TAM 212 – Dynamics

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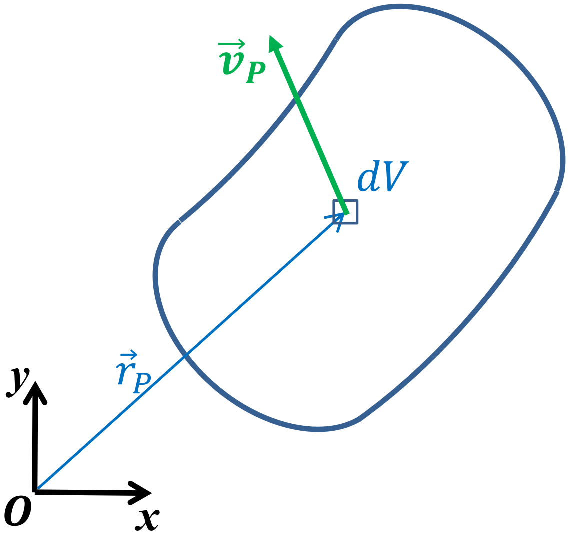
Summer 2019

Energy

$E = T + V$

Point mass: $T = \frac{1}{2} m v^2$

General body: $T = \iiint\limits_B \frac{1}{2} \rho v_P^2 dV$



Rigid body:

Gravitational potential: $V = mgh_C$ (must be C)

Work

Work-energy principle: $W = \Delta E = E_f - E_i$

Work done by force $\vec{F}$:

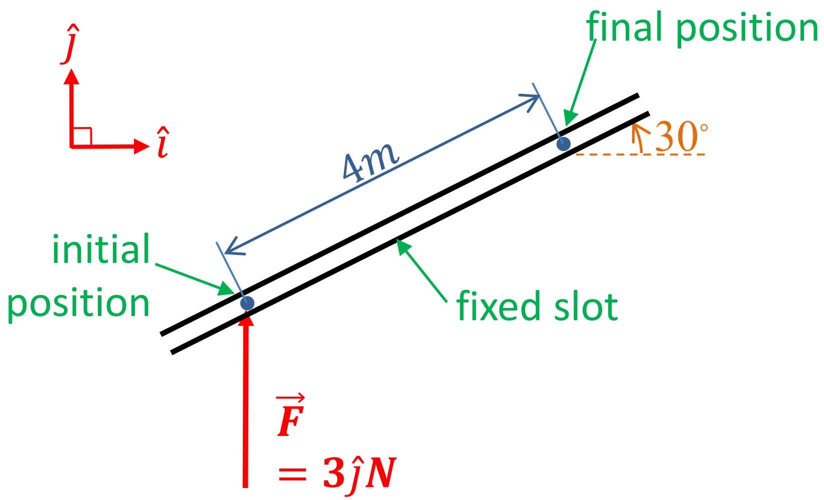
$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \int_{t_i}^{t_f} \vec{F} \cdot \vec{v} dt$$

Units: $[W] = [E] = J$

Which is not a unit of power?

A. $\boldsymbol{Nm\, s^{-1}}$ **B.** $\boldsymbol{Js^{-1}}$ **C.** $\boldsymbol{Jm^{-1}}$ **D.** $\boldsymbol{kgm^2s^{-3}}$

Example: Constant Force $\vec{F}$



$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \int_{t_i}^{t_f} \vec{F} \cdot \vec{v} dt$$

Work done by $\vec{F}$

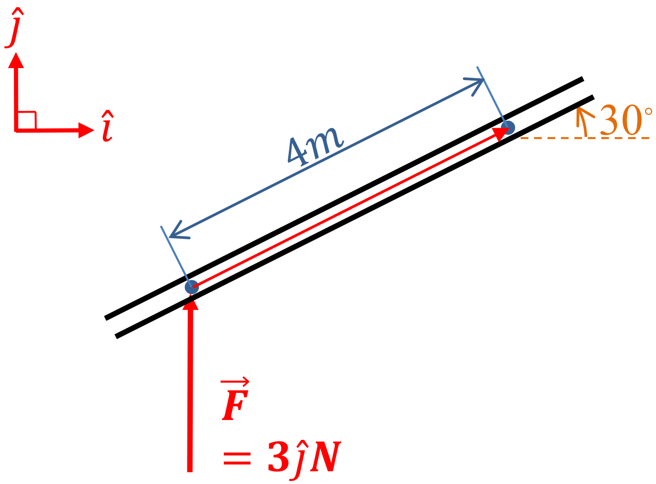
$W =$
- A. $12\ J$
- B. $6 \sin 30^\circ\ J$
- C. $6\ J$
- D. $-12\ J$
- E. $6\sqrt{3}\ J$

Constant Force

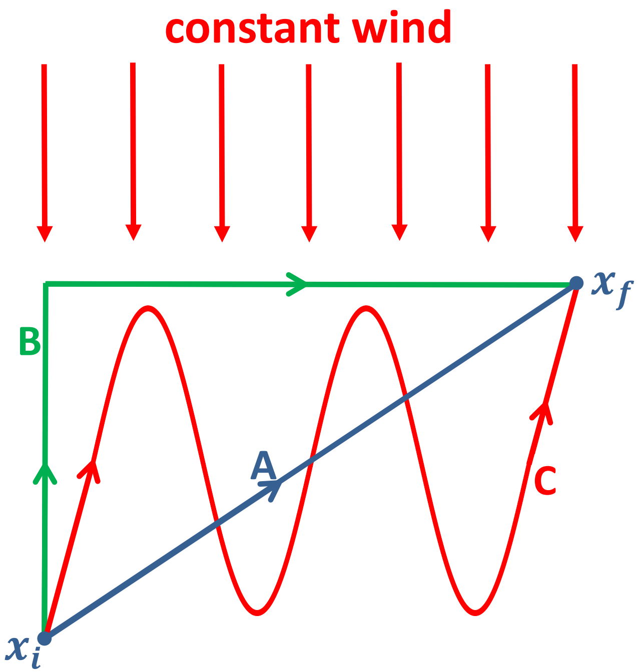
$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \vec{F} \cdot \int_{\vec{r}_i}^{\vec{r}_f} d\vec{r} = \vec{F} \cdot (\vec{r}_f - \vec{r}_i)$$

$$W = \vec{F} \cdot \Delta\vec{r} = (F_x, F_y) \cdot (\Delta r_x, \Delta r_y) = F_x \Delta r_x + F_y \Delta r_y$$

Example:



Example: Riding bike against the wind



$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

Work done by wind on different paths:

A. A = B = C

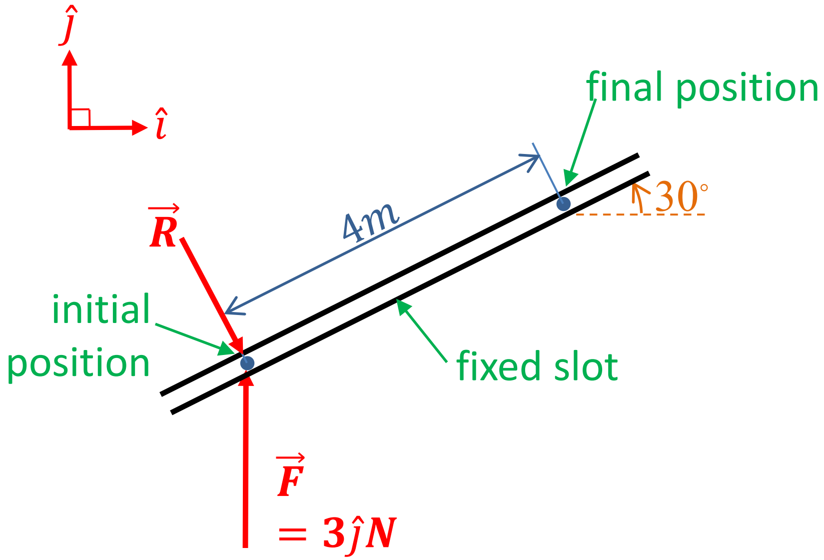
B. A = B ≠ C

C. A ≠ B = C

D. A = C ≠ B

E. All different

Example: Constant Force $\vec{F}$



Work done by $\vec{F}$

$W = 6\,J$

Assume: - negligible gravitational force

- frictionless
- constraint force $\vec{R}$ of slot on mass

Energy increase of mass ΔE :

A. >6 J

B. 6 J

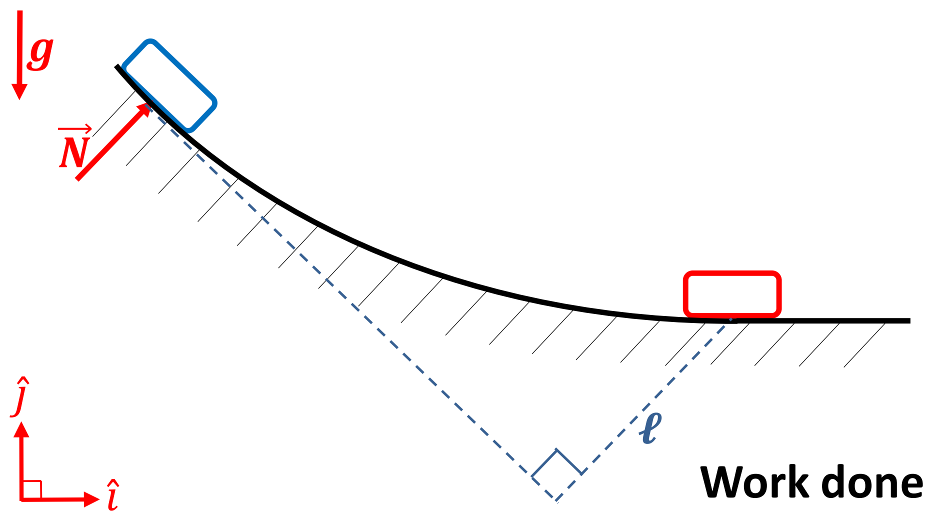
C. <6 J

D. need to find $\vec{R}$ to know

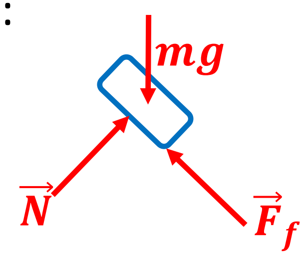
Forces of Constraint

Constraint forces do zero work because there is no net motion (displacement) in the constraint direction.

Example:



FBD:



Work done by $\vec{N}$ is

A. **positive**

B. **zero**

C. **negative**

D. **answer depends on ℓ, m, g**

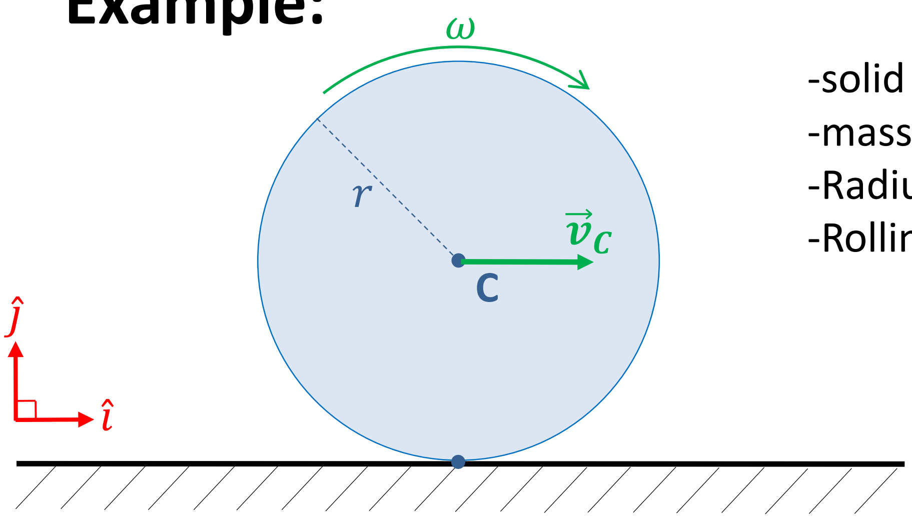
Rigid Body Energy

Remember:

$$T = \frac{1}{2}mv_C^2 + \frac{1}{2}I_C\omega^2$$

$$V = mgh_C$$

Example:



- -solid disk
- -mass m
- -Radius r
- -Rolling without slip

$T =$ **A.** mv_C^2

B. $\frac{3}{4}mr^2\omega^2$

C. $\frac{1}{4}mv_C^2$

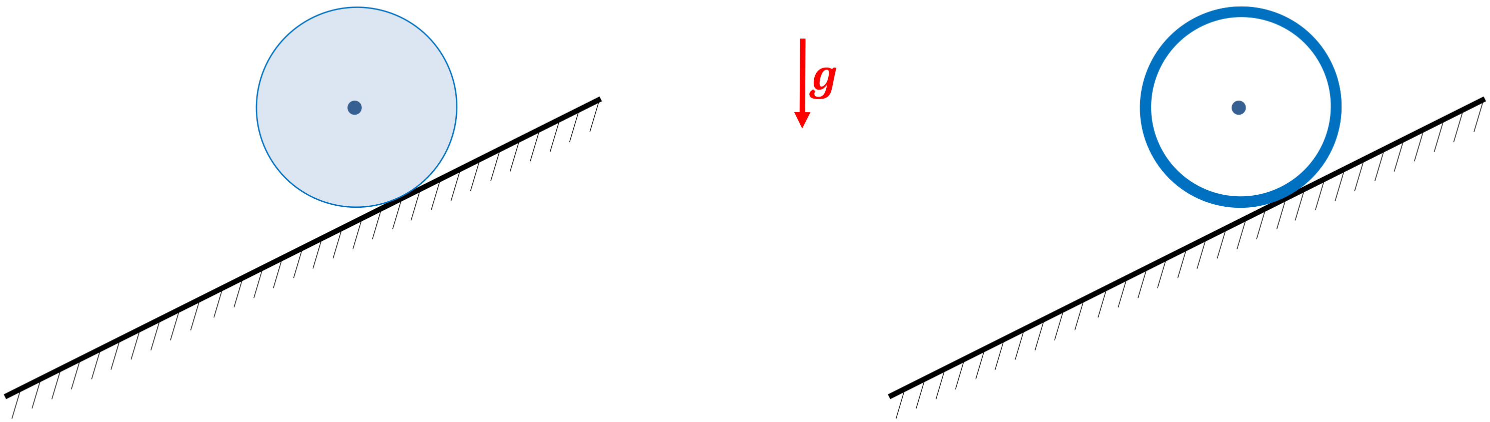
D. $\frac{1}{2}mv_C^2 + \frac{1}{2}mr^2\omega^2$

E. $\frac{2}{3}mv_C^2$

Example

A. Solid Cylinder

B. Hollow Cylinder



Both cylinders have same mass, same radius, same slope, start at rest, roll without slip.

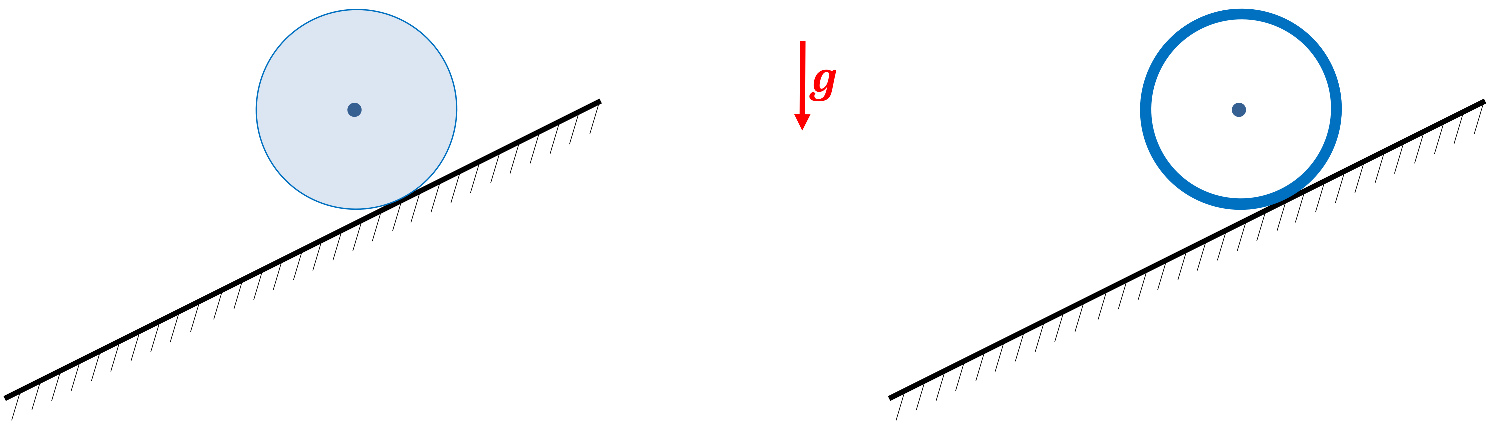
Which cylinder reaches the bottom first?

- **A. A**
- **B. B**
- **C. Same**
- **D. Can’t tell**

Example

A. Solid Cylinder

B. Hollow Cylinder



Both cylinders have same mass, same radius, same slope, start at rest, roll without slip.

How much faster is the solid cylinder?

- **A. 5%**
- **B. 15%**
- **C. 45%**
- **D. 65%**
- **E. Depends on r**

Work Done by Friction

Recall: $W = \Delta E = E_f - E_i$ (increase in energy)

$W_F =$ work done by friction force $\vec{F}_f$:

A. $W_F < 0$ always

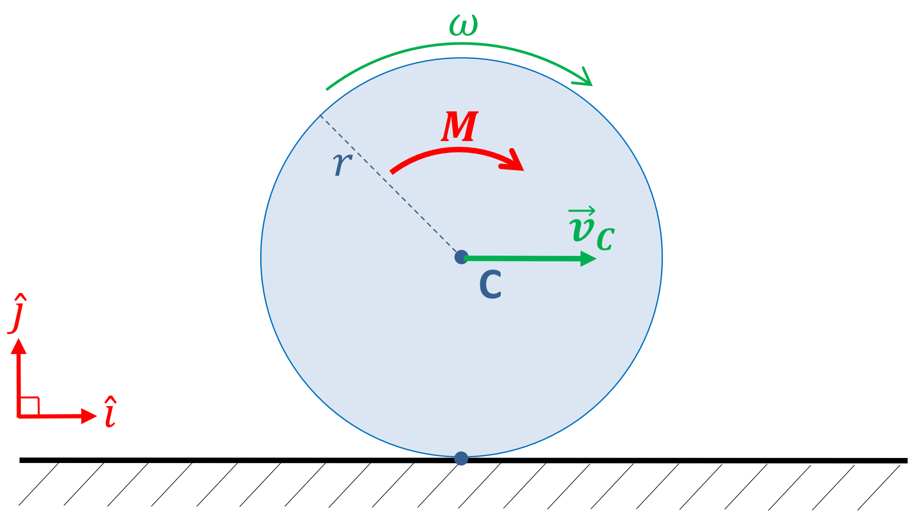
B. $W_F \leq 0$ always

C. $W_F > 0$ or $W_F < 0$, but $W_F \neq 0$

D. $W_F = 0$ always

E. W_F can be postive, negative or zero

Example: Rolling without Slip



Which way does friction force $\vec{F}_f$ act?

A. →

B. ←

C. zero

D. can't tell

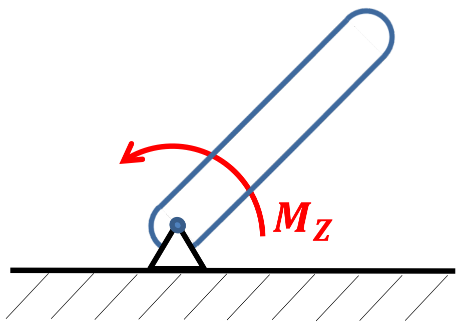
What is work done by $\vec{F}_f$?

A. positive

B. negative

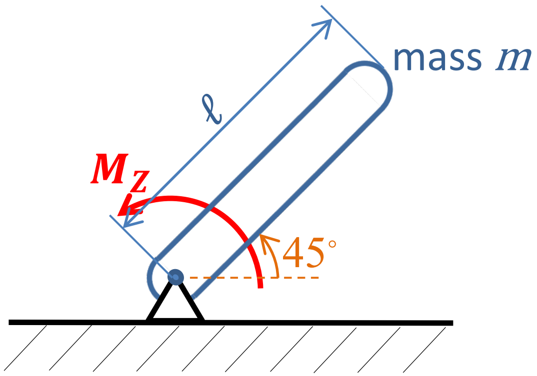
C. zero

Work Done by a Moment



$$W = \int_{\theta_i}^{\theta_f} M_Z d\theta = \int_{t_i}^{t_f} M_Z \dot{\theta} dt$$

Example:



Rod starts at rest , falls to horizontal.

Friction moment of 2 Nm

Work done by friction is: