TAM 212 – Dynamics

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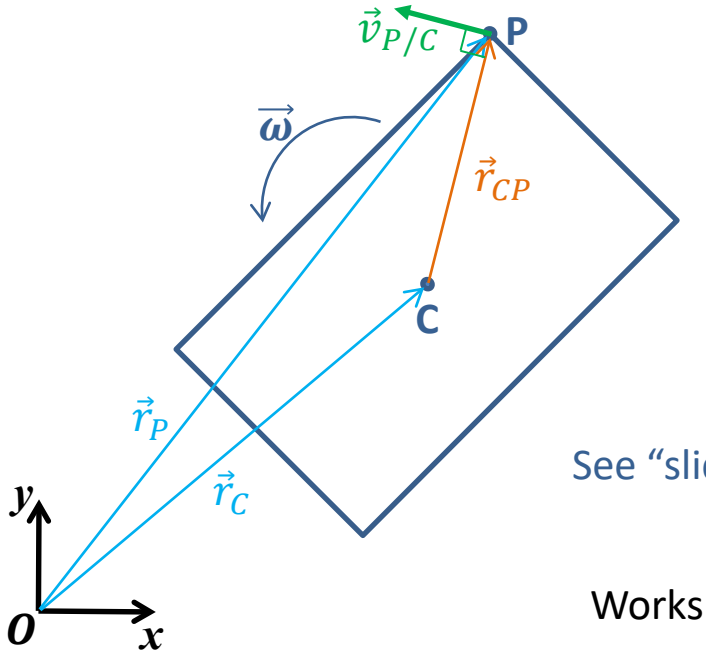
Summer 2019

Recap

- Rotating Rigid Body Motion

Today

- Rigid Body Motion, Constraints



points → position, velocity, acceleration

bodies → angular velocity & acceleration

$\vec{v}_{P/C}$ = velocity of P relative to C

$= \vec{v}_P - \vec{v}_C$

$$\vec{v}_{P/C} = \vec{\omega} \times \vec{r}_{CP}$$

$$\vec{v}_P = \vec{v}_C + \vec{\omega} \times \vec{r}_{CP}$$

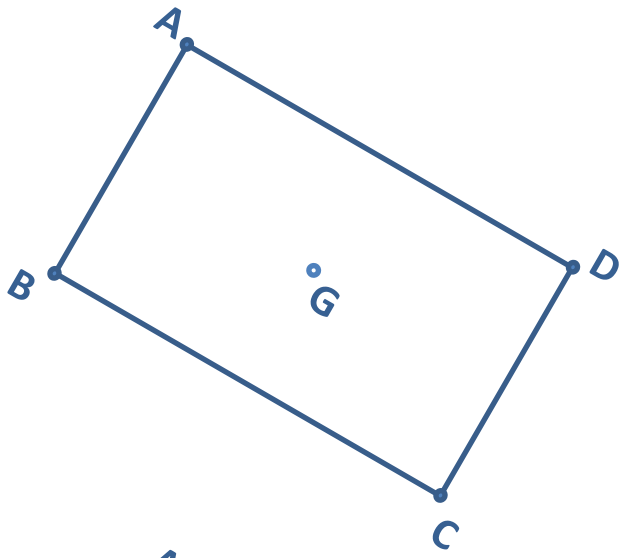
relative velocity equation

See “slide” animation in #rkg-fd

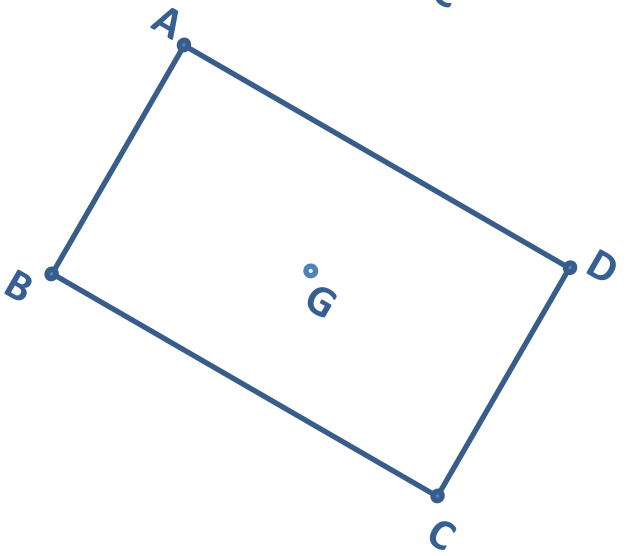
Works for any two points on RB: $\vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ}$ ($\vec{v}_{Q/P}$)

Example

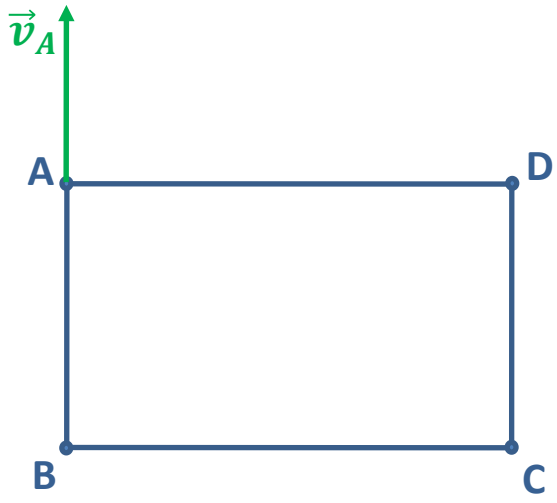
$\vec{v}_D = \vec{v}_G + \vec{\omega} \times \vec{r}_{GD}$

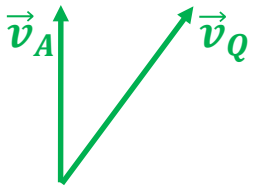


$\vec{v}_D = \vec{v}_C + \vec{\omega} \times \vec{r}_{CD}$



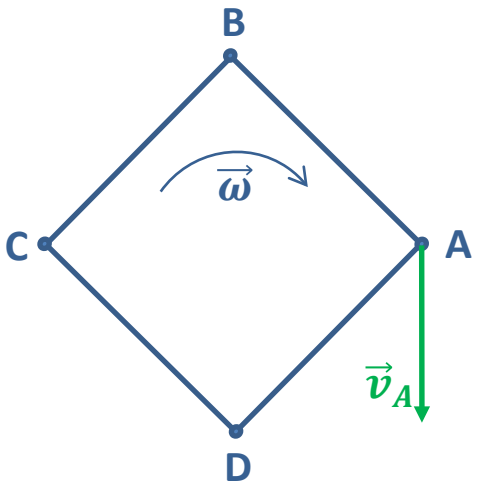
Example

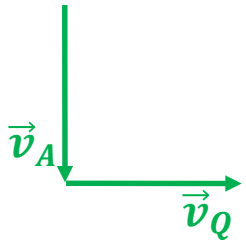




Which point is Q? Point A, B, C, or D

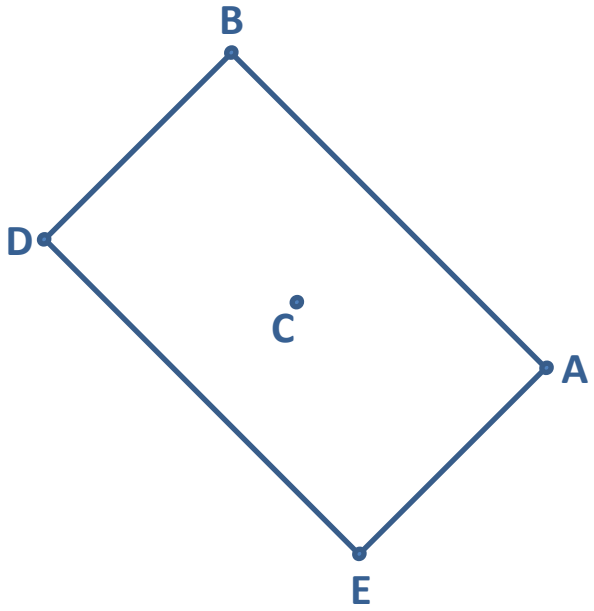
Example

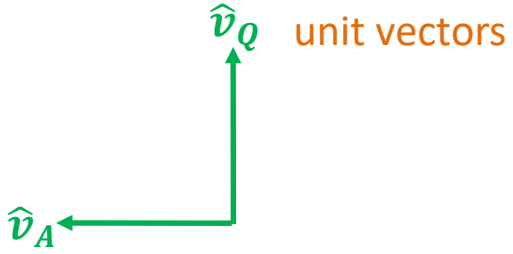




Which point is Q? Point A, B, C, or D

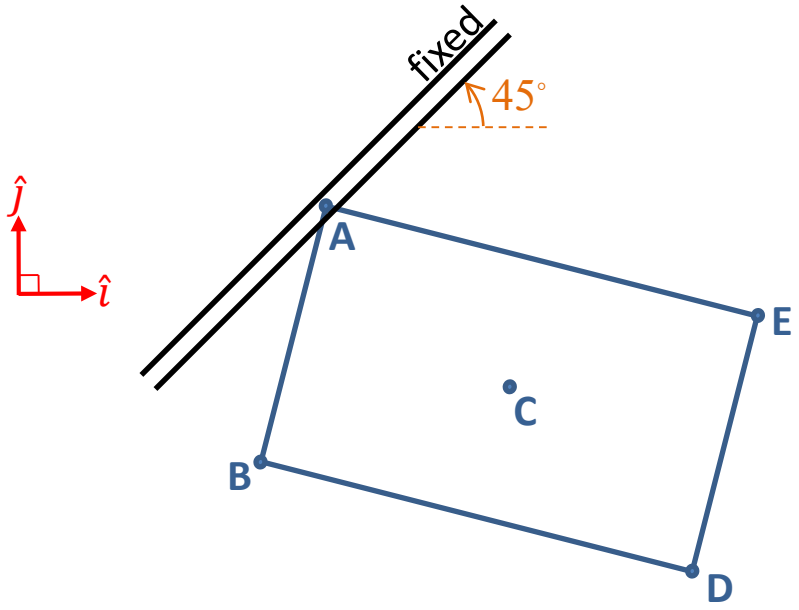
Example





Which point cannot be Q?
Point B, C, D, or E
(don't choose A which is a trivial solution)

Constrained Motion



$\vec{r}_{AE} = 4\hat{\imath} - \hat{\jmath}$ m

$\vec{v}_E = \hat{\imath} + 7\hat{\jmath}$ m/s

Point A constrained to the 45° slot

Two methods for constrained velocities

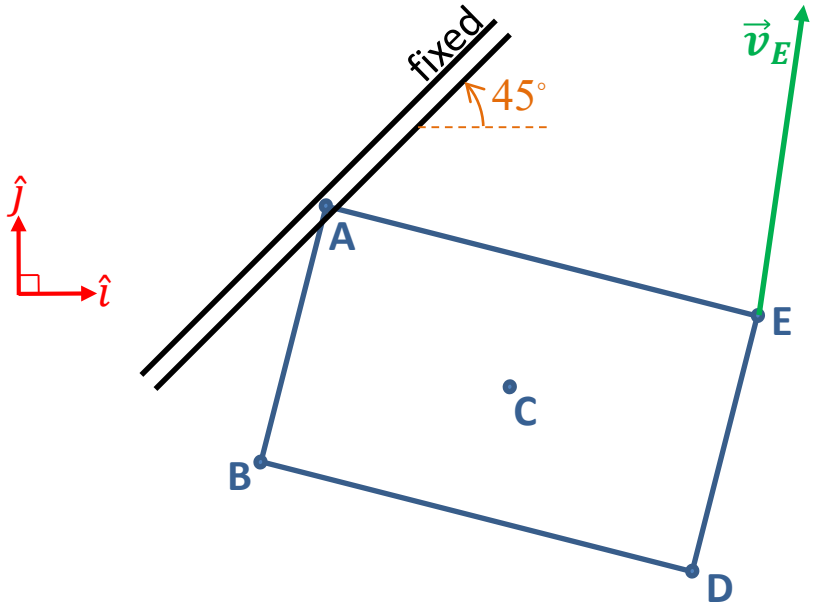
1. **Which way can it move?**

$$\Rightarrow \hat{u}_1 \Rightarrow \vec{v} = v_u \hat{u}_1$$

2. **Which way can't it move?**

$$\Rightarrow \hat{u}_2 \Rightarrow \vec{v} \cdot \hat{u}_2 = 0$$

1. Which way can it move?

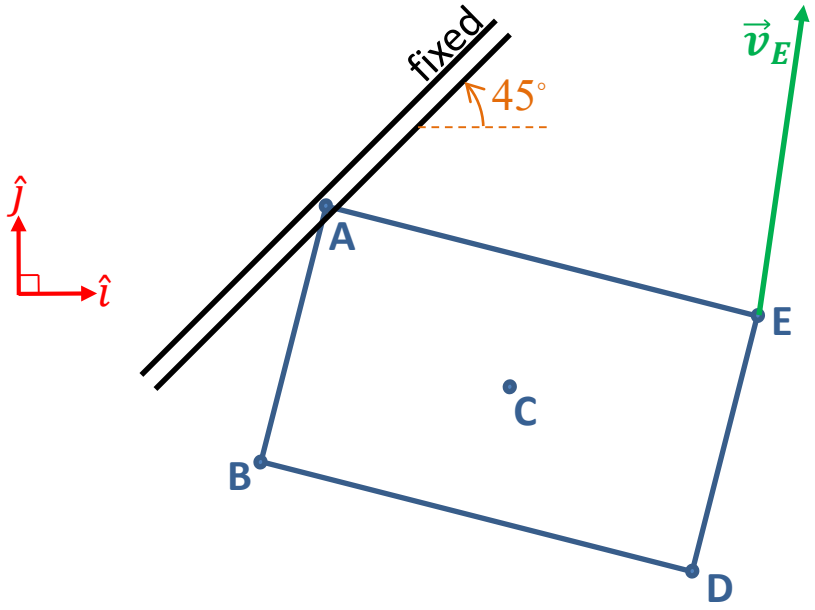


$\vec{r}_{AE} = 4\hat{\imath} - \hat{\jmath}$ m

$\vec{v}_E = \hat{\imath} + 7\hat{\jmath}$ m/s

Point A constrained to the 45° slot

2. Which way can't it move?

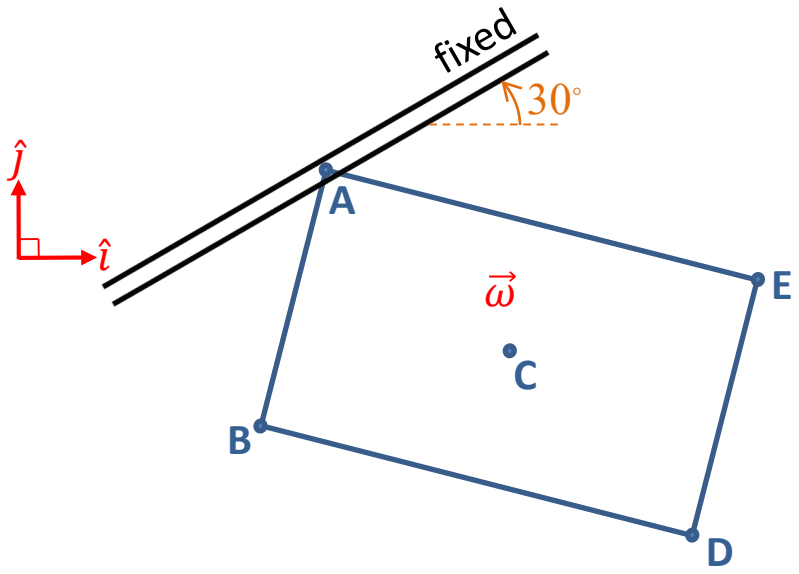


$\vec{r}_{AE} = 4\hat{\imath} - \hat{\jmath}$ m

$\vec{v}_E = \hat{\imath} + 7\hat{\jmath}$ m/s

Point A constrained to the 45° slot

Constrained Motion



$\vec{r}_{AE} = 4\hat{\imath} - \hat{\jmath}$ m

$\vec{v}_E = 3\hat{\imath} + 12\hat{\jmath}$ m/s

Point A constrained to the 30° slot

Which direction is $\vec{\omega}$?

A. CW

B. CCW

C. neither

D. can't tell

Which direction is $\vec{v}_A$?

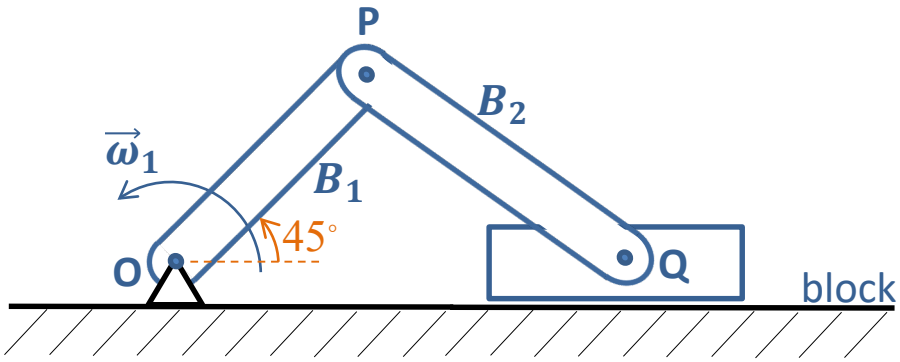
A. ↗

B. ↙

C. not moving

D. can't tell

Example



$OP = 3\sqrt{2}$ m

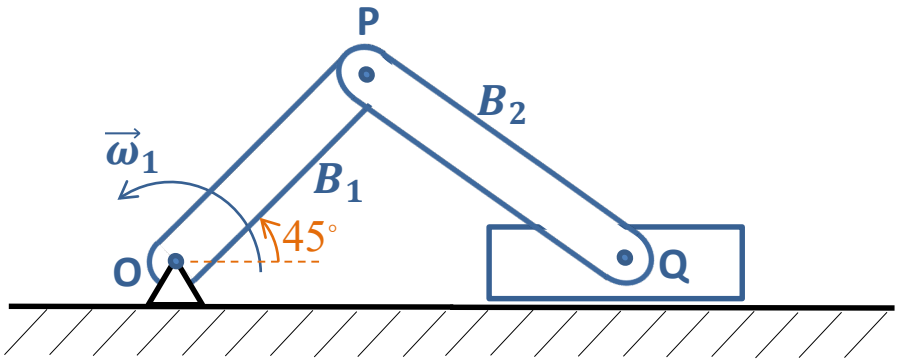
$PQ = 5$ m

$\vec{\omega}_1 = 4\hat{k}$ rad/s

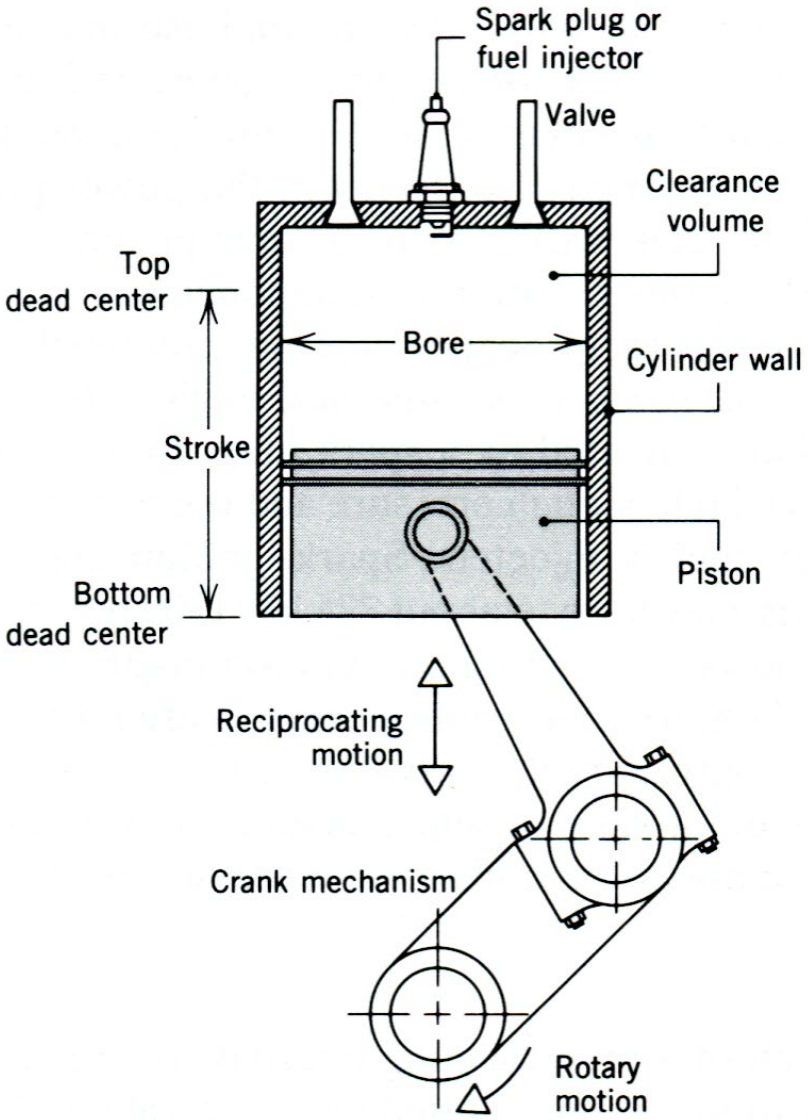
block slides along ground

1. Predict

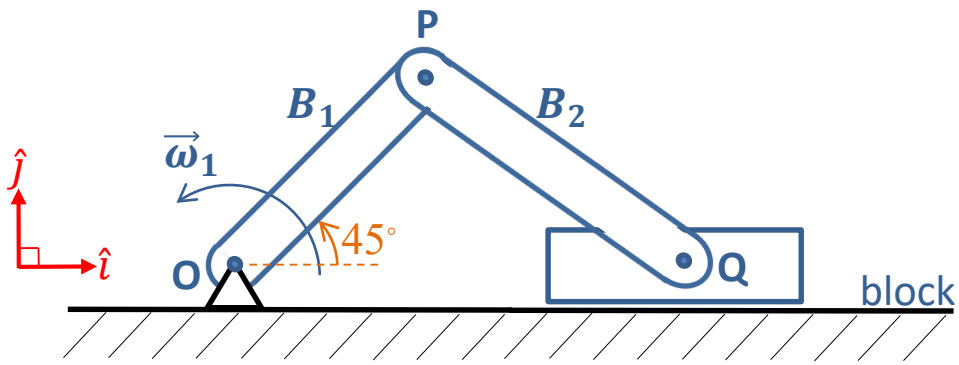
$\vec{\omega}_2 = \omega_{2Z}\hat{k}$



Slider-crank mechanism, arrangement of mechanical parts designed to convert straight-line motion to rotary motion, as in a reciprocating piston engine, or to convert rotary motion to straight-line motion, as in a reciprocating piston pump.



Example



$OP = 3\sqrt{2}$ m

$PQ = 5$ m

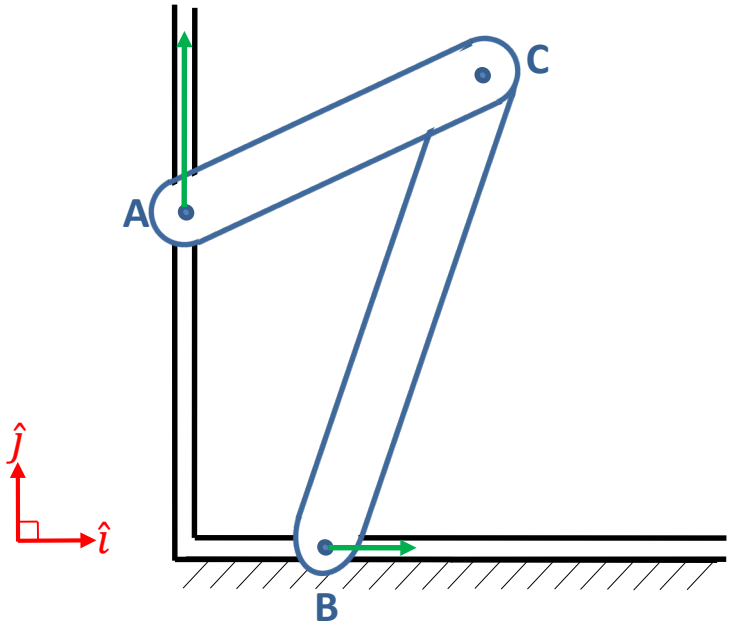
$\vec{\omega}_1 = 4\hat{k}$ rad/s

block slides along ground

2. Calculate

3. Reflect Does this make sense?

Example



$\vec{r}_{AC} = (2, 1)$ m

$\vec{r}_{BC} = (1, 3)$ m

$\vec{v}_A = (0, 2)$ m/s

$\vec{v}_B = (1, 0)$ m/s

What is $\vec{v}_C$?

1. Predict