Recap

1. Positions have coordinates: $x = 3\,m$, $y = -2\,m$; $r = 4\,m$, $\theta = 30^\circ$

2. Vectors have components: $\vec{v} = 2\hat{\imath} + 3\hat{\jmath}\ m/s$; $\vec{r} = 6\hat{e}_r\ km$

3. Units go with values: $x = 3m$, not "r is in m"

4. Vectors have direction and magnitude, not position ("free")

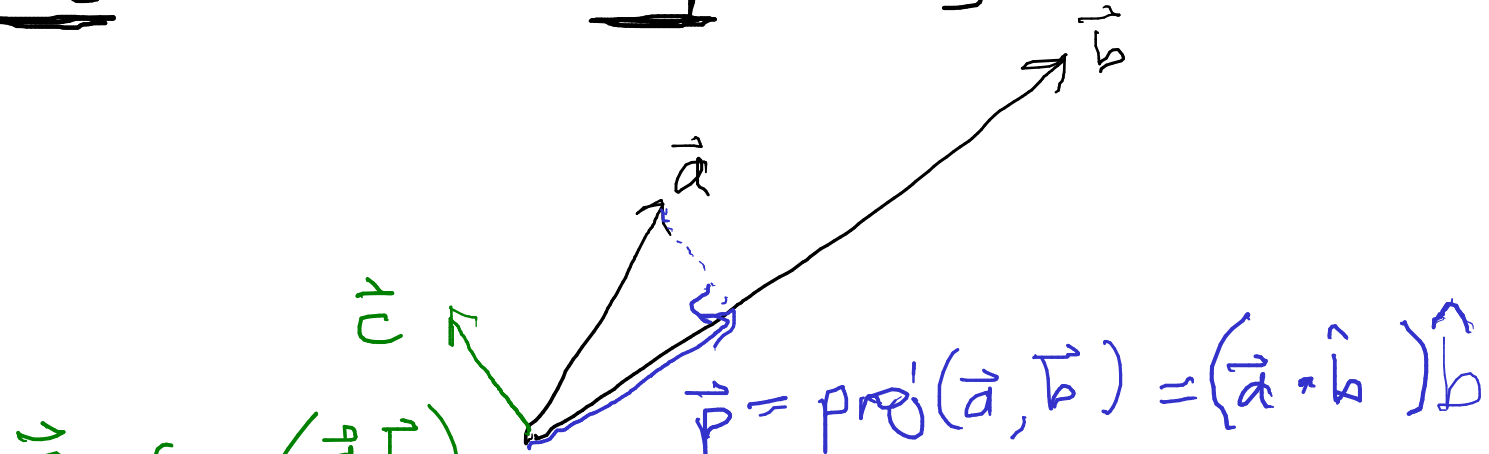
$\vec{v}$ ↑ $\hat{v}$ ↑ v ↑

$$\vec{v} = v\hat{v}$$

$$\underbrace{3\hat{\imath} + 4\hat{\jmath}}_{\vec{v}} = \underbrace{5}_{v}\underbrace{(0.6\hat{\imath} + 0.8\hat{\jmath})}_{\hat{v}}$$

5. Unit vectors don't have units (dimensionless)

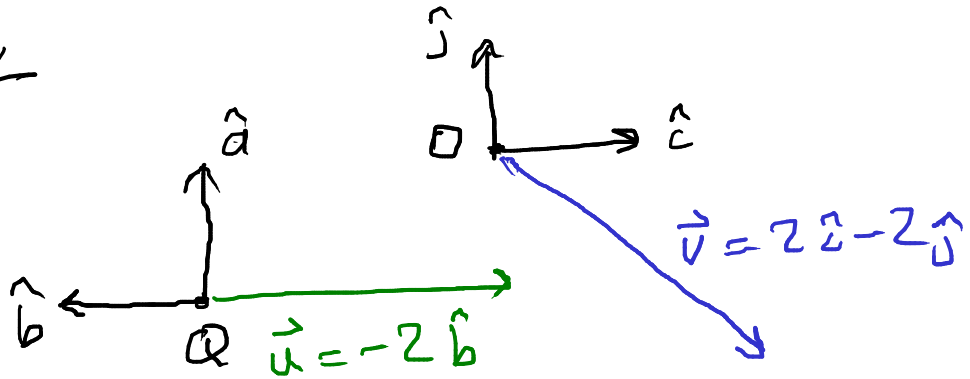
(6) Projections and complementary projections



Today: - Final notes on vectors

- Differentiating vectors

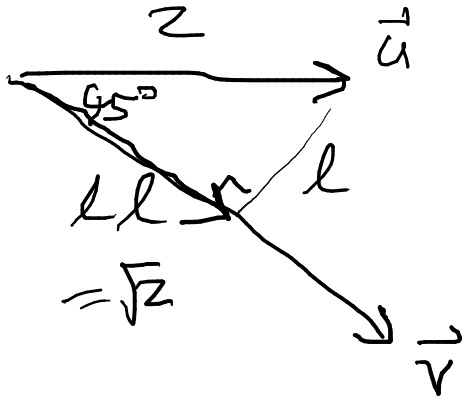
Ex



$$\vec{r}_{OQ} = -2\hat{c} - \hat{j}$$

$\text{proj}(\vec{u}, \vec{v}) =$

A	$\hat{c} + \hat{a}$
B	$-\hat{c} + \hat{j}$
C	$\hat{a} + \hat{b}$
D	$\hat{j} - \hat{a}$
E	$-\hat{j} - \hat{b}$



$$\ell^2 + \ell^2 = 2^2 = 4$$

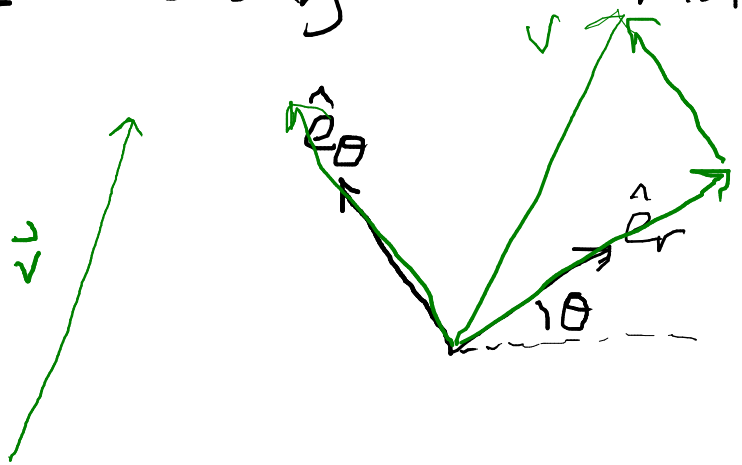
$$2\ell^2 = 4$$

$$\ell^2 = 2$$

$$\ell = \sqrt{2}$$

Also $\quad -\hat{j} - 2\hat{b} - \hat{c}$

Ex change of basis



$\vec{v} = \underline{\quad}\,\hat{e}_r + \underline{\quad}\,\hat{e}_\theta$

Scalar

E

A $\vec{v} \times \hat{e}_r$

B $\vec{v} \cdot \hat{e}_r$

C $v \cos\theta$

D $v \sin\theta$

E $\text{proj}(\vec{v}, \hat{e}_r)$

vector

Calculus and Vectors

Derivative of a vector w.r.t. time t

$$\vec{v}(t) = 2t^2\hat{i} + 3t\hat{j} \text{ m/s}$$ where t is in seconds

$$= \left(2(t/s)^2\hat{i} + 3(t/s)\hat{j}\right) \text{m/s}$$

$\vec{v}$

$$t = 2 \text{ min} = 120 \text{ s} \qquad \frac{t}{s} = \frac{2 \text{ min}}{s}$$

$$\vec{v}(2 \text{ min}) = 28800\hat{i} + 360\hat{j} \text{ m/s}$$

$$\frac{d}{dt}\vec{v} = \dot{\vec{v}} = \frac{d}{dt}\left(2t^2\hat{i} + 3t\hat{j}\right)$$

$$= \frac{d}{dt}(2t^2)\hat{i} + 2t^2\frac{d}{dt}(\hat{i}) + \frac{d}{dt}(3t)\hat{j} + 3t\frac{d}{dt}(\hat{j})$$

($\frac{d}{dt}(\hat{i}) \to 0$, $\frac{d}{dt}(\hat{j}) \to 0$)

$$= 4t\hat{i} + 3\hat{j}$$

What is the direction of $\vec{a}$ when $\vec{a}(t)$ goes through $\vec{a}(0)$?