TAM 212 Class 6

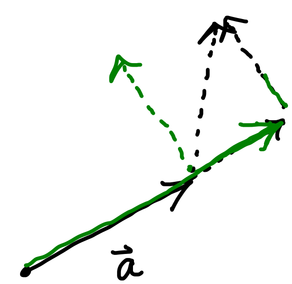
Position, Velocity, & Acceleration Vectors

James Scholar Meeting Time 4-5pm on Fridays

Review

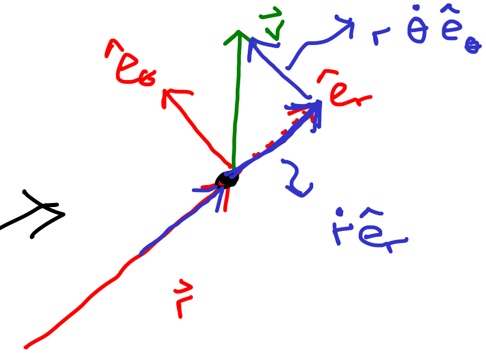
① time derivative of a vector $\vec{a}$:

$$\dot{\vec{a}} = \frac{d}{dt}(a\hat{a}) = \underbrace{\dot{a}\hat{a}}_{\text{parallel to } \vec{a}\text{, changes in magnitude of } \vec{a}} + \underbrace{a\dot{\hat{a}}}_{\text{perpendicular to } \vec{a}\text{, changes to direction of } \vec{a}}$$



② in 2D

	cartesian	polar
position	$\vec{r} = x\hat{i} + y\hat{j}$	$\vec{r} = r\hat{e}_r$
velocity	$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$
acceleration	?	?

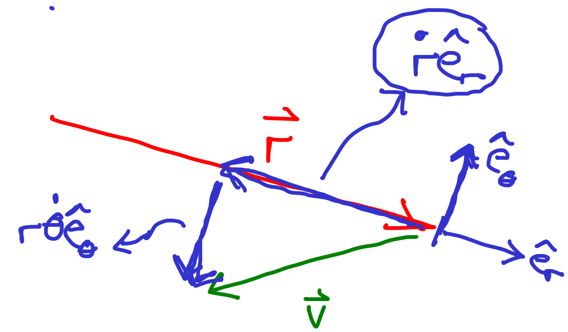


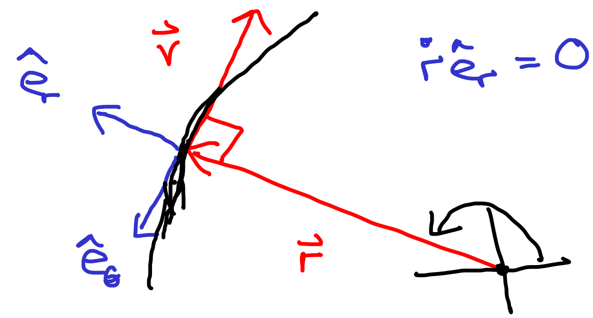
iclicker: At the instant shown the mag. of $\vec{r}$ is

Ⓐ increasing

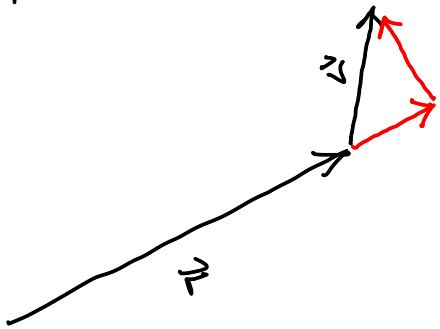
Ⓑ decreasing

Ⓒ staying the same





At the instant shown, the direction of the particle (direction of $\vec{r}$)

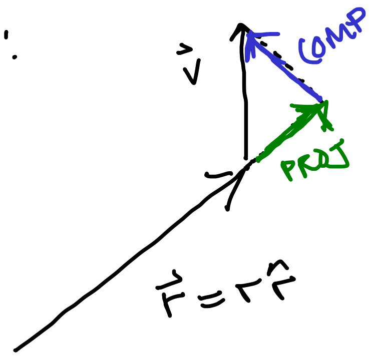


(A) is changing?

(B) is not changing?

Q: In cartesian coords, what part of $\vec{v}$ reflects changes to the magnitude of $\vec{r}$?

A:



the PROJECTION of $\vec{v}$ onto $\vec{r}$

$$= (\vec{v} \cdot \hat{r})\,\hat{r}$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$$

$$= \frac{(\dot{x}x + \dot{y}y)}{r^2}(x\hat{i} + y\hat{j})$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{i} + y\hat{j}}{r} \qquad r = \sqrt{x^2 + y^2}$$

Q: In cartesian coords, what part of $\vec{v}$ reflects changes to direction of $\vec{r}$?

COMP PROJ: $\vec{v} - (\vec{v} \cdot \hat{r})\hat{r}$

Example) A vector and its time derivative are given by

$\vec{a} = \hat{\imath} - 2\hat{\jmath}$ m, $\dot{\vec{a}} = 8\hat{\imath}$ m/s.

Find the rate of change of the length of vector $\vec{a}$ $\Longrightarrow$ FIND $\dot{a}$

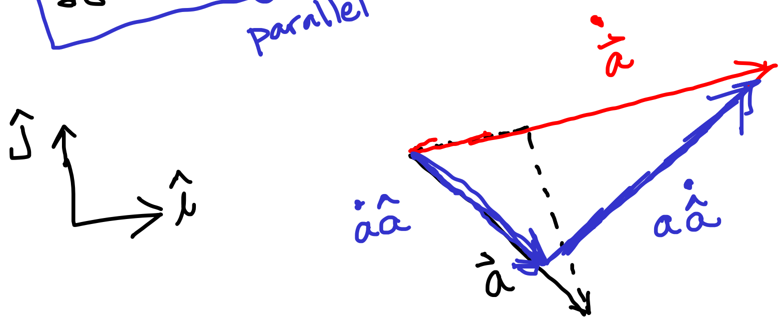
$$\vec{a} = a\hat{a}$$

$$\boxed{\frac{d}{dt}\vec{a} = \underbrace{\dot{a}\hat{a}}_{\text{parallel}} + a\dot{\hat{a}}}$$

$$\boxed{\dot{a} = \dot{\vec{a}} \cdot \hat{a}}$$

$$= 8\hat{\imath} \cdot \frac{1}{\sqrt{5}}(\hat{\imath} - 2\hat{\jmath})$$

$$= 8/\sqrt{5}$$



Acceleration Vector $\vec{a} \equiv \dot{\vec{v}}$

cartesian: $\vec{v} = \dot{x}\,\hat{\imath} + \dot{y}\,\hat{\jmath}$

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}\,\hat{\imath} + \cancel{\dot{x}\,\dot{\hat{\imath}}} + \ddot{y}\,\hat{\jmath} + \cancel{\dot{y}\,\dot{\hat{\jmath}}} = \ddot{x}\,\hat{\imath} + \ddot{y}\,\hat{\jmath}$$

polar: $\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

$$\vec{a} = \dot{\vec{v}} = \frac{d}{dt}\left(\dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta\right)$$

$$= \ddot{r}\,\hat{e}_r + \dot{r}\,\dot{\hat{e}}_r + \frac{d}{dt}(r\dot{\theta})\,\hat{e}_\theta + (r\dot{\theta})\,\dot{\hat{e}}_\theta$$

$$= \ddot{r}\,\hat{e}_r + \dot{r}\dot{\theta}\,\hat{e}_\theta + (\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{e}_\theta + (r\dot{\theta})(-\dot{\theta})\,\hat{e}_r$$

$$= (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$$

$(\ddot{r} - r\dot{\theta}^2)$ ↓ centripetal acceleration

$(r\ddot{\theta} + 2\dot{r}\dot{\theta})$ ↓ Coriolis acceleration

recall:

$\hat{e}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\dot{\hat{e}}_r = \dot{\theta}(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath})$

$= \dot{\theta}\,\hat{e}_\theta$

$\hat{e}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

$\dot{\hat{e}}_\theta = \dot{\theta}(-\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath})$

$= -\dot{\theta}\,\hat{e}_r$