Recap

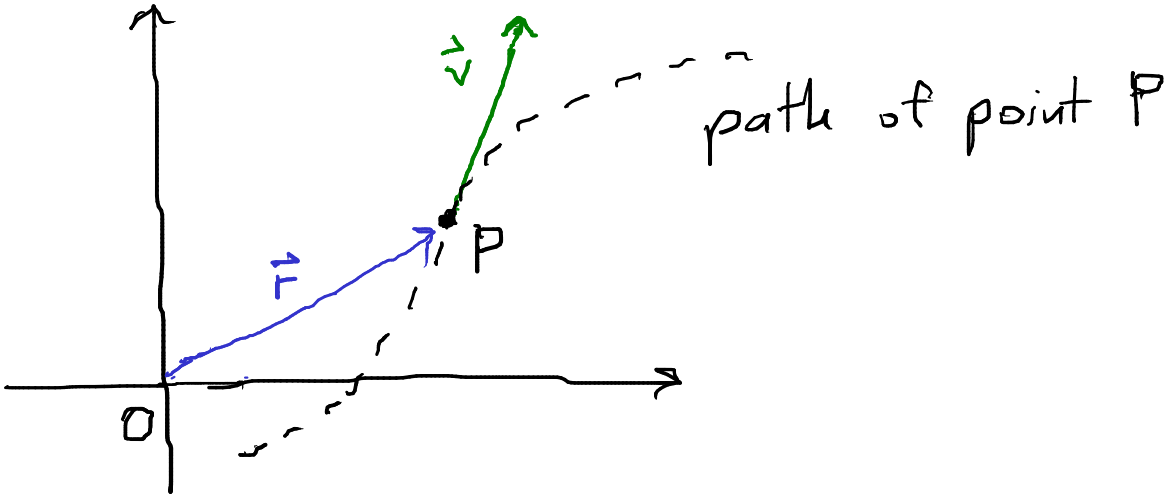
	Cartesian	Polar
position P	x, y	r, θ
position vec.	$\vec{r} = x\hat{i} + y\hat{j}$	$\vec{r} = r\hat{e}_r$
velocity	$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$
acceleration	$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$	$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

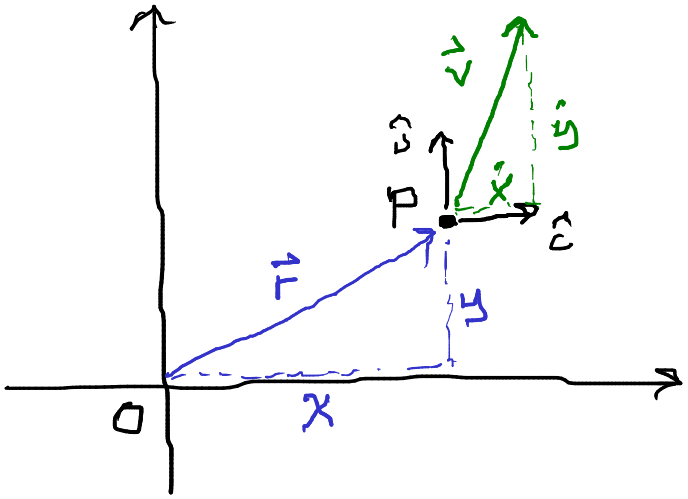
radial

centripetal

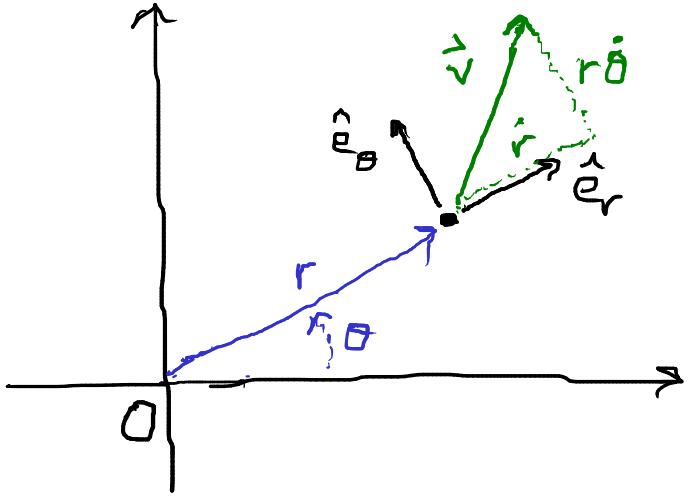
angular

Coriolis





Cartesian



Polar

Today:
- cylindrical and spherical coordinates
- integrating vectors
- rocket launch example
- graphical acceleration
- motion on a path (chain rule)

ex A particle P has velocity

$$\vec{v}(t) = 6t\,\hat{i} - 2\hat{j} \text{ m/s}$$

and at time $t = 1$ s it has position vec

$$\vec{r}(1) = 4\hat{j} \text{ m}$$

where is the particle at $t = 2$ s?

$\vec{r}(2) =$

A. $2\hat{i}$ m

B. $20\hat{i} - 2\hat{j}$ m

C. $25\hat{i} + \hat{j}$ m

D. $24\hat{i} - 4\hat{j}$ m

E. ~~$23\hat{i} + 2\hat{j}$ m~~

$4\hat{i} + 2\hat{j}$ m

$\vec{a} = \vec{v}(t) = 6t\,\hat{i} - 2\,\hat{j}$ m/s² $\hat{i}, \hat{j}$ constant in t

$\vec{r}(1) = 4\,\hat{j}$ m

$\vec{r}(t) = (3t^2 + C_1)\,\hat{i} + (-2t + C_2)\,\hat{j}$ ↓ int + C

$\vec{r}(1) = 4\,\hat{j} = (3 + C_1)\,\hat{i} + (-2 + C_2)\,\hat{j}$ ↓ 1 vec eqn ⇒ 2 scal eqn

$\hat{i}: \; 0 = 3 + C_1 \Rightarrow C_1 = -3$

$\hat{j}: \; 4 = -2 + C_2 \Rightarrow C_2 = 6$ ↓ solve for C

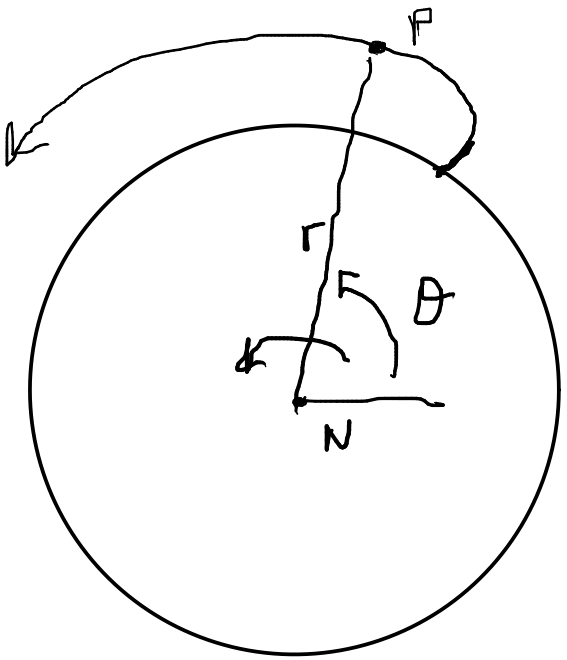
$\vec{r}(t) = (3t^2 - 3)\,\hat{i} + (-2t + 6)\,\hat{j}$

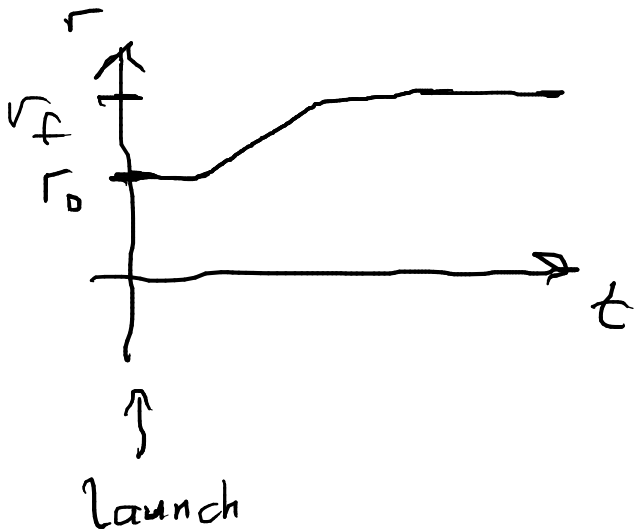
$\vec{r}(2) = 9\,\hat{i} + 2\,\hat{j}$ m

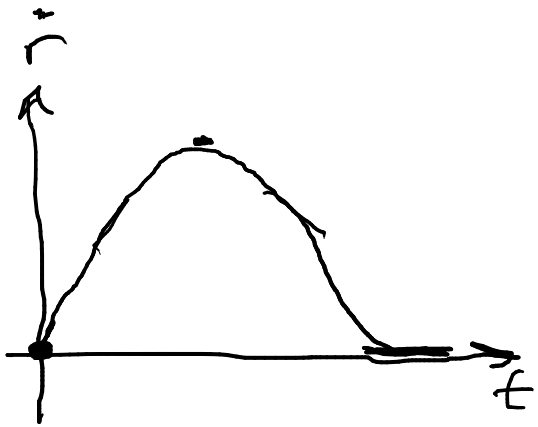
Rocket launches to orbit

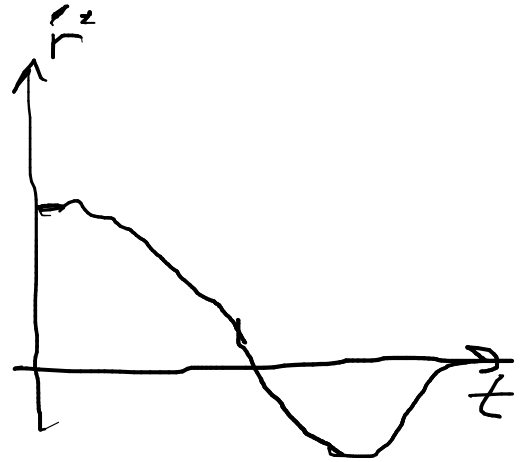
A. CCW

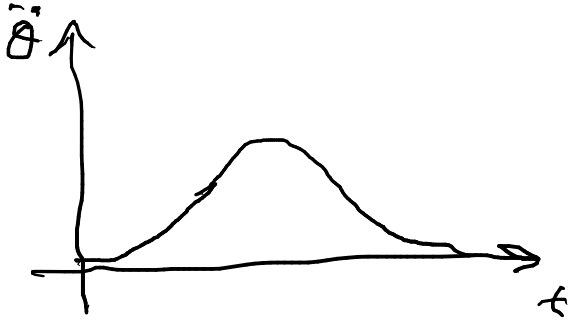
B. CW











$r_0 = 6400$ km

$r_f = 6600$ km

$\omega_0 \approx$

A. 10^{-8} rad/s

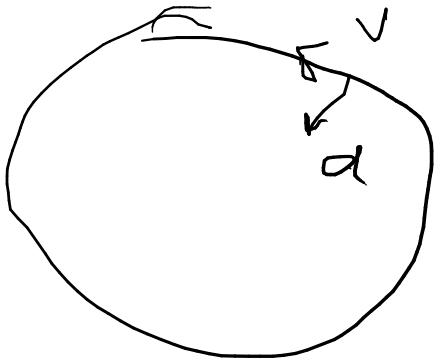
B. 10^{-6} rad/s

C. 10^{-4} rad/s

D. 10^{-2} rad/s

E. 1 rad/s

$$\omega = \frac{2\pi}{24\text{h}} = 7.3 \times 10^{-5} \text{ rad/s}$$



$\omega_f \approx$

A. 10^{-7} rad/s

B. 10^{-5} rad/s

C. 10^{-3} rad/s

D. 10^{-1} rad/s

E. 10 rad/s