TAM 212 Class 9: Tangential/Normal Coords.

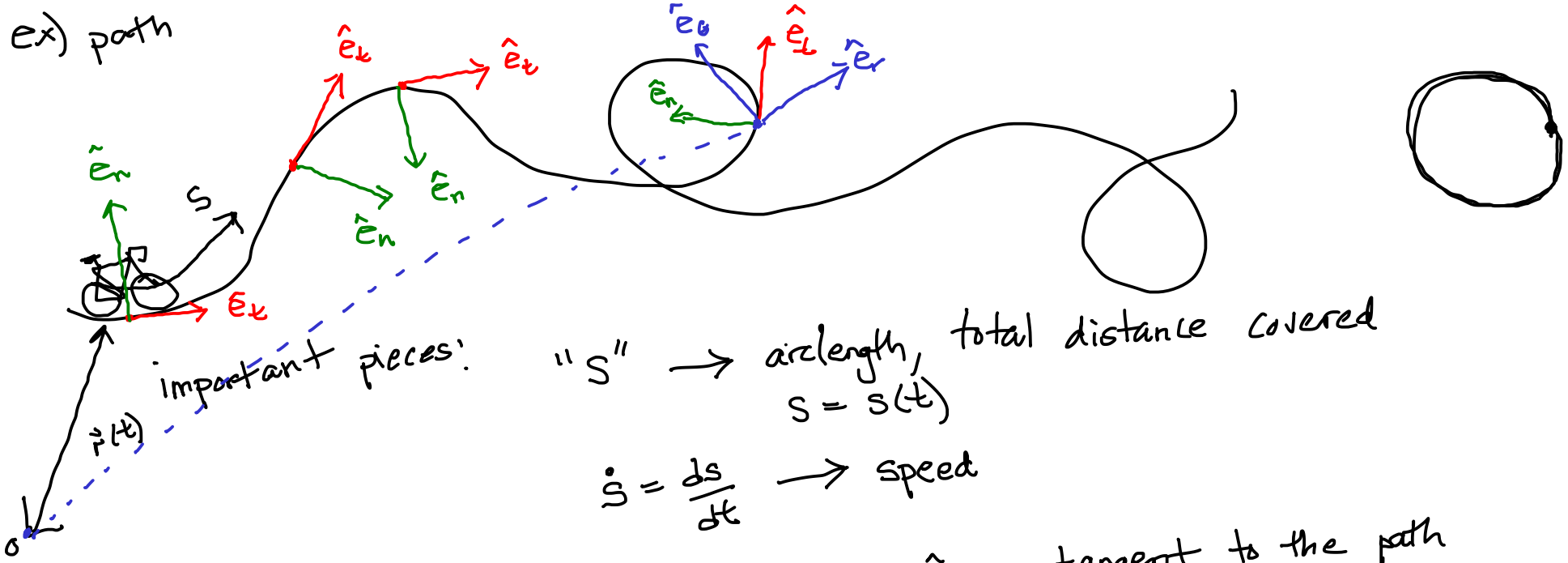
Review: 2D coords

	cartesian	polar	tangential/normal
coords	x, y	r, θ	s
pos	$\vec{r} = x\hat{\imath} + y\hat{\jmath}$	$\vec{r} = r\hat{e}_r$	—
vel.	$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	$\vec{v} = \dot{s}\hat{e}_t$
accel.	$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$	$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$	

TANG/NORMAL:

* are defined by the trajectory of the position vector (the path)
* ARE NOT THE SAME AS POLAR COORDINATES!!

ex) path



Important pieces:

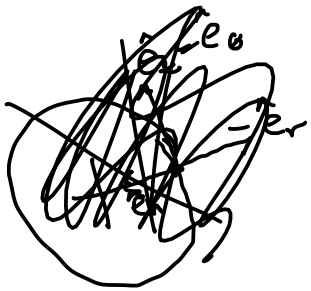
"s" → arclength, total distance covered

$s = s(t)$

$\dot{s} = \frac{ds}{dt}$ → speed

Basis Vectors: $\hat{e}_t$ = tangent to the path

$\hat{e}_n$ = normal to the path, always points to the INSIDE of curve



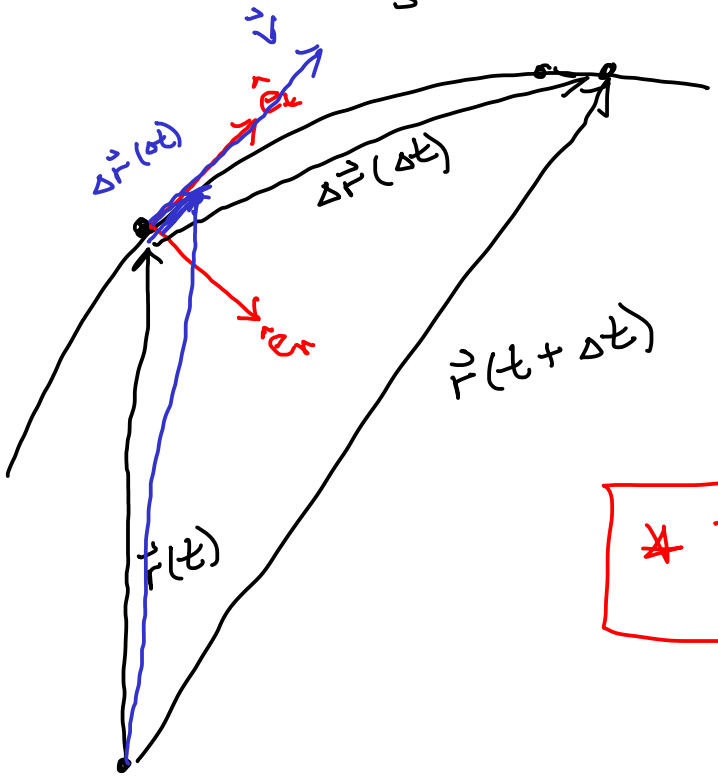
Velocity Vector in T/N?

$$\vec{v} = v\,\hat{v}$$

- v: magnitude, $\dot{s}$
- $\hat{v}$: direction, $\hat{e}_t$

$$\boxed{\vec{v} = \dot{s}\,\hat{e}_t}$$

also: $\hat{e}_t = \dfrac{\vec{v}}{v} = \hat{v}$



as $\Delta t \to 0$

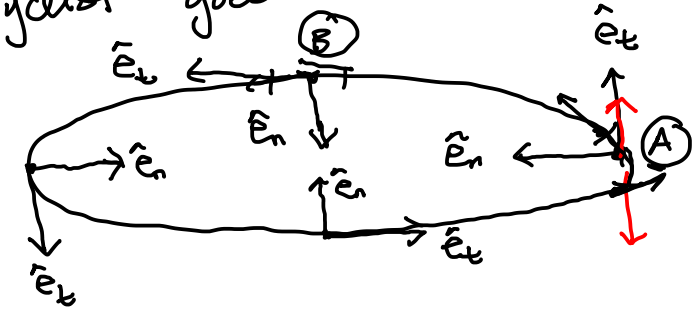
1) $\Delta\vec{r}(\Delta t)$ becomes tangent to the path (along $\hat{e}_t$)

2) $|\Delta\vec{r}|/\Delta s = 1$

* The velocity is always tangent to the path

iclicker:

Cyclist goes around an oval track at constant speed.



The acceleration $\vec{a}$ has components:

A) only along $\hat{e}_t$

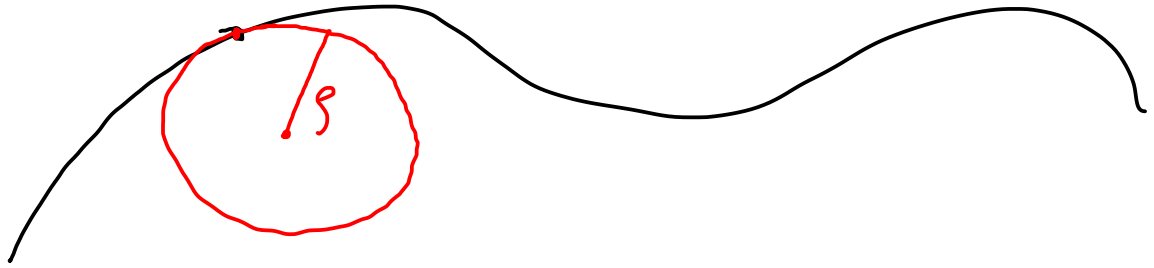
B) only along $\hat{e}_n$

C) along both

Is the magnitude of the acceleration larger at (A) or (B)

Magnitude of the acceleration is larger when the curvature of the path is larger.

Curvature:



best fit circle
"osculating circle"

radius ρ

$|\text{curvature}| = \kappa = 1/\rho$