TAM 212 – Dynamics

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Summer 2019

Recap

- Rigid Bodies in Contact, Gears

Today

- Rigid Body Acceleration, Gears

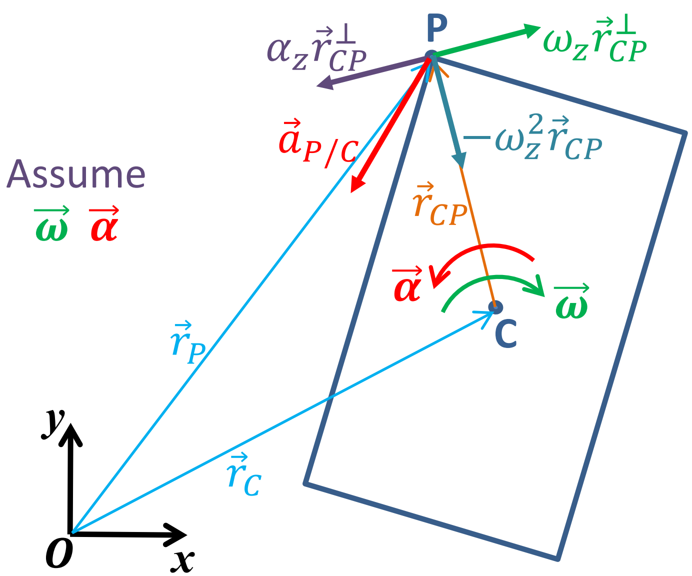
stick = no relative velocity at contact

$$\vec{v}_A = \vec{v}_B \qquad v_{An} = v_{Bn}$$

slip = relative velocity at contact

$$\vec{v}_A \neq \vec{v}_B \qquad v_{An} = v_{Bn}$$

Gears: $\boxed{\dfrac{\omega_{1z}}{\omega_{2z}} = -\dfrac{r_2}{r_1}}$



$$\vec{r}_P = \vec{r}_C + \vec{r}_{CP}$$

$$\vec{v}_P = \vec{v}_C + \vec{\omega} \times \vec{r}_{CP}$$

$$\vec{a}_P = \vec{a}_C + \vec{\alpha} \times \vec{r}_{CP} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{CP})$$

For 2D x-y plane:

$$\vec{r}_P = \vec{r}_C + \vec{r}_{CP}$$

$$\vec{v}_P = \vec{v}_C + \omega_z \vec{r}_{CP}^{\perp}$$

$$\vec{a}_P = \vec{a}_C + \alpha_z \vec{r}_{CP}^{\perp} - \omega_z^2 \vec{r}_{CP}$$

Standard Sign Conventions

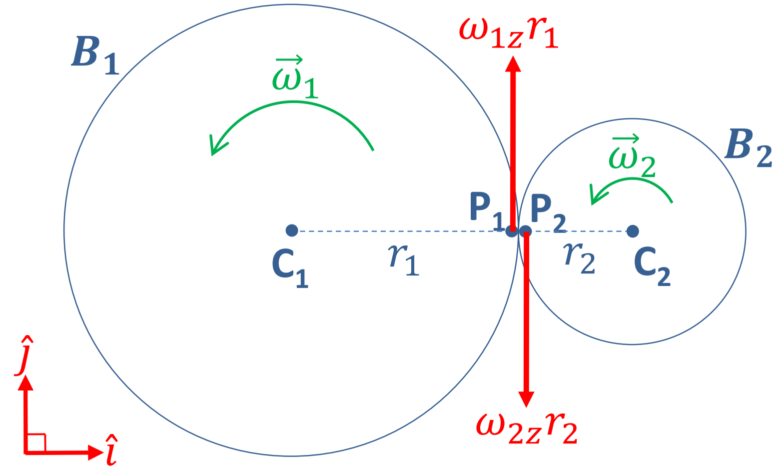
$$\vec{v}_{P_1} = \vec{v}_{C_1} + \vec{\omega}_1 \times \vec{r}_{C_1P_1}$$
$$= 0 + (0, 0, \omega_{1z}) \times (r_1, 0, 0)$$
$$= (0, \omega_{1z} r_1, 0)$$

$$\vec{v}_{P_2} = \vec{v}_{C_2} + \vec{\omega}_2 \times \vec{r}_{C_2P_2}$$
$$= 0 + (0, 0, \omega_{2z}) \times (-r_2, 0, 0)$$
$$= (0, -\omega_{2z} r_2, 0)$$

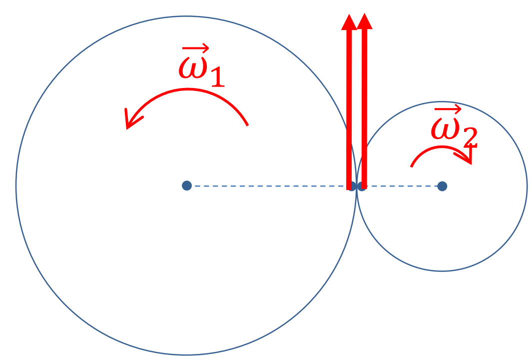
$$\boxed{\vec{v}_{P_1} = \vec{v}_{P_2}}$$

$$\omega_{1z} r_1 = -\omega_{2z} r_2$$

$$\boxed{\frac{\omega_{1z}}{\omega_{2z}} = -\frac{r_2}{r_1}}$$

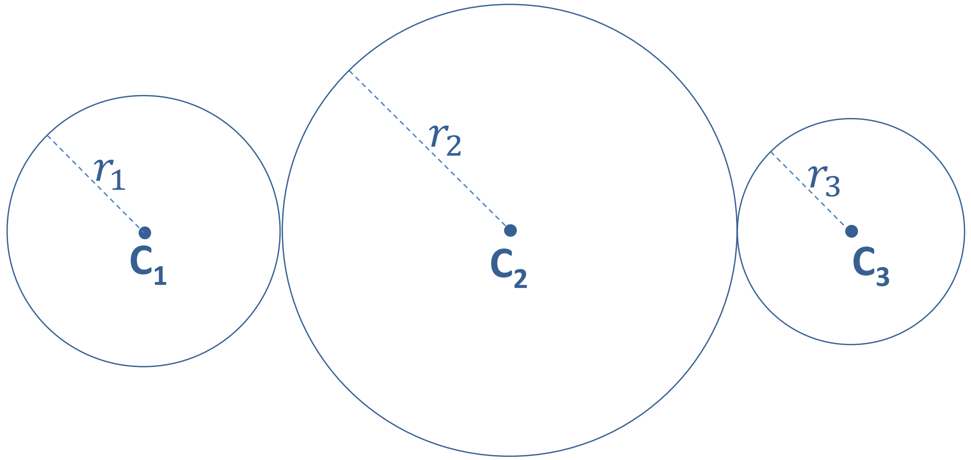


Standard sign convention uses CCW as positive direction for $\vec{\omega}$ and let the resulting sign of ω to determine the actual direction.



Alternative sign convention

Example



$$\left|\frac{\omega_{3Z}}{\omega_{1Z}}\right| =$$

A. $\frac{r_1}{r_3}$

B. $\frac{r_3}{r_1}$

C. $\frac{r_1 r_3}{r_2}$

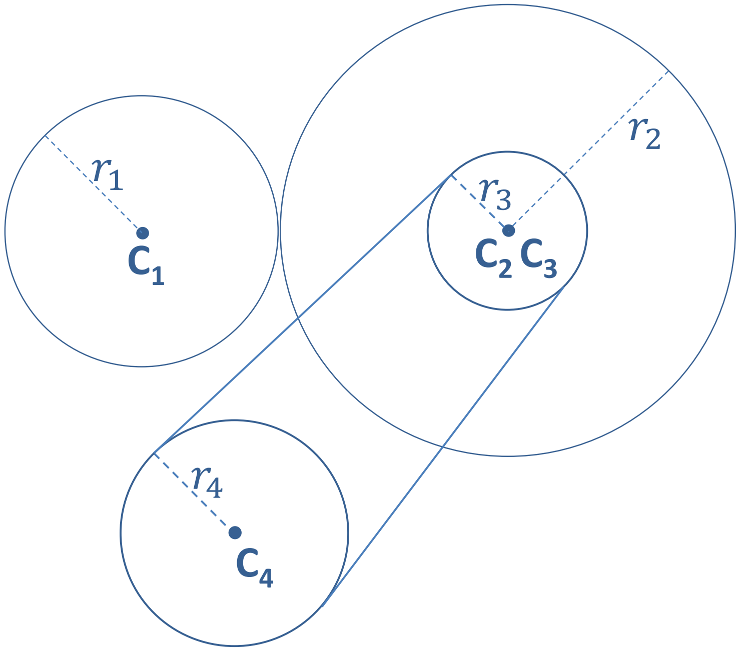
D. $\frac{r_2}{r_1 r_3}$

$$\frac{\omega_{3Z}}{\omega_{1Z}} = \square \underline{\qquad\qquad}$$

A. Positive sign

B. Negative sign

Example



$$\left|\frac{\omega_{4Z}}{\omega_{1Z}}\right| =$$

A. $\dfrac{r_1 r_2}{r_3 r_4}$

B. $\dfrac{r_1 r_4}{r_2 r_3}$

C. $\dfrac{r_2 r_3}{r_1 r_4}$

D. $\dfrac{r_1 r_3}{r_2 r_4}$

E. $\dfrac{r_3 r_4}{r_1 r_2}$

$$\frac{\omega_{4Z}}{\omega_{1Z}} = \square \underline{\qquad\qquad}$$

A. Positive sign

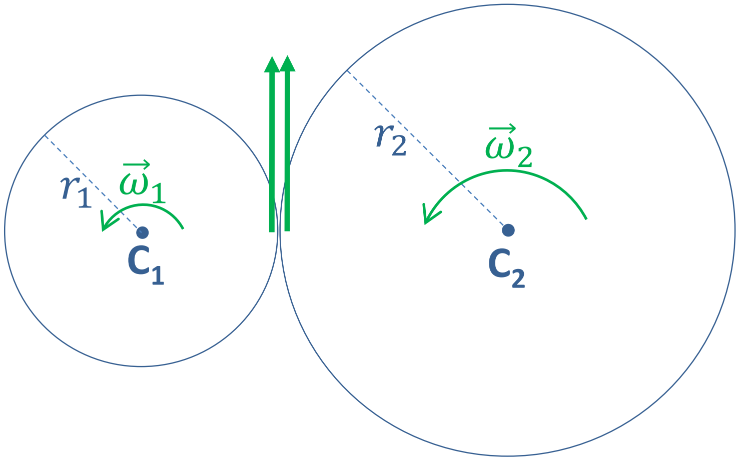
B. Negative sign

Acceleration in contact ("no slip")

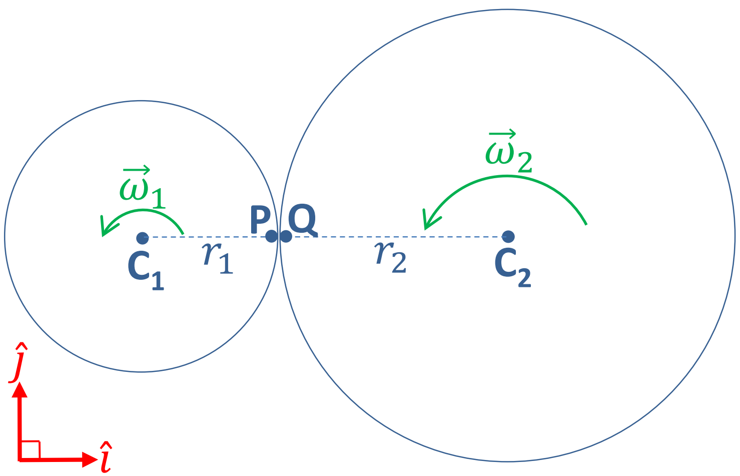
1. Differentiate velocity expressions

$$\frac{\omega_{2z}}{\omega_{1z}} = -\frac{r_1}{r_2}$$

$$\omega_{1z} r_1 = -\omega_{2z} r_2$$

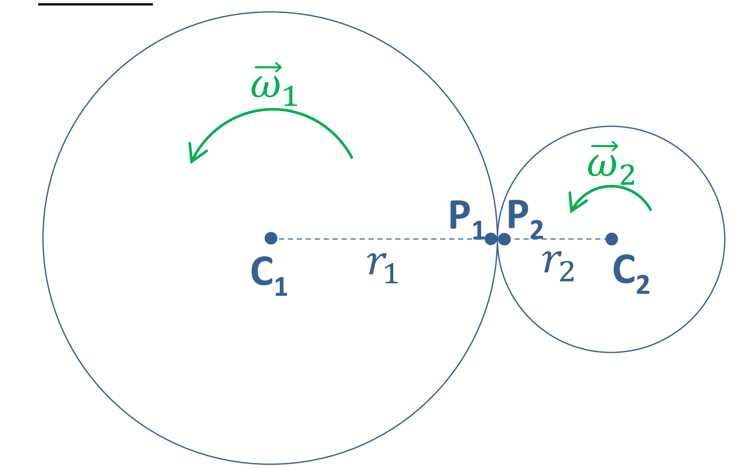


2. Contacting points have equal tangential acceleration



Gear Drives

Gears

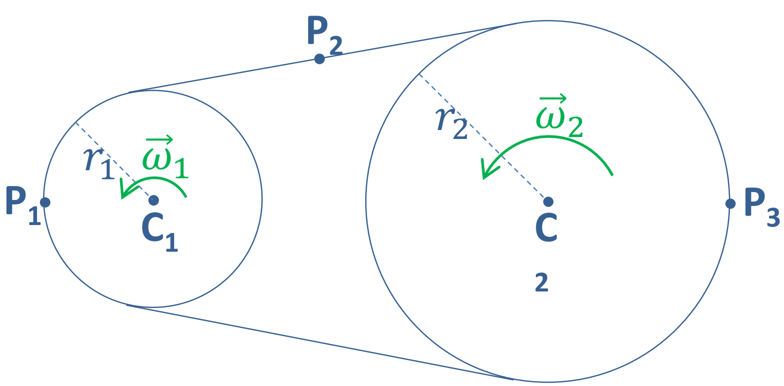


$\vec{v}_{P_1} = \vec{v}_{P_2}$ ← no slip

$$\frac{\omega_{2z}}{\omega_{1z}} = -\frac{r_1}{r_2}$$

larger gear spins slower
opposite directions

Chains

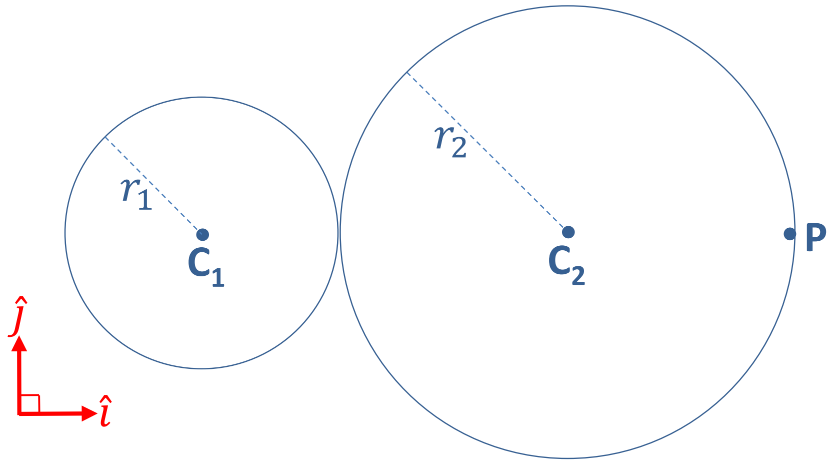


$v_{P_1} = v_{P_2} = v_{P_3}$ ← same speeds

$$\frac{\omega_{2z}}{\omega_{1z}} = \frac{r_1}{r_2}$$

larger gear spins slower
same directions

Example



$$r_1 = 2\text{ m}$$

$$r_2 = 4\text{ m}$$

$$\vec{\omega}_1 = 2\hat{k}\text{ rad/s}$$

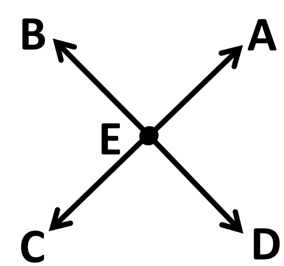
$$\vec{\alpha}_1 = -4\hat{k}\text{ rad/s}^2$$

$$\vec{a}_P = \left(a_{P_x}, a_{P_y}\right)$$

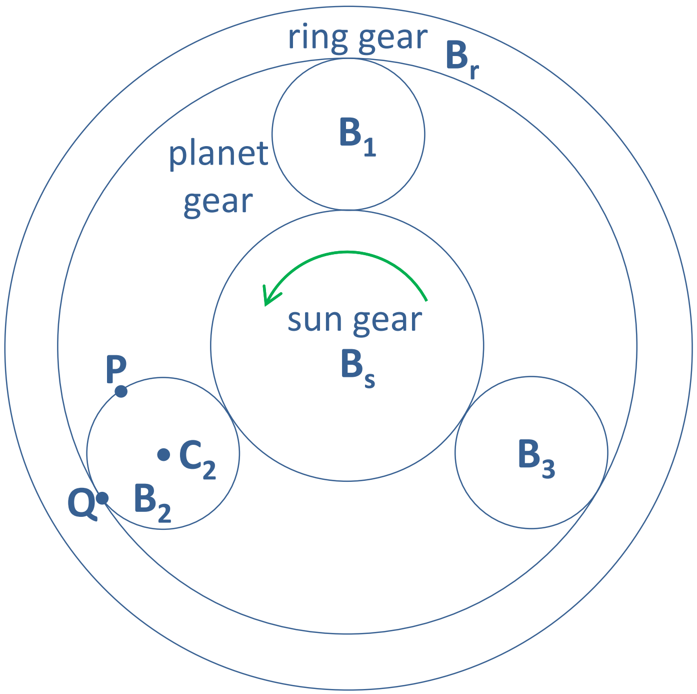
$\left|a_{P_x}\right| =$ **A.** 1 **B.** 2 **C.** 4 **D.** 8 **E.** 16 (m/s²)

$\left|a_{P_y}\right| =$ **A.** 1 **B.** 2 **C.** 4 **D.** 8 **E.** 16 (m/s²)

What is the direction of $\vec{a}_P$?



Example

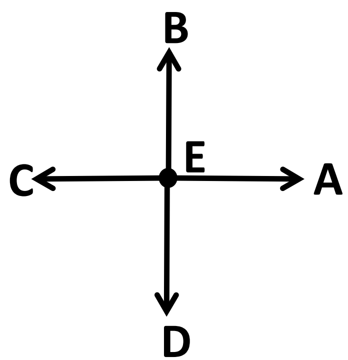


Planetary gears

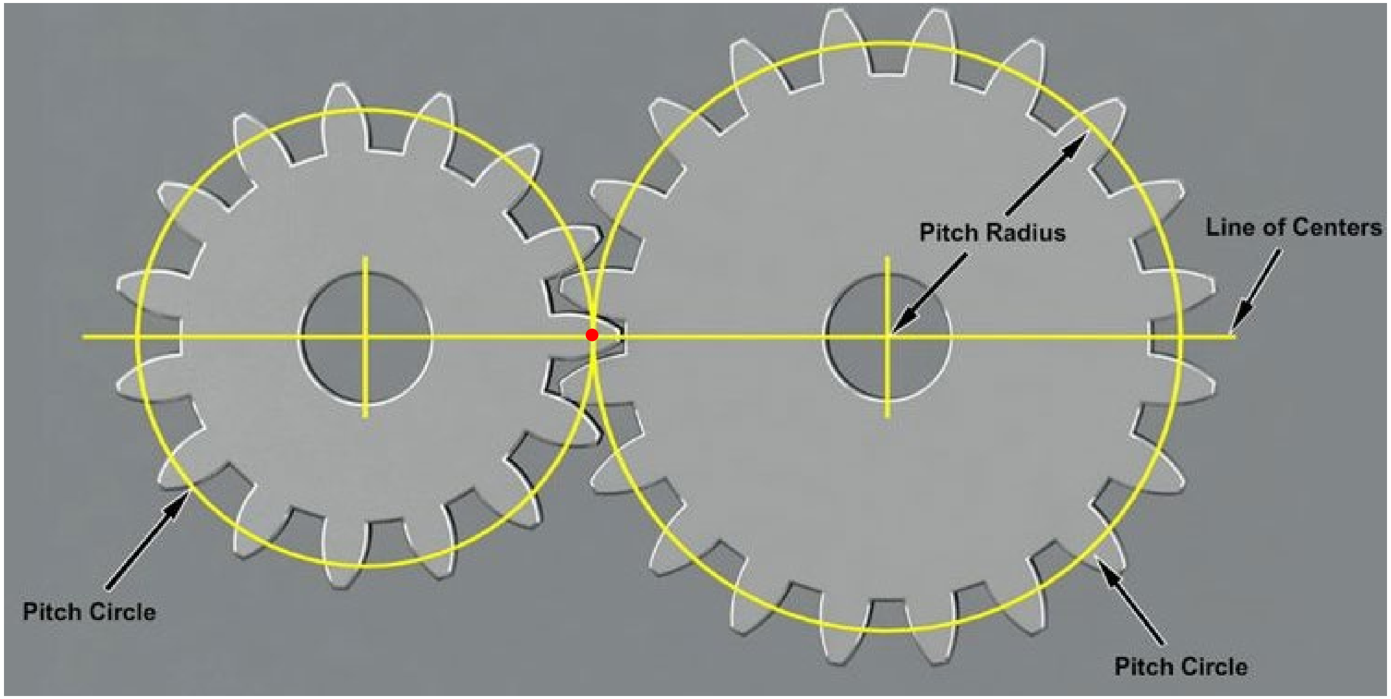
ring fixed

$\omega_{sZ} = 2$ rad/s

What is the direction of $\vec{v}_P$?



Contact Point



Speed and torque are transmitted at the **contact point**. The contact point is between the gear teeth along a line that passes through the line of centers of the two gears. This is a simplified view of the complex interaction between two gears. In this view, adopted in TAM 212, the contact point between two pitch circles is “<u>not moving</u>” and the transferring of torque is ideal (without losses).