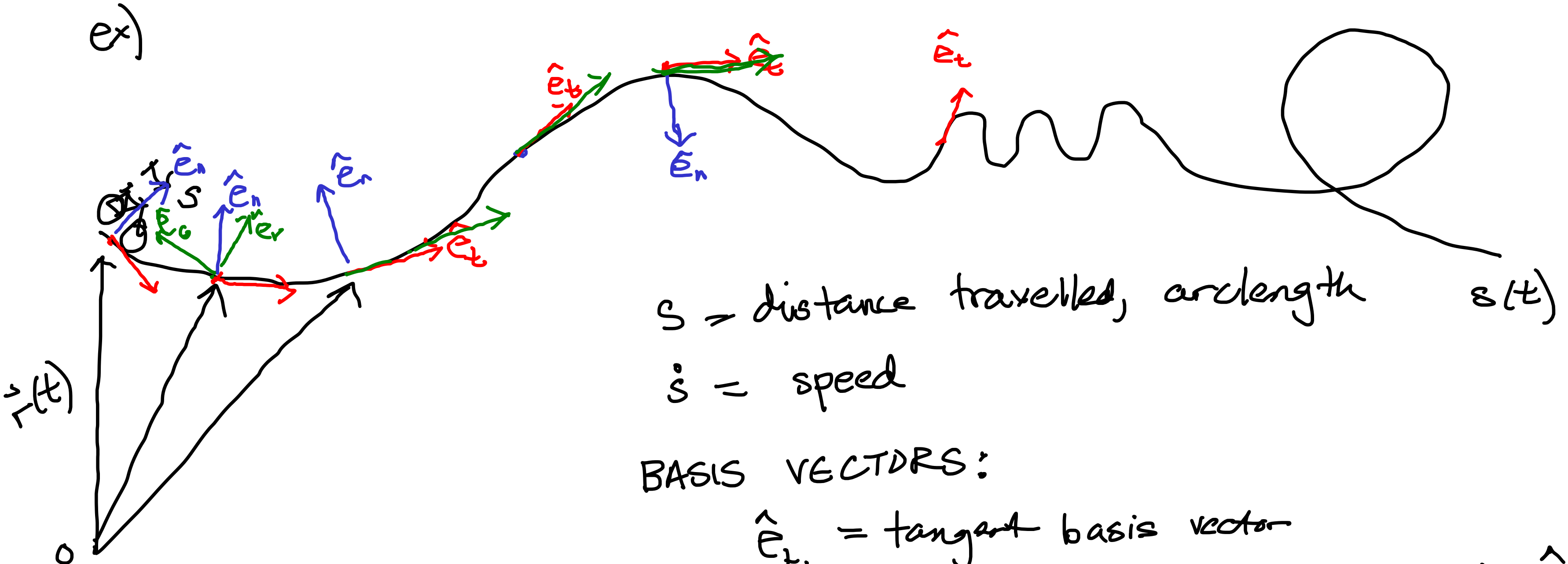
TAM 212 : Class 10 TANG/NORM COORDS

Review

	cartesian	polar	T/N
coords	x, y	r, θ	s (sort of)
pos	$x\hat{\imath} + y\hat{\jmath}$	$r\hat{e}_r$	—
vel	$\dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$	$\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	$\vec{v} = \dot{s}\hat{e}_t$
accel.	$\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$	$(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$	$\vec{a}$

T/N:
* defined by the path entirely
* NOT THE SAME AS POLAR COORDINATES!!

ex)



s = distance travelled, arclength $\quad s(t)$

$\dot{s}$ = speed

BASIS VECTORS:

$\hat{e}_t$ = tangent basis vector

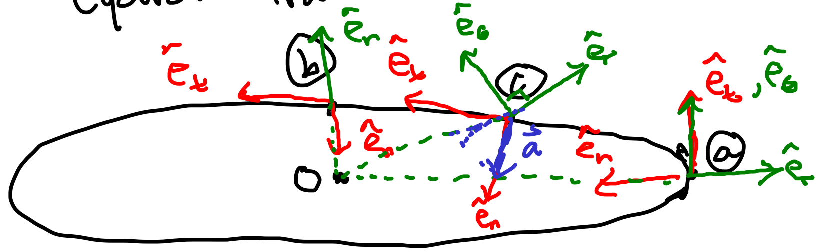
$\hat{e}_n$ = normal basis vector, $\perp$ to $\hat{e}_t$ and points to inside of curve ALWAYS

$$\vec{v} = \dot{s}\,\hat{e}_t = v\,\hat{e}_t$$

$$\hat{e}_t = \frac{\vec{v}}{v}$$

$\vec{v}$ is ALWAYS tangent to the path

ex) Cyclist travels around oval track, at a constant speed



TRUE OR FALSE
A B

1) At point ⓐ $\quad \hat{e}_t = \hat{e}_\theta \quad$ TRUE

2) At point ⓑ $\quad \hat{e}_r = \hat{e}_n \quad$ FALSE

(TRUE $\hat{e}_r = -\hat{e}_n$)

3) At point ⓐ, $\vec{a} \perp \vec{v}$ $\quad$ TRUE

4) At point ⓒ, $\vec{v}$ has both $\hat{e}_r$ and $\hat{e}_\theta$ comp. $\quad$ TRUE

5) At point Ⓒ, $\vec{a}$ is $\perp$ to $\vec{v}$ TRUE

6) At point Ⓒ, $\vec{a}$ has both $\hat{e}_r$ and $\hat{e}_\theta$ comp. TRUE

Acceleration

$\rightarrow$ cyclist travels at constant speed $\dot{s}$ $\Rightarrow$ $\vec{a}$ is always along $\hat{e}_n$

$\rightarrow$ where is the magnitude of $\vec{a}$ largest? Largest where the curvature is largest.

define CURVATURE (is a vector) $\equiv \dfrac{d\hat{e}_t}{ds}$ how much the tangent vector changes direction with distance travelled.

Q: What is the direction of curvature $\dfrac{d\hat{e}_t}{ds}$?

A: Direction is $\hat{e}_n$

Q: What is the magnitude of $\dfrac{d\hat{e}_t}{ds}$?

A: $K \equiv \dfrac{1}{\rho}$

$\rho \equiv$ radius of "best fit" circle

OSCULATING circle

$$\text{CURVATURE VECTOR} \quad \frac{d\hat{e}_t}{ds} = K\hat{e}_n = \frac{1}{\rho}\hat{e}_n$$

Now we're ready:

$$\vec{v} = \dot{s}\,\hat{e}_t$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\dot{s}\hat{e}_t\right)$$

$$= \ddot{s}\hat{e}_t + \dot{s}\,\dot{\hat{e}}_t$$

Note: $\dot{\hat{e}}_t = \frac{d\hat{e}_t}{dt} = \frac{d\hat{e}_t}{ds}\frac{ds}{dt} = (\kappa\hat{e}_n)\,\dot{s} = \dot{s}\kappa\hat{e}_n$

$$\boxed{\vec{a} = \ddot{s}\hat{e}_t + (\dot{s})^2\kappa\hat{e}_n}$$

Accel. in T/N coords.

$\ddot{s}\hat{e}_t$ → parallel to $\vec{v}$: changes to speed

$(\dot{s})^2\kappa\hat{e}_n$ → perpendicular to $\vec{v}$: changes in direction of $\vec{v}$