TAM 212 CLASS 5 : More on Vector Derivatives.

① James Scholar Info Meeting
Thurs Febr 5th 12-1pm $\boxed{\text{143 MEB}}$

② HW1 PL – due tonight 11:59 pm
HW2 PL → become available

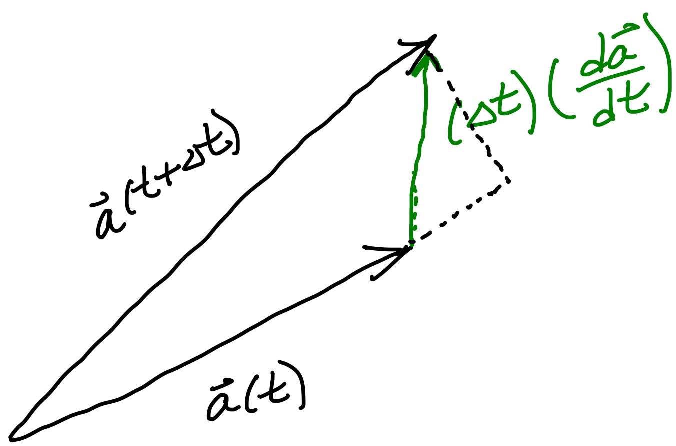
Report 1 – MON 11:59pm

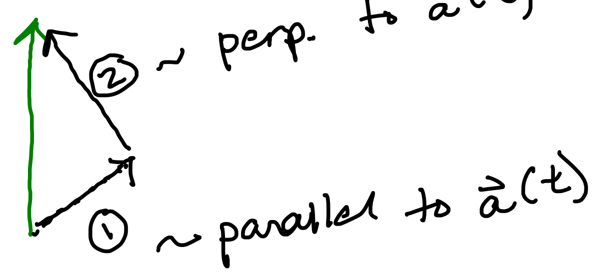
Review

① general concept: derivative of $\vec{a}$, $\dot{\vec{a}}$, has two parts

$$\vec{a} = a\hat{a}$$

$$\frac{d}{dt}\vec{a} = \underbrace{\dot{a}\hat{a}}_{①} + \underbrace{a\dot{\hat{a}}}_{②} \quad \sim \frac{d}{dt}(\hat{a})$$





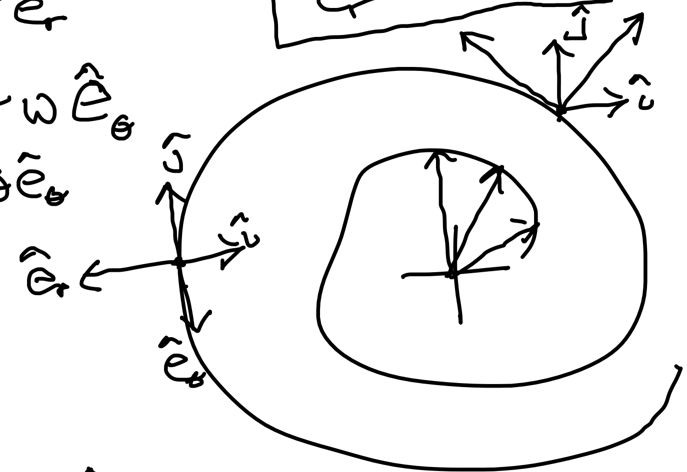
② 2D systems

	CARTESIAN	POLAR
position $\vec r$	$\vec r = x\hat i + y\hat j$	$\vec r = r\hat e_r$
velocity $\vec v$	$\vec v = \dot x\hat i + \dot y\hat j$	$\dot{\vec r} = \dot r\hat e_r + r\dot{\hat e}_r$

$$= \dot r\hat e_r + r\omega\hat e_\theta$$

$$= \dot r\hat e_r + r\dot\theta\hat e_\theta$$

$$\boxed{\dot{\hat e}_r = \omega\hat e_\theta}$$



$$\vec v = \dot{\vec r}$$

$$\omega = \dot\theta = \frac{d\theta}{dt}$$

How did we show $\dot{\hat e}_r = \omega\hat e_\theta$

write $\hat e_r = \cos\theta\hat i + \sin\theta\hat j$

$$\frac{d\hat e_r}{dt} = -\dot\theta\sin\theta\hat i + \dot\theta\cos\theta\hat j = \omega\hat e_\theta$$

$$= -\omega\sin\theta\hat i + \omega\cos\theta\hat j$$

POLAR: $\boxed{\vec r = r\hat e_r}$

$$\Rightarrow \vec v = \underbrace{\dot r\hat e_r}_{\text{parallel part}} + \underbrace{r\dot\theta\hat e_\theta} \to \text{perpendicular}$$

$v = \omega r$

I Clicker:

(1) a vector $\vec{r}$ follows a trajectory

$$\vec{r}(t) = (\sin t)(\hat{i} + \hat{j})$$

The velocity $\vec{v}(t)$ has:

(a) only $\hat{e}_r$ component

(b) only $\hat{e}_\theta$ component

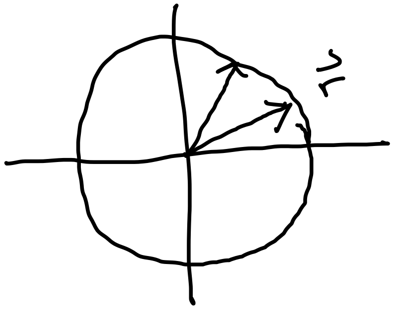
(c) Both

(d) Neither

$$\vec{r} = r\hat{e}_r$$

$$\vec{v} = \underbrace{\dot{r}\hat{e}_r}_{} + \underbrace{r\dot{\theta}}_{}\hat{e}_\theta$$

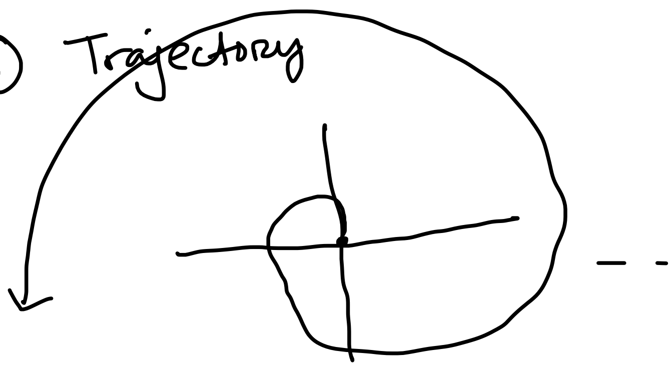
② Trajectory: $r = \text{constant}$
$\theta = \omega t$



vector $\vec{v}$ has
(a) only $\hat{e}_r$ component
(b) only $\hat{e}_\theta$ component
(c) BOTH
(d) neither

$$\vec{v} = \overset{0}{\cancel{\dot{r}}}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

③ Trajectory



vector $\vec{v}$ has
(a) $\hat{e}_r$ comp. only
(b) $\hat{e}_\theta$ comp. only
(c) BOTH

Ex) A particle moves in the plane w/ polar coords so that at a particular moment:

$$v = 5 \text{ m/s}$$
$$\dot{\theta} = 1 \text{ rad/sec}$$
$$\dot{r} = -3 \text{ m/s}.$$

Find $r = \|\vec{r}\|$

$$\vec{r} = r\hat{e}_r$$
$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$
$$v = \|\vec{v}\| = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$$

$$5 = \sqrt{9 + r^2(1)^2}$$
$$r = 4 \text{ m}$$