TAM 212 – Dynamics

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Recap

- Friction

Today

- Linear and Angular Momentum

A. Slip

1. Relative motion

$v_{Px} \neq 0$ or $a_{Px} \neq 0$

2. $F = \mu N$

3. $\hat{F}$ opposes motion

($\hat{v}_P$ or $\hat{a}_P$)

B. Stick

1. No relative motion

$v_{Px} = 0$ or $a_{Px} = 0$

2. $F \leq \mu N$

(magnitudes)

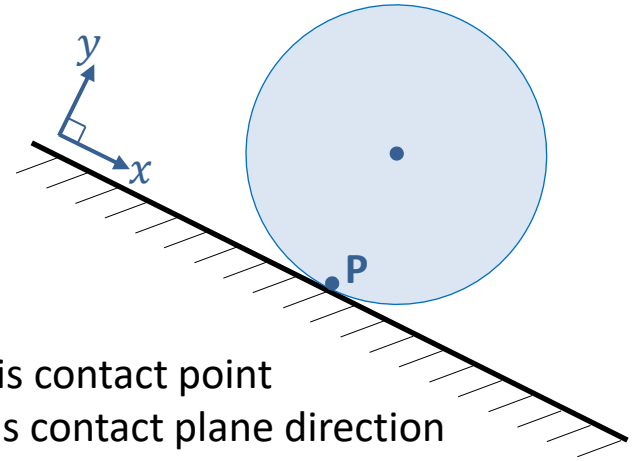
C. Transition

1. No relative motion

$v_{Px} = 0$ or $a_{Px} = 0$

2. Critical friction force

$F = \mu N$



P is contact point

x is contact plane direction

Solution Procedure (known case)

- Determine case (stick, transition, slip left, slip right)
- FBD, equations depending on case, solve

Solution Procedure (unknown case)

- Try stick: ① Assume $v_{Px} = 0, a_{Px} = 0, N, F \leq \mu N$
  ② Solve for motion, N, F
  ③ Check $|F| \leq \mu |N|$ (actual friction $\leq$ available friction)

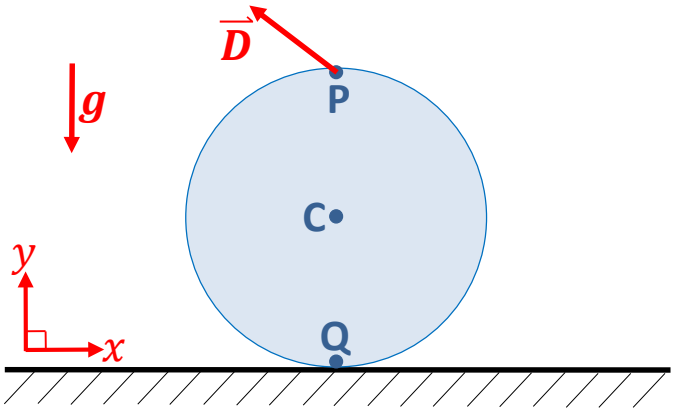
- Try slip left: ① Assume $F = \mu N$ to the right
  ② Solve for motion of contact point, N, F
  ③ Check $v_{Px} \neq 0$, $a_{Px} \neq 0$ and $\hat{F}$ opposes motion ($\hat{v}$ or $\hat{a}$)

- Try slip right: ① Assume $F = \mu N$ to the left
  ②③ same as slip left

$\hat{v}$ if in motion
$\hat{a}$ if start from rest

Example

A uniform rigid disk of mass $m = 7$ kg and radius $r = 4$ m starts at rest on a flat ground as shown. Force $\vec{D} = -44\hat{\imath} + 34\hat{\jmath}$ N acts at point P on the top edge, and g = 9.8 m/s^2 acts vertically. The coefficient of friction between the block and ground is $\mu = 0.25$.



What is the angular acceleration $\vec{\alpha}$ of the disk?

$\vec{\alpha} =$ _____ $\hat{k}$ rad/s^2

Momentum of a Point Mass

Linear Momentum: $\vec{P} = m\vec{v}$

Angular Momentum: $\vec{L}_O = \vec{r}_{OP} \times \vec{P}$
$= m\vec{r}_{OP} \times \vec{v}$
$= m\vec{r}_{OP} \times (\vec{\omega} \times \vec{r}_{OP})$

Principle of Impulse-Momentum

Initial Momentum	+	Impulse	=	Final Momentum

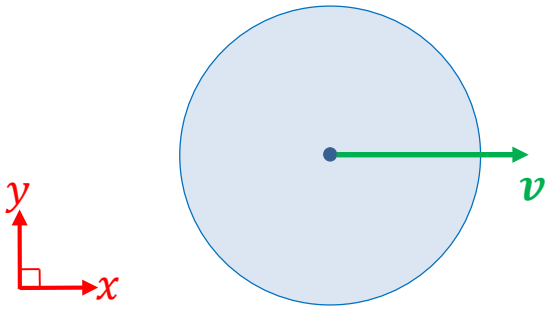
Linear Momentum: $\vec{P}_i + \Delta\vec{P} = \vec{P}_f$

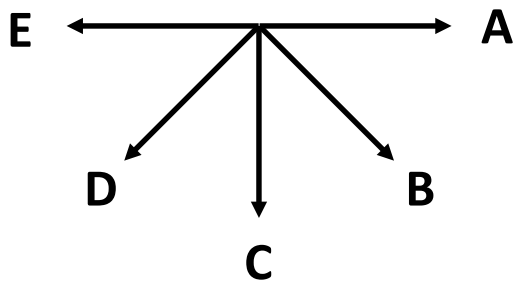
Angular Momentum: $\vec{L}_{Oi} + \Delta\vec{L}_O = \vec{L}_{Of}$

From Newton's Equation:

Example

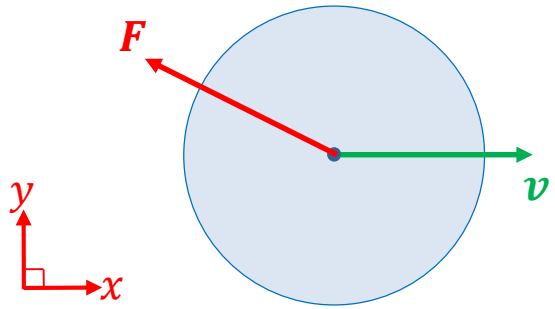
A particle is initially moving to the right at 5 m/s. Which direction should an impulse be applied to redirect the particle downward?





Example

A particle is initially moving to the right at 5 m/s. A constant force $\vec{F} = -\hat{\imath} + \widehat{2j}$ N is applied to the particle. How long does it take to redirect the particle's motion to upward only?



Conservation Law

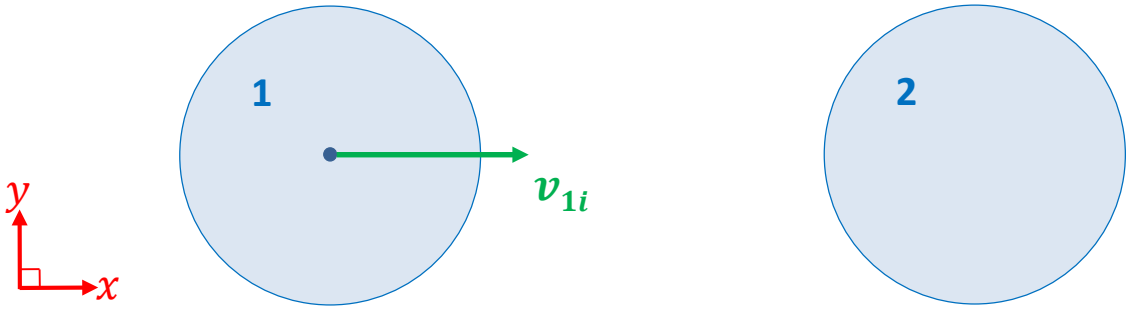
If $\Sigma\vec{F} = 0 = \dot{\vec{P}}$, then $\vec{P}$ is a constant.

If $\Sigma\vec{M}_{OZ} = 0 = \dot{\vec{L}}_{OZ}$, then $\vec{L}_{OZ}$ is a constant.

Coefficient of restitution: $e = \frac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}}$

Example

Particle 1 is initially moving to the right at 5 m/s and comes into contact with Particle 2 initially at rest. If the two particles stick together after collision, what is the terminal velocity of the two particles?



Example

A tethered particle is moving on a horizontal smooth surface with an initial speed of 7 m/s. If the tethered is shortened by a factor of 3, what is the resulting speed of the particle?