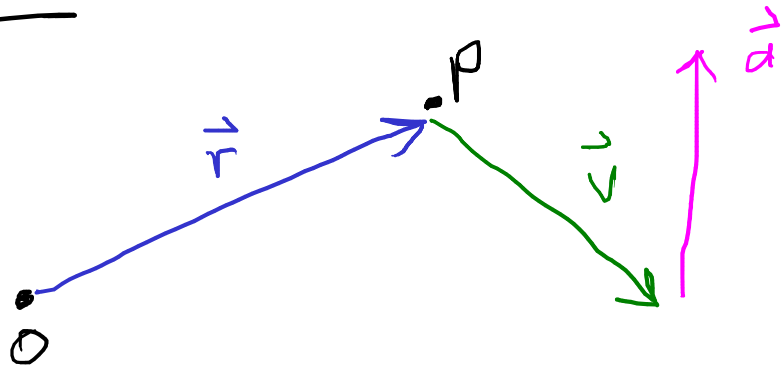
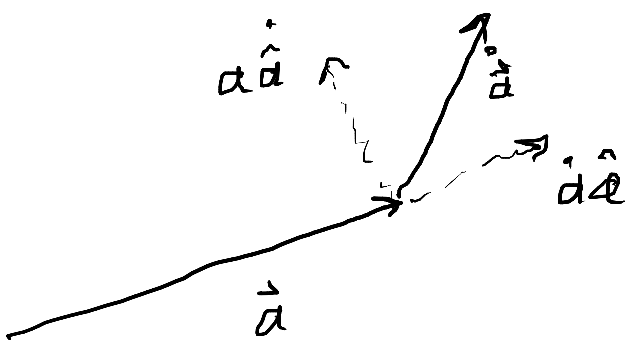
Recap



Any vector: $\vec a = a\hat a$

$\dot{\vec a} = \dot a \hat a + a \dot{\hat a}$

$\dot{\vec a} \cdot \hat a$ ... orth to $\hat a$



Always: $\dot{\hat a} \cdot \hat a = 0$

polar : $\dot{\hat{e}}_r = \dot{\theta}\,\hat{e}_\theta$ $\qquad \dot{\theta} = \omega$

$\dot{\hat{e}}_\theta = -\dot{\theta}\,\hat{e}_r$

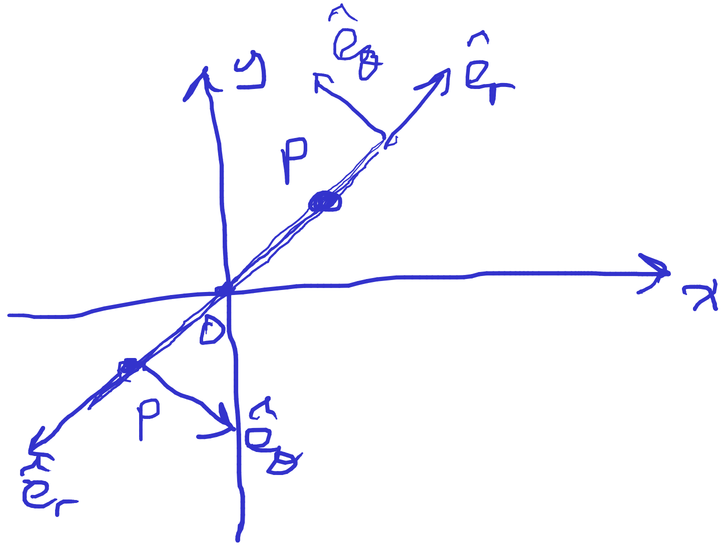
$\vec{r} = r\,\hat{e}_r \longrightarrow \dot{\vec{r}} = \dot{r}\,\hat{e}_r + r\,\dot{\hat{e}}_r$

$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

Today : acceleration in Cartesian and polar

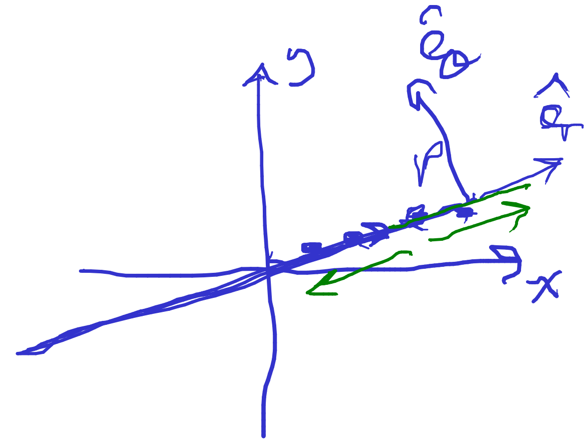
1/ $\vec{r}(t) = (\sin t)(\hat{\imath} + \hat{\jmath})$

$\vec{v}$ has
A. only $\hat{e}_r$ component
B. only $\hat{e}_\theta$ component
C. both
D. neither



$\vec{r}(t) = (\cos t)(2\hat{\imath} + \hat{\jmath})$

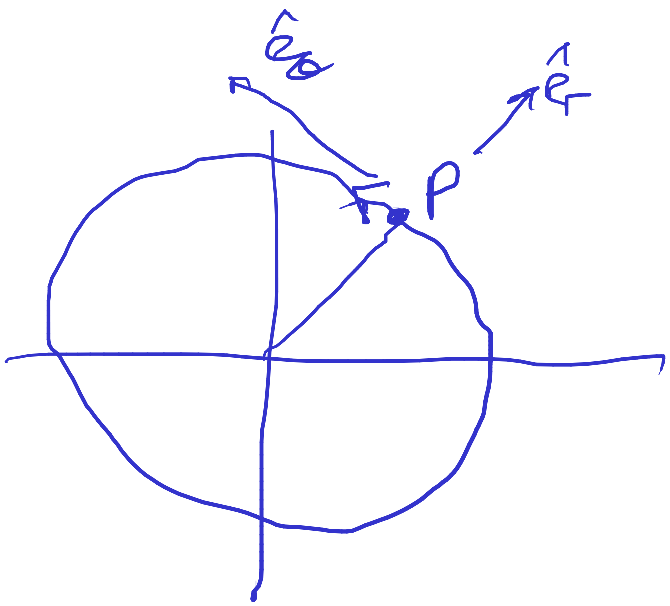
A → P ↑



2/ $r(t) = \text{const}$

$\theta(t) = \omega t$

const



$\vec{v}$ has

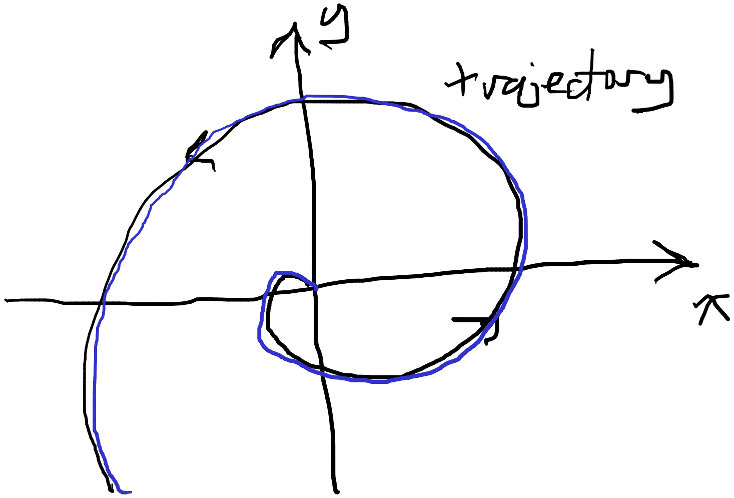
A. only $\hat{e}_r$ component

B. only $\hat{e}_\theta$ component

C. both

D. neither

3/



$\vec{v}$ has

A. only $\hat{e}_r$ component

B. only $\hat{e}_\theta$ component

C. both

D. neither

$$\vec{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

4/ A particle observed at a particular instant has:

speed

$v = 5$ m/s

$\dot{\theta} = 1$ rad/s

$\dot{r} = -3$ m/s

What is r closest to?

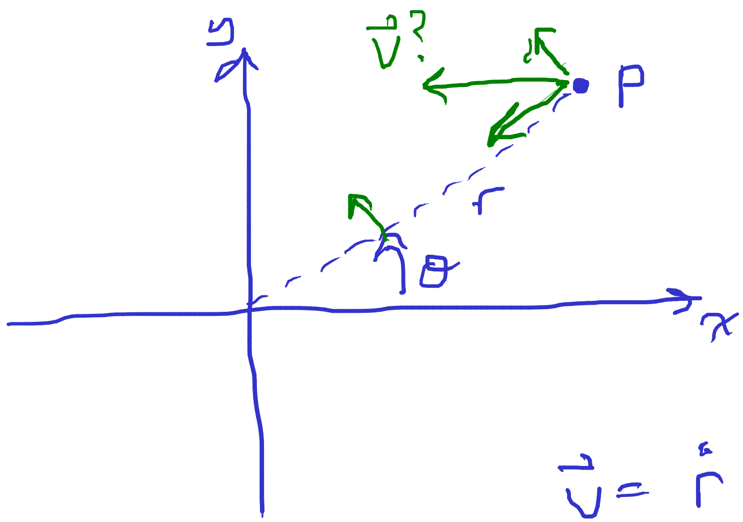
A: 1 m

B: 2 m

C: 3 m

D: 4 m

E: 5 m



$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$$

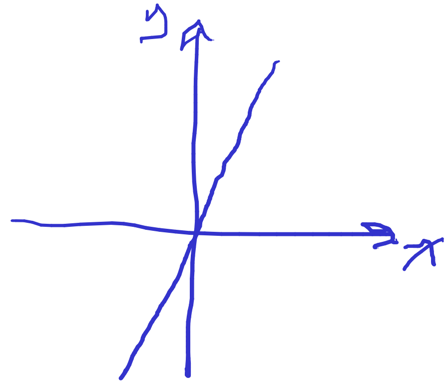
Acceleration

	Cartesian	Polar
position	$\vec{r} = x\hat{i} + y\hat{j}$	$\vec{r} = r\hat{e}_r$
velocity	$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$
acceleration	$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$	$(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$$\vec{a} = \frac{d}{dt}(\vec{v}) = \ddot{r}\hat{e}_r + \underbrace{\dot{r}\dot{\hat{e}}_r}_{\dot{\theta}\hat{e}_\theta} + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + \underbrace{r\dot{\theta}\dot{\hat{e}}_\theta}_{-\dot{\theta}\hat{e}_r}$$

$$= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

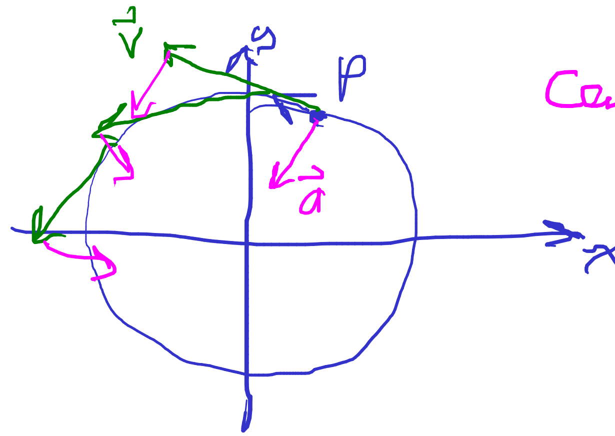
$\vec{a} = \ddot{r}\hat{e}_r$



$\vec{a} = -r\dot{\theta}^2\hat{e}_r$

const r

const $\dot{\theta} = \omega$



Centripetal acceleration

center seeking

$\vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta$

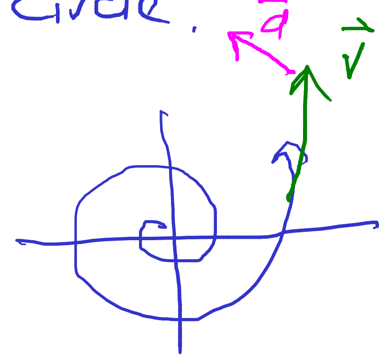
accelerating around a circle.

$\vec{a} = -r\dot{\theta}^2\hat{e}_r + 2\dot{r}\dot{\theta}\hat{e}_\theta$

$\dot{r} = \text{const}$

$\dot{\theta} = \text{const}$

Coriolis acceleration



Spiral