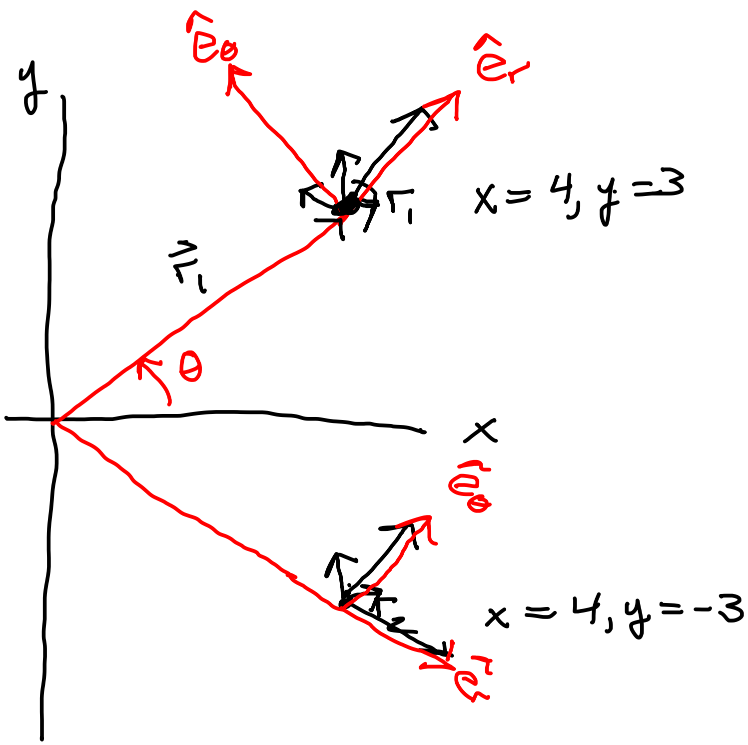
TAM 212: ① Coordinate Systems, Basis Vectors
② Projection, Complementary Projection

in 2D	cartesian	polar
coordinates	x, y	r, θ
basis vectors	$\hat{\imath}, \hat{\jmath}$	$\hat{e}_r, \hat{e}_\theta$
position vector	$\vec{r} = x\hat{\imath} + y\hat{\jmath}$	$\vec{r} = r\hat{e}_r$

NOT $\vec{r} =$

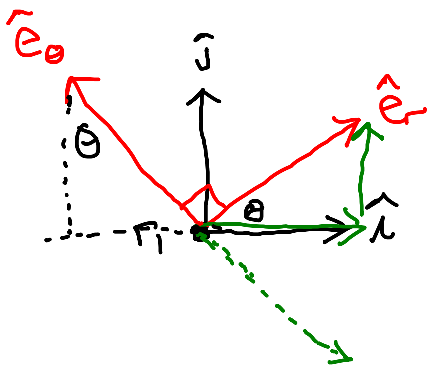


position vector

$$\vec{r}_1 = 4\hat{\imath} + 3\hat{\jmath} \qquad \vec{r}_1 = r_1\hat{e}_r = 5\hat{e}_r$$

$$\vec{r}_2 = 4\hat{\imath} - 3\hat{\jmath} \qquad \vec{r}_2 = r_2\hat{e}_r = 5\hat{e}_r$$

We can always find an expression for $\hat{e}_r, \hat{e}_\theta$ at any point in terms of θ



$$\hat{e}_r = \cos\theta\ \hat{\imath} + \sin\theta\ \hat{j}$$

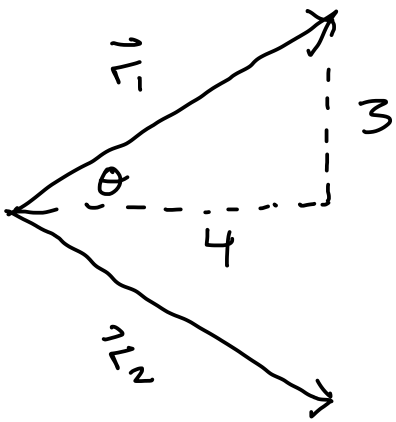
$$\hat{e}_\theta = -\sin\theta\ \hat{\imath} + \cos\theta\ \hat{j}$$

useful trick!!

So at r_1 $\quad \vec{r}_1 = r\,\hat{e}_r \qquad x = 4,\ y = 3 \quad \Rightarrow r = 5$

$$\hat{e}_r = \frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{j}$$

$$\vec{r}_1 = r\hat{e}_r = 5\left(\frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{j}\right) = 4\hat{\imath} + 3\hat{j}$$

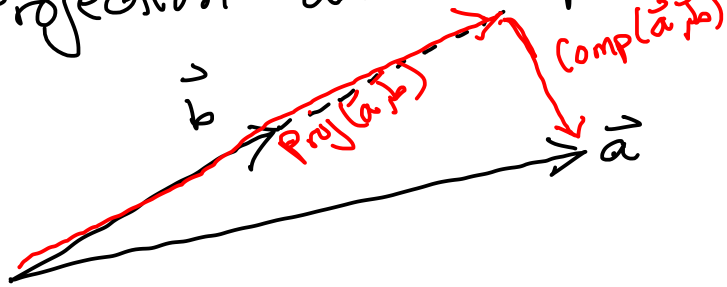


Dot Product is Independent of Basis

$$\vec{r}_1 \cdot \vec{r}_2 = \|\vec{r}_1\|\ \|\vec{r}_2\| \cos\theta$$

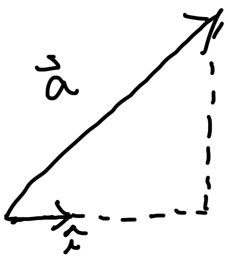
But must be evaluated using same basis for both vectors.

Projection and Complementary Projection of vector $\vec{a}$ onto $\vec{b}$



Proj$(\vec{a},\vec{b})$: How much of $\vec{a}$ lies along $\vec{b}$?

ex) $\vec{a} = 3\hat{i} + 4\hat{j}$; $\vec{b} = \hat{i}$ } What is ~~the~~ proj. of $\vec{a}$ onto $\vec{b}$?



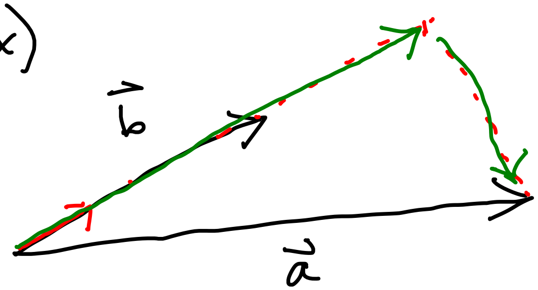
Proj$(\vec{a},\hat{i}) = 3\hat{i}$

Comp$(\vec{a},\hat{i}) = 4\hat{j}$

Proj$(\vec{a},\hat{i}) = (\vec{a}\cdot\hat{i})\,\hat{i}$

Comp$(\vec{a},\hat{i}) = \vec{a} - (\vec{a}\cdot\hat{i})\,\hat{i}$

ex)



$$\hat{b} = \frac{\vec{b}}{b}$$

$$\text{Proj}(\vec{a}, \vec{b}) = \frac{(\vec{a} \cdot \vec{b})}{b} \frac{\vec{b}}{b}$$

$$= (\vec{a} \cdot \hat{b}) \hat{b}$$

$$\text{Comp}(\vec{a}, \vec{b}) = \vec{a} - (\vec{a} \cdot \hat{b}) \hat{b}$$

Coordinate Systems in 3D

	cartesian	cyllindrical	spherical
coords	x, y, z	r, θ, z	r, θ, ϕ
basis vectors	$\hat{i}, \hat{j}, \hat{k}$	$\hat{e}_r, \hat{e}_\theta, \hat{e}_z \ (\hat{k})$	$\hat{e}_r, \hat{e}_\theta, \hat{e}_\phi$
position	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$	$\vec{r} = r\hat{e}_r + z\hat{e}_z$ $r = \sqrt{x^2+y^2}$	$\vec{r} = r\hat{e}_r$ $r = \sqrt{x^2+y^2+z^2}$