TAM 212 – Dynamics

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Recap

- Energy and Friction

Today

- Friction

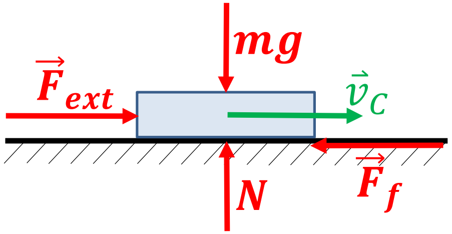
Work-energy principle: $W = \Delta E = E_f - E_i$

Work done by force $\vec{F}$: $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \int_{t_i}^{t_f} \vec{F} \cdot \vec{v} dt$

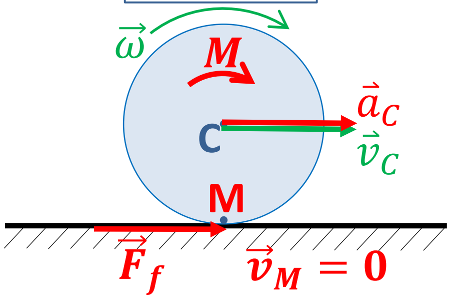
Work done by moment M_Z: $W = \int_{\theta_i}^{\theta_f} M_Z d\theta = \int_{t_i}^{t_f} M_Z \dot{\theta} dt$

W_F can be negative, zero or postive:

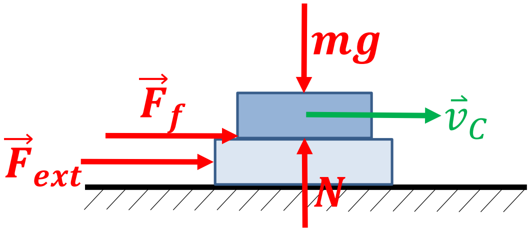
$\boxed{W_F < 0}$



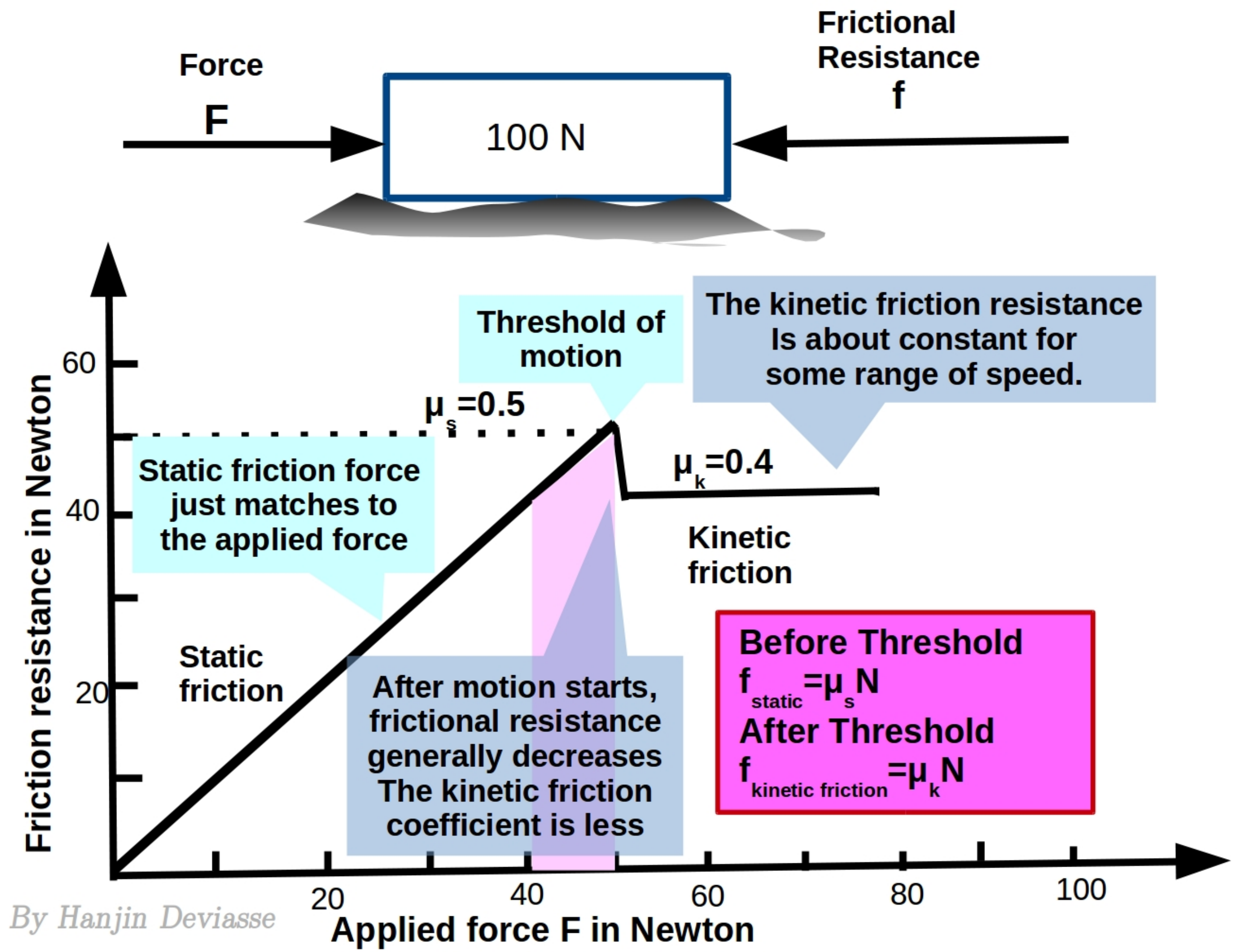
$\boxed{W_F = 0}$



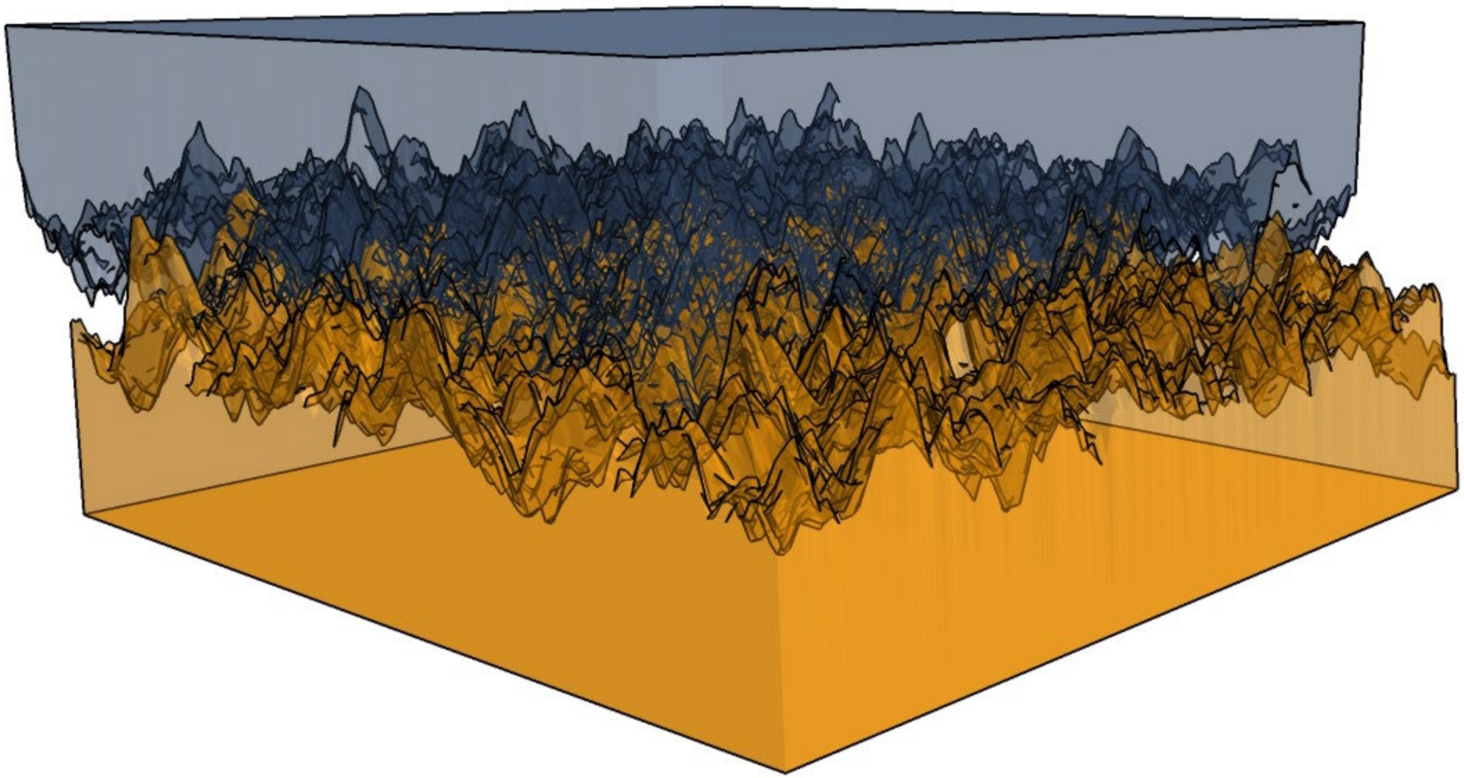
$\boxed{W_F > 0}$



Static vs Kinetic Friction



Simulated blocks with fractal rough surfaces, exhibiting static frictional interactions



Static friction can be thought of as arising from surface roughness

Static & Kinetic Friction Coefficients

Material	Coefficient of Static Friction μ_s	Coefficient of Kinetic Friction μ_k
Rubber on Glass	2.0+	2.0
Rubber on Concrete	1.0	0.8
Steel on Steel	0.74	0.57
Wood on Wood	0.25 – 0.5	0.2
Metal on Metal	0.15	0.06
Ice on Ice	0.1	0.03
Synovial Joints in Humans	0.01	0.003

Example

An empty cart is being rolled across a warehouse floor. If the cart was filled, the force of friction between the cart and the floor would

A. **Decrease**
B. **Increase**
C. **Remain the same**

Example

Sand is often placed on an icy road because the sand:

A. Decreases the coefficient of friction between the tires of a car and the road
B. Increases the coefficient of friction between the tires of a car and the road
C. Decrease the gravitational force on a car
D. Increases the normal force of a car on the road

Dry/Coulomb Friction (2D) - Multiple Cases

A. Slip

1. Relative motion

   $v_{Px} \neq 0$ or $a_{Px} \neq 0$

2. $F = \mu N$

3. $\hat{F}$ opposes motion

   ($\hat{v}_P$ or $\hat{a}_P$)

B. Stick

1. No relative motion

   $v_{Px} = 0$ or $a_{Px} = 0$

2. $F \leq \mu N$

   (magnitudes)

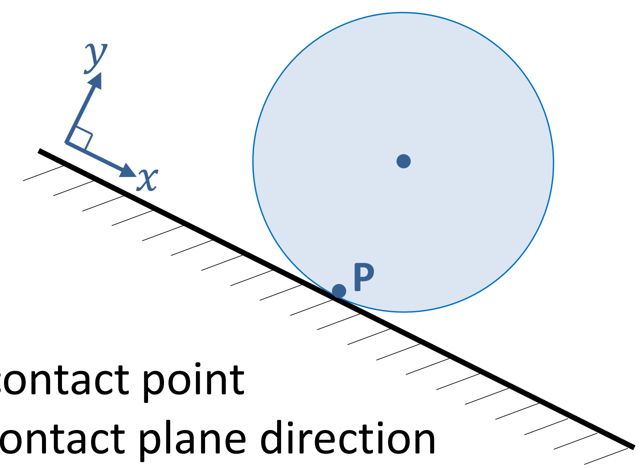
C. Transition

1. No relative motion

   $v_{Px} = 0$ or $a_{Px} = 0$

2. Critical friction force

   $F = \mu N$



P is contact point
x is contact plane direction

Solution Procedure (known case)

- Determine case (stick, transition, slip left, slip right)
- FBD, equations depending on case, solve

Solution Procedure (unknown case)

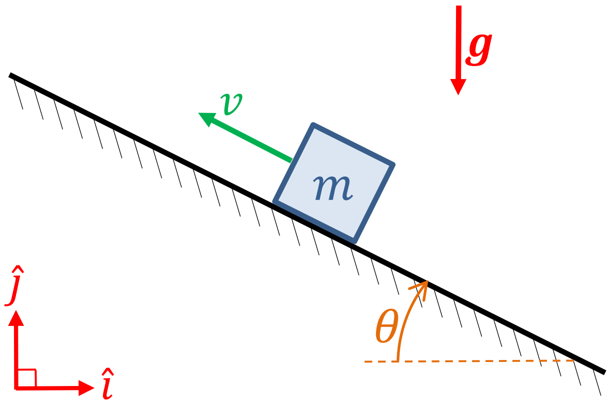
- Try stick: ① Assume $v_{Px} = 0$, $a_{Px} = 0, N, F \leq \mu N$
  ② Solve for motion, N, F
  ③ Check $|F| \leq \mu|N|$

- Try slip left: ① Assume $F = \mu N$ to the right
  ② Solve for motion of contact point, N, F
  ③ Check $v_{Px} \neq 0$, $a_{Px} \neq 0$ and $\hat{F}$ opposes motion ($\hat{v}$ or $\hat{a}$)

- Try slip right: ① Assume $F = \mu N$ to the left
  ②③ same as slip left

Example

A block of mass $m = 8$ kg is sliding up a sloped ground with speed $v = 9$ m/s. The ground is at an angle of $\theta = 30^o$ from horizontal, the coefficient of friction between the block and ground is $\mu = 0.25$, and gravity $g = 9.8$ m/s^2 acts vertically.



What is the acceleration $\vec{a}$ of the block?

$\vec{a} =$ ____________ $\hat{\imath}$ + ____________ $\hat{\jmath}$ m/s^2

Which case?

- **A. Slip**
- **B. Stick**
- **C. Transition**

How many unknown variables?

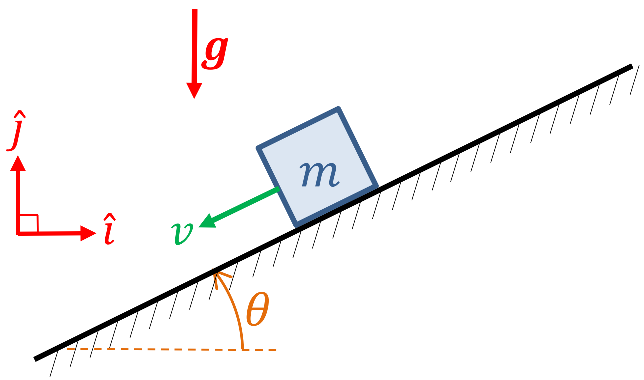
- **A. 1**
- **B. 2**
- **C. 3**
- **D. 4**
- **E. 5**

What is equation (3)?

- **A.** $ma_x = F + mg\sin\theta$
- **B.** $ma_x = -F + mg\sin\theta$
- **C.** $ma_x = F - mg\sin\theta$
- **D.** $ma_x = -F - mg\sin\theta$

Example

A block of mass $m = 8$ kg is sliding down a sloped ground with speed $v = 5$ m/s. The ground is at an angle of $\theta = 25^{\circ}$ from horizontal, the coefficient of friction between the block and ground is $\mu = 0.5$, and gravity $g = 9.8$ m/s^2 acts vertically.



What is the acceleration $\vec{a}$ of the block?

$\vec{a} =$ __________ $\hat{\imath}$ + __________ $\hat{\jmath}$ m/s^2

Given that g is down and $\vec{v}$ is down the slope:

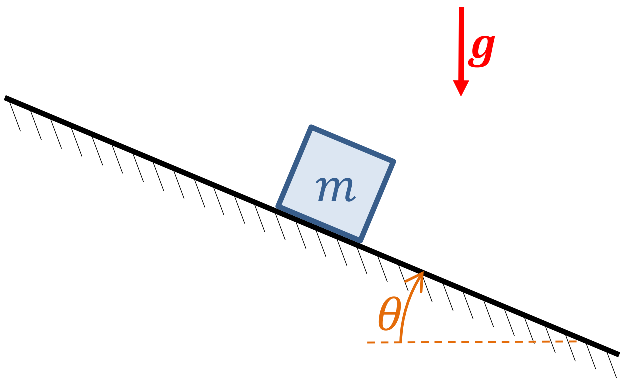
A. $\vec{a}$ must be down the slope gravity

B. $\vec{a}$ must be up the slope friction

C. $\vec{a}$ might be up or down the slope

Example

A block of mass $m = 9$ kg starts at rest on a sloped ground. The ground is at an angle of $\theta = 25^o$ from horizontal and gravity $g = 9.8$ m/s^2 acts vertically.



What is the minimum coefficient of friction so that the block will not slide?

$\mu =$ ___________

Which case?

A. **Slip**
B. **Stick**
C. **Transition**

Invalid (not useful) equation?

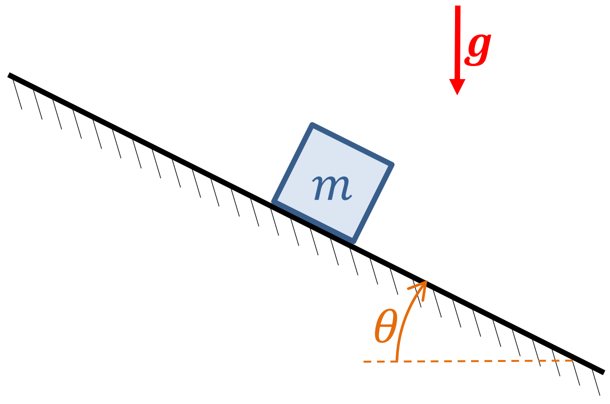
A. $F = \mu N$
B. $a = 0$
C. $v = 0$
D. $F \leq \mu N$
E. $a \leq 0$

μ **depends on:**

A. m, θ, g
B. m, θ
C. θ, g
D. m, g
E. θ

Example

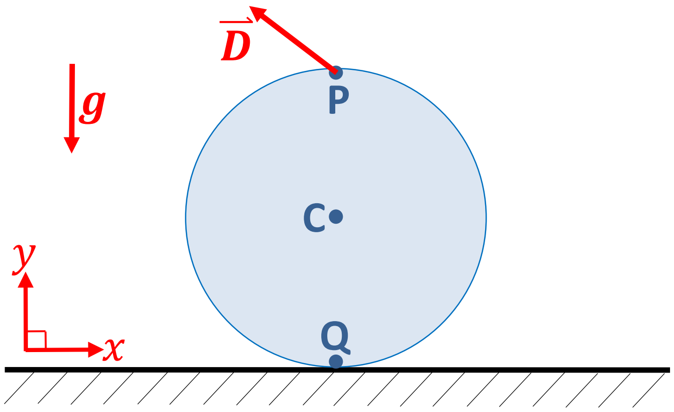
A block of mass $m = 3$ kg starts at rest on a sloped ground. The ground is at an angle of $\theta = 30^{\circ}$ from horizontal, the coefficient of friction between the block and ground is $\mu = 0.5$, and gravity $g = 9.8$ m/s^2 acts vertically.



What type of motion does the block experience?

Example

A uniform rigid disk of mass $m = 7$ kg and radius $r = 4$ m starts at rest on a flat ground as shown. Force $\vec{D} = -44\hat{\imath} + 34\hat{\jmath}$ N acts at point P on the top edge, and g = 9.8 m/s^2 acts vertically. The coefficient of friction between the block and ground is $\mu = 0.25$.



What is the angular acceleration $\vec{\alpha}$ of the disk?

$\vec{\alpha} =$ ___________$\hat{k}$ rad/s^2