TAM 212 Class 7 : Acceleration Vectors

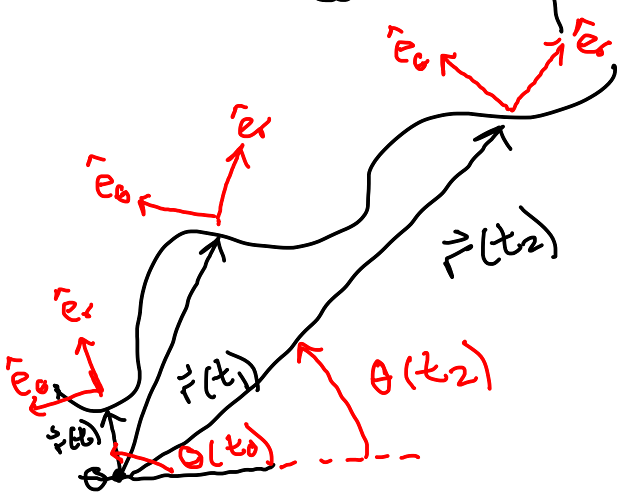
ANNOUNCEMENTS

1. PL2 due Friday 11:59pm
   Report 2 due Monday 11:59 pm, PL 2-13
2. James Scholar Honors:
   F 4-5pm MEL 2005

Review: in 2D

	cartesian	polar
position	$\vec r = x\hat\imath + y\hat\jmath$	$\vec r = r\hat e_r$
velocity	$\vec v = \dot x\hat\imath + \dot y\hat\jmath$	$\vec v = \dot r\hat e_r + r\dot\theta\,\hat e_\theta$
accel.	$\vec a = \ddot x\hat\imath + \ddot y\hat\jmath$	$\boxed{\vec a} = (\ddot r - r\dot\theta^2)\hat e_r + (r\ddot\theta + 2\dot r\dot\theta)\hat e_\theta$

$$\vec a = (\underbrace{\ddot r}_{\text{radial}} - \underbrace{r\dot\theta^2}_{\text{centripetal "center-seeking"}})\hat e_r + (\underbrace{r\ddot\theta}_{\text{angular}} + \underbrace{2\dot r\dot\theta}_{\text{coriolis}})\hat e_\theta$$



How did we obtain $\vec a$? differentiate $\vec v$

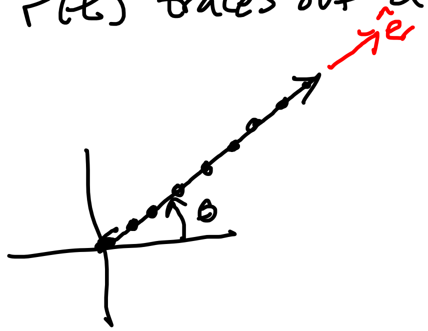
used: $\hat e_r = \cos\theta\,\hat\imath + \sin\theta\,\hat\jmath \Rightarrow$

$\hat e_\theta = -\sin\theta\,\hat\imath + \cos\theta\,\hat\jmath \Rightarrow$

$$\boxed{\begin{aligned}\dot{\hat e}_r &= \dot\theta\,\hat e_\theta \\ \dot{\hat e}_\theta &= -\dot\theta\,\hat e_r\end{aligned}}$$

What are all the terms in $\vec{a}$? (polar)

ex) $\vec{r}(t)$ traces out a straight line at constant θ

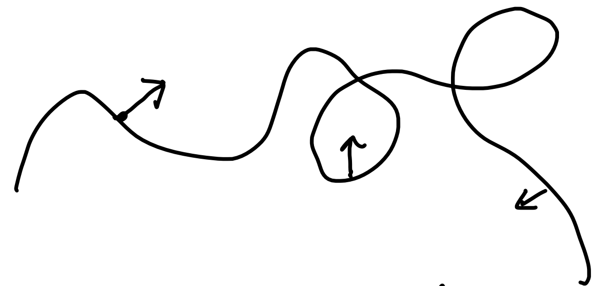


$\vec{r} = r\hat{e}_r$

$\vec{v} = \dot{r}\hat{e}_r + \cancel{r\dot{\theta}\hat{e}_\theta}$

$\vec{a} = \ddot{r}\,\hat{e}_r + \cancel{\dot{r}}\,\dot{\hat{e}}_r$

$\vec{a} = \ddot{r}\hat{e}_r$ radial



ex) $\vec{r}(t)$ traces out circular path at constant speed

$r = \text{constant}$
$\dot{\theta} = \text{constants}$

CCW
$\dot{\theta} > 0$

$\vec{r} = r\hat{e}_r$

$\vec{v} = \cancel{\dot{r}\hat{e}_r} + r\dot{\theta}\,\hat{e}_\theta$

$\vec{a} = \frac{d}{dt}(r\dot{\theta})\hat{e}_\theta + (r\dot{\theta})\dot{\hat{e}}_\theta$

$= (\cancel{\dot{r}\dot{\theta}} + \cancel{r\ddot{\theta}})\hat{e}_\theta + (r\dot{\theta})(-\dot{\theta}\hat{e}_r)$

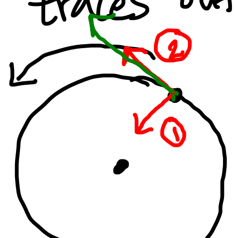
$= -r\dot{\theta}^2\,\hat{e}_r$ centripetal

$r\omega^2$

ex) $\vec{r}(t)$ traces out a circular path at increasing speed

$r = \text{constants}$
$\dot{\theta} > 0$

ccw



$\vec{r} = r\hat{e}_r$

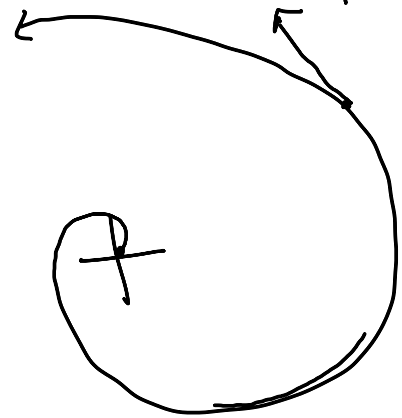
$\vec{v} = \cancel{\dot{r}\hat{e}_r} + \underline{r\dot{\theta}}\;\hat{e}_\theta$

$\vec{a} = \frac{d}{dt}(r\dot{\theta})\hat{e}_\theta + (r\dot{\theta})\dot{\hat{e}}_\theta$

$= (\cancel{\dot{r}\dot{\theta}} + r\ddot{\theta})\hat{e}_\theta - r\dot{\theta}^2\,\hat{e}_r$

2 angular $\rightarrow$ $= r\ddot{\theta}\,\hat{e}_\theta - r\dot{\theta}^2\,\hat{e}_r$ $\rightarrow$ centripetal 1

$\vec{r}(t)$ traces out a spiral so that $\dot{r}$ = constant, $\dot{\theta}$ = constant



$$\vec{r} = r\hat{e}_r$$

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\hat{e}_r + (\cancel{r\ddot{\theta}} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$= \underbrace{-r\dot{\theta}^2}_{\text{centripetal}}\hat{e}_r + \underbrace{2\dot{r}\dot{\theta}}_{\text{coriolis acceleration}}\hat{e}_\theta$$

comes from increase of speed needed to keep $\dot{r}$ constant