# TAM 212 Class 8: Acceleration Vector Examples

iclicker question:

A) TRUE

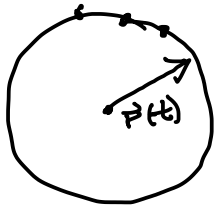
B) FALSE

C) WTF?

$\dot{r} = \dot{\vec{r}}$ ?

$\frac{d}{dt}(r)$ $\quad$ $\frac{d}{dt}(\vec{r})$

$\frac{d}{dt}(\|\vec{r}\|)$ $\quad$ $\frac{d}{dt}(r\hat{e}_r)$

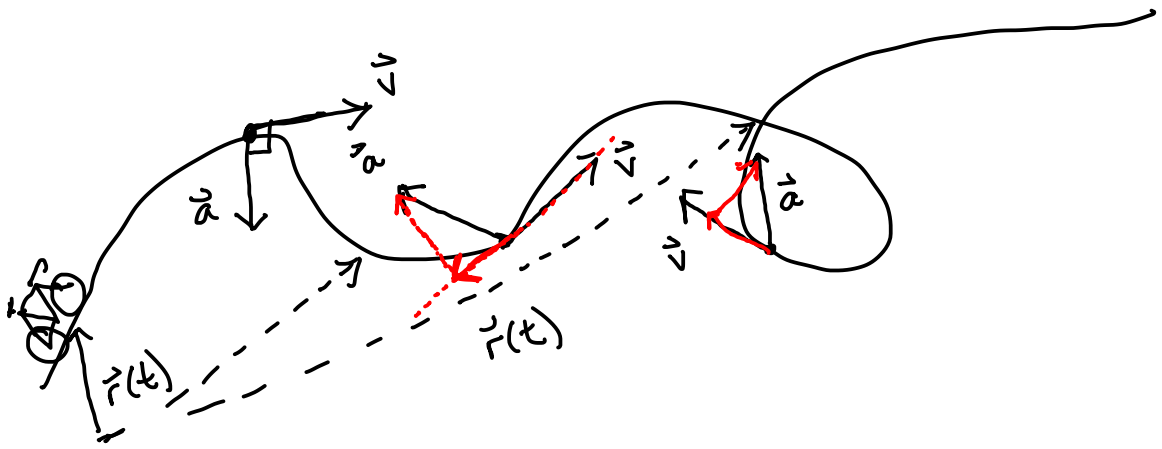



$\dot{r} = 0$

$\dot{\vec{r}} = \dot{\vec{v}} \neq 0$

| | cartesian | polar |
|---|---|---|
| coords | $x, y$ | $r, \theta$ |
| pos | $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ | $\vec{r} = r\hat{e}_r$ |
| vel | $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$ | $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ |
| accel. | $\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$ | $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$ |

$$\text{speed} = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$$



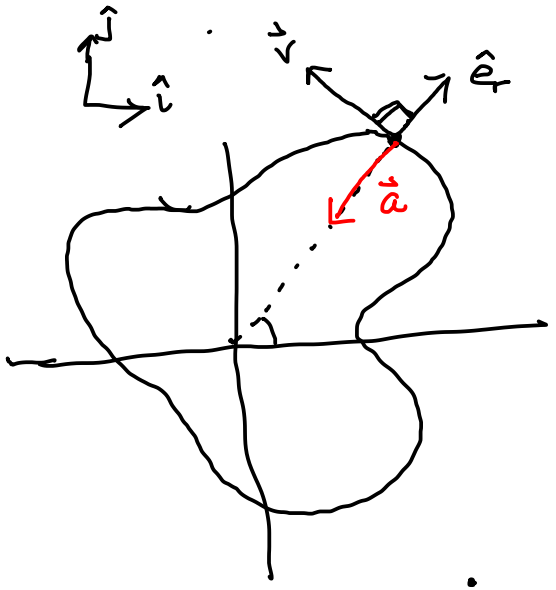

At the instant shown, the cyclist is

A) speeding up

B) slowing down

C) staying the same

**Example:** A particle P moves at constant speed in the plane as shown. At this instant, the angular velocity of P is $\omega = -2$ rad/sec. The accel. vector is $\vec{a} = a_x \hat{\imath} + a_y \hat{\jmath}$ $m/s^2$.




Which is true?

A) $a_x < 0$ $m/s^2$, $a_y < 0$ $m/s^2$

B) $a_x < 0$ $m/s^2$, $a_y > 0$ $m/s^2$

C) $a_x > 0$ $m/s^2$, $a_y < 0$ $m/s^2$

D) $a_x > 0$ $m/s^2$, $a_y > 0$ $m/s^2$

particle $\vec{r} = r\hat{e}_r$

$\vec{v} = \cancel{\dot{r}\hat{e}_r} + r\dot{\theta}\hat{e}_\theta \Rightarrow \boxed{\dot{r} = 0}$

$\vec{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\hat{e}_r + (\cancel{r\ddot{\theta}} + \cancel{2\dot{r}\dot{\theta}})\hat{e}_\theta$

$\vec{a} = -r\dot{\theta}^2\hat{e}_r$

constant speed $\Rightarrow$ $r\dot{\theta} = \text{constant}$

$\frac{d}{dt}(r\dot{\theta}) = 0$

$\cancel{\dot{r}\dot{\theta}} + r\ddot{\theta} = 0$

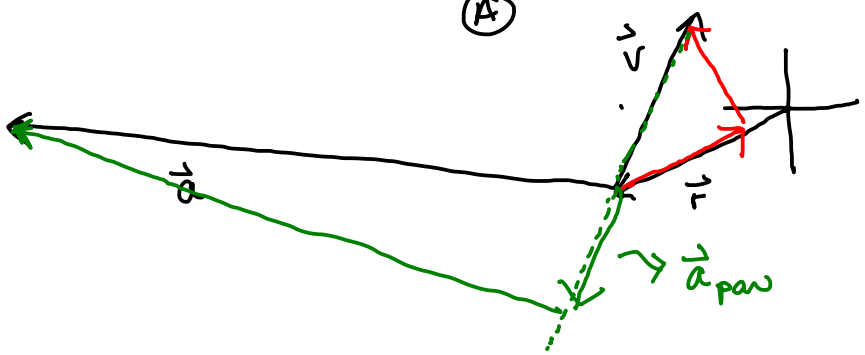
Example) A particle P has position, velocity, and acceleration given by

$$\vec{r} = -3\hat{i} - 2\hat{j} \text{ m}$$
$$\vec{v} = \hat{i} + 4\hat{j} \text{ m/s}$$
$$\vec{a} = -9\hat{i} + 2\hat{j} \text{ m/s}^2$$

(1) Is the particle moving closer to (A) or further away (B) from the origin?

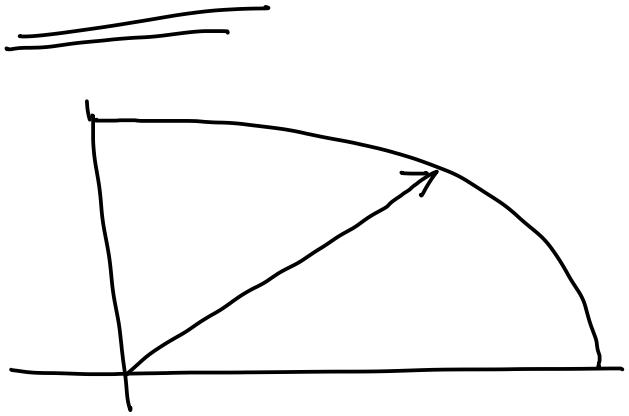
$\vec{r} \cdot \vec{v} < 0$

(2) Is it speeding up (A) or slowing down (B)?




$\|\vec{a}_{par}\| = \text{Proj}(\vec{a}, \vec{v})$

$\vec{a} \cdot \vec{v} < 0$




$$\vec{r} = r\hat{e}_r$$
$$\dot{\vec{r}} = \frac{d}{dt}\left[r(\theta)\,\hat{e}_r\right]$$
$$= \frac{d}{dt}\left[r(\theta)\right]\hat{e}_r + r(\theta)\,\dot{\hat{e}}_r$$
$$= \frac{dr}{d\theta}\,\dot{\theta}\,\hat{e}_r + r(\theta)\,\dot{\hat{e}}_r$$