TAM 212 – Dynamics

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Summer 2019

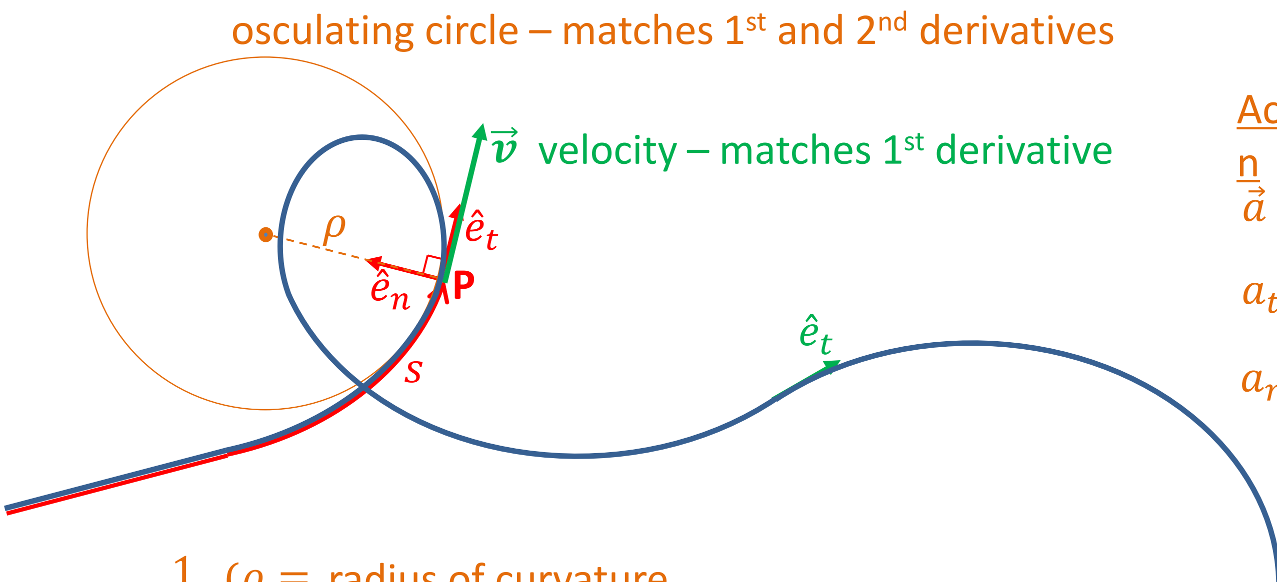
Recap
- Tangential-normal basis

Today
- Particle Kinetics

$s =$ arc length

$\dot{s} = v =$ speed

$\ddot{s} =$ rate of change of speed



$$\rho = \frac{1}{\kappa} \begin{cases} \rho = \text{radius of curvature} \\ \kappa = \text{curvature} \end{cases}$$

Velocity

$$\vec{v} = \dot{s}\hat{e}_t = v_t\hat{e}_t + v_n\hat{e}_n$$

$$v_t = \dot{s} \qquad v_n = 0$$

Acceleration

$$\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n = a_t\hat{e}_t + a_n\hat{e}_n$$

$$a_t = \ddot{s}$$

$$a_n = \frac{\dot{s}^2}{\rho} = \text{centripetal acceleration}$$

Example

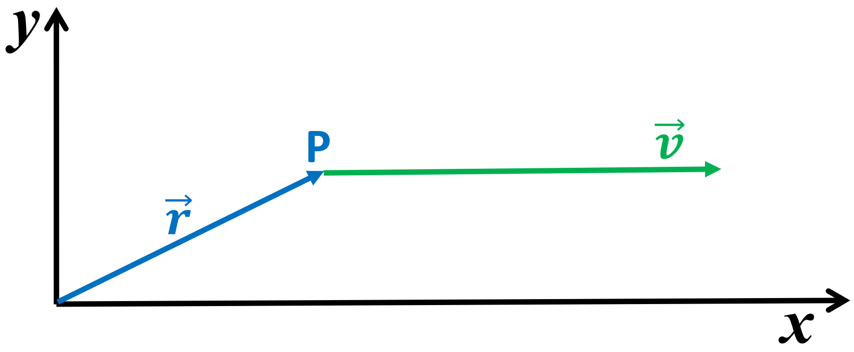
Car is speeding up and has:

$$\vec{r} = 4\hat{\imath} + 2\hat{\jmath} \ \text{ m}$$

$$\vec{v} = 6\hat{\imath} \ \text{ m/s}$$

$$a = 5 \ \text{ m/s}^2$$

$$\rho = 12 \ \text{ m}$$



$$\dot{\boldsymbol{v}} = ?$$

$$v = \dot{s}$$

$$\dot{v} = \ddot{s}$$

$$\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

Particle Kinetics

Kinematics $\longrightarrow$ $\vec{r}, \vec{v}, \vec{a}$ (no forces)

Kinetics $\longrightarrow$ $\vec{F} = m\vec{a}$, forces, moments

Statics

$$\sum \vec{F} = 0$$

Kinematics

$$\vec{a}$$

Kinetics

$$\sum \vec{F} = m\vec{a}$$

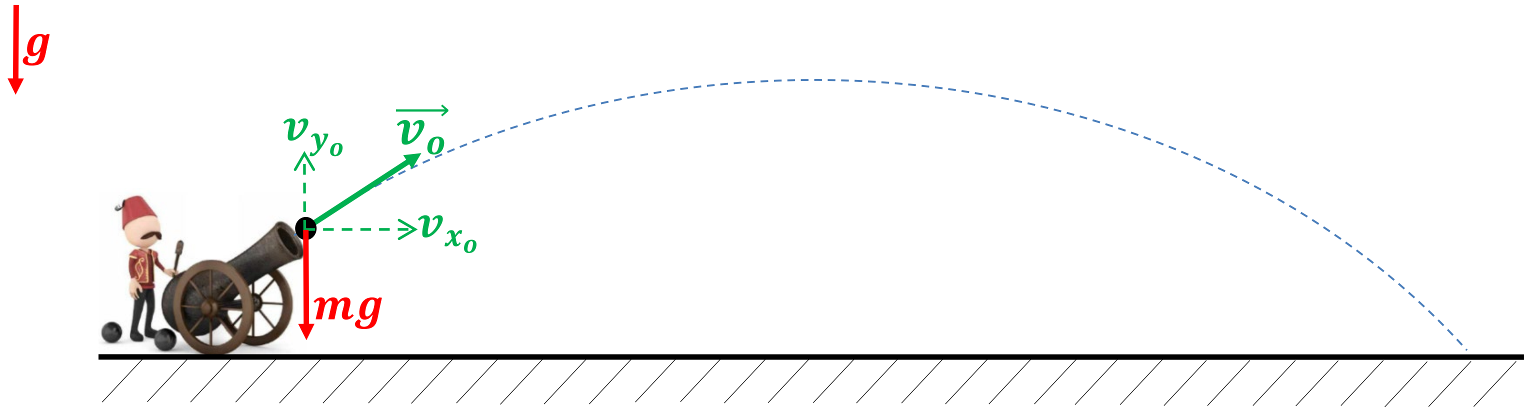
Classical Mechanics: "All models are wrong. Some models are useful." - George Box

Classical Mechanics: $\sum \vec{F} = m\vec{a}$

Method of assumed forces: $\vec{F} \rightarrow \vec{a}$ (simulation)

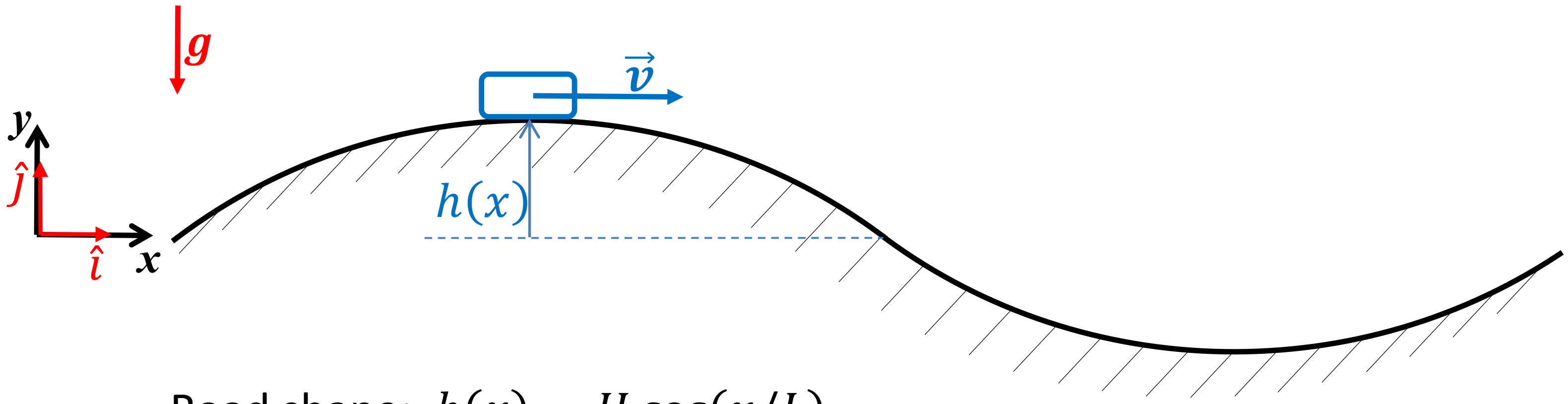
Method of assumed motion: $\vec{a} \rightarrow \vec{F}$ (measurement)

Example: Assumed Forces: Cannon



Example

Assumed motion: car on road



Road shape: $h(x) = H\cos(x/L)$

N = vertical force of the road on the car

Solution steps

Method of assumed forces: $\vec{F} \rightarrow \vec{a}$ (simulation)

Method of assumed motion: $\vec{a} \rightarrow \vec{F}$ (measurement)

Solution steps

1. System diagram – coordinates
2. Free body diagrams (FBD) – forces
3. Kinematics $\rightarrow \vec{a}$
4. Kinetics: Newton $\vec{F} = m\vec{a}$
5. Algebra: solve for $\vec{F}$ or $\vec{a}$

Thinking about $\vec{F} = m\vec{a}$

Vector Form

$$\vec{F} = m\vec{a}$$

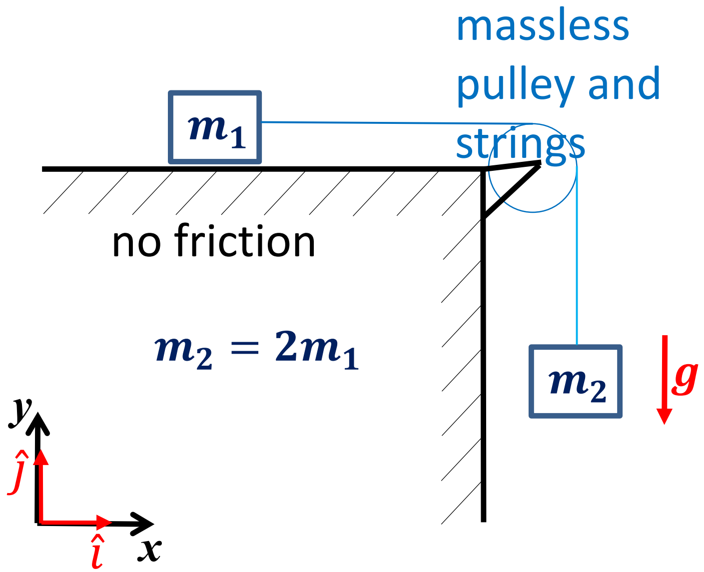
$$\begin{cases} F_x = ma_x \\ F_y = ma_y \\ F_z = ma_z \end{cases}$$

Conceptual

$$\vec{F} = m\vec{a}$$

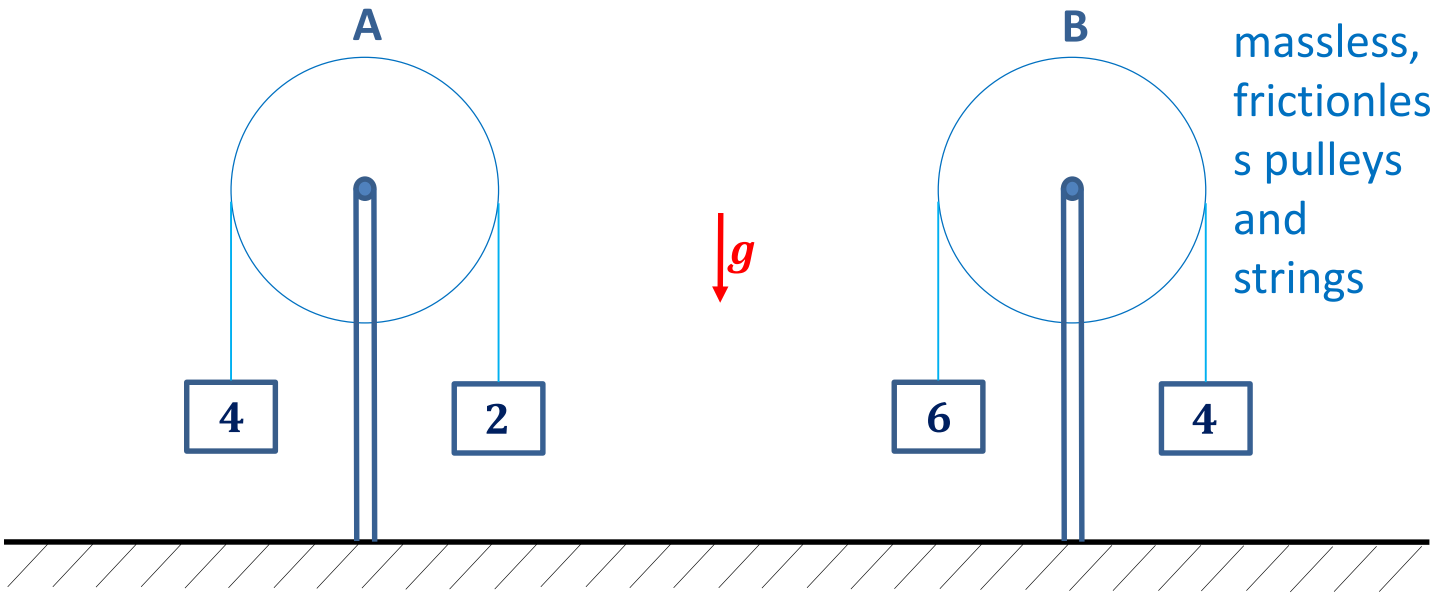
What is the acceleration?

Example: Multiple masses



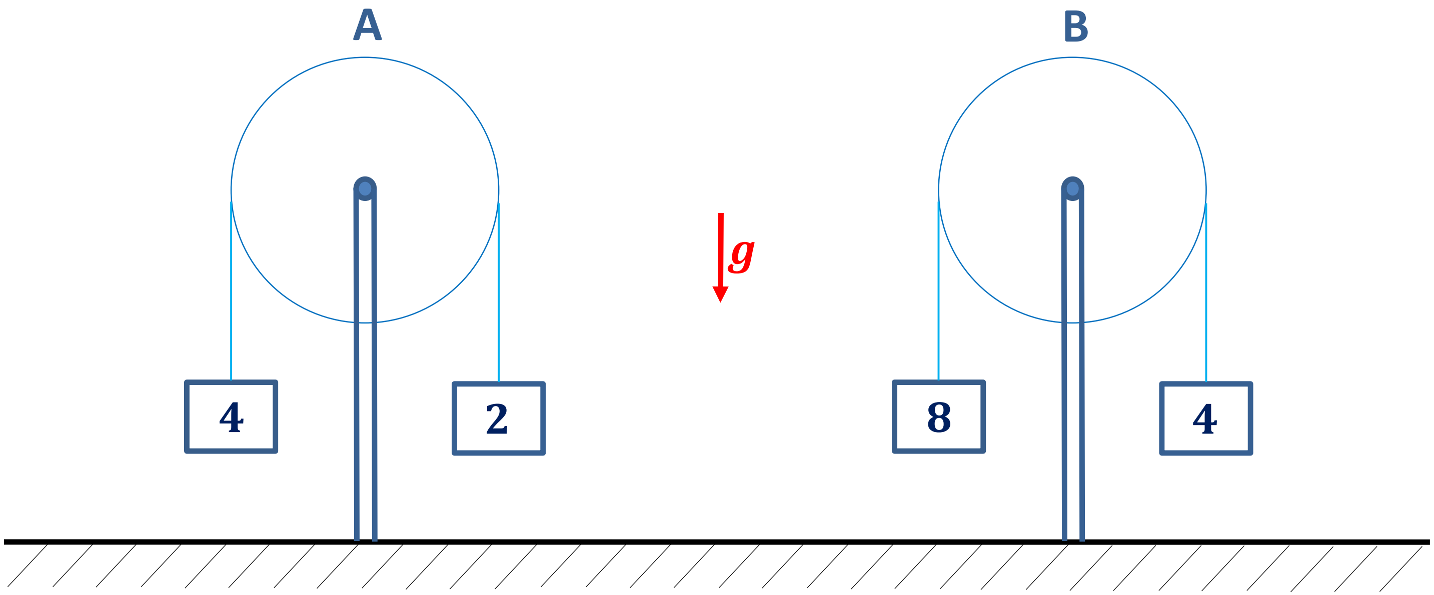
<u>Conceptual</u>

Example

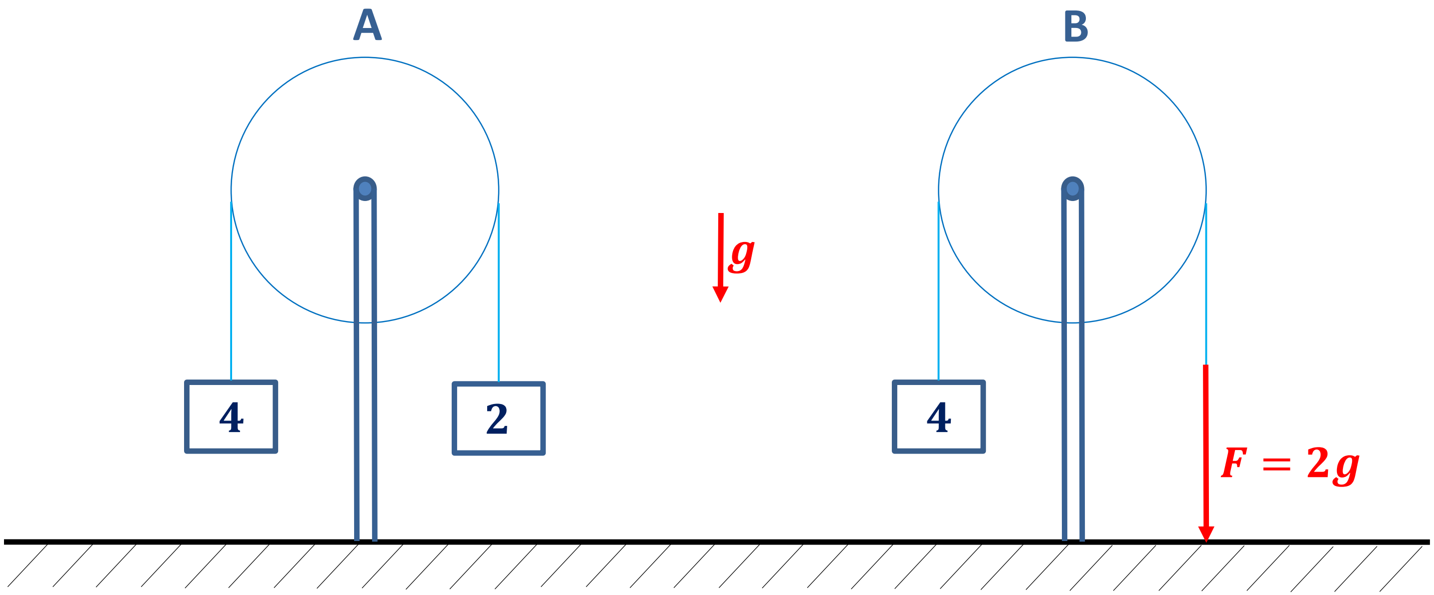


massless, frictionless pulleys and strings

Example

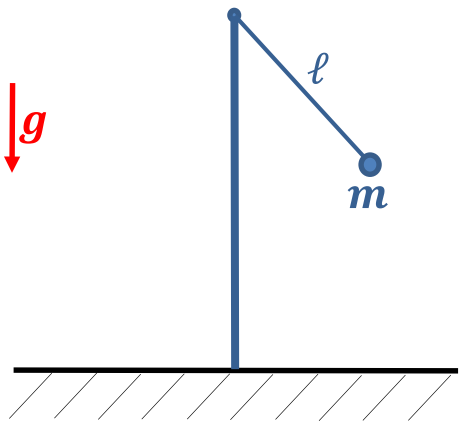


Example



Kiiking

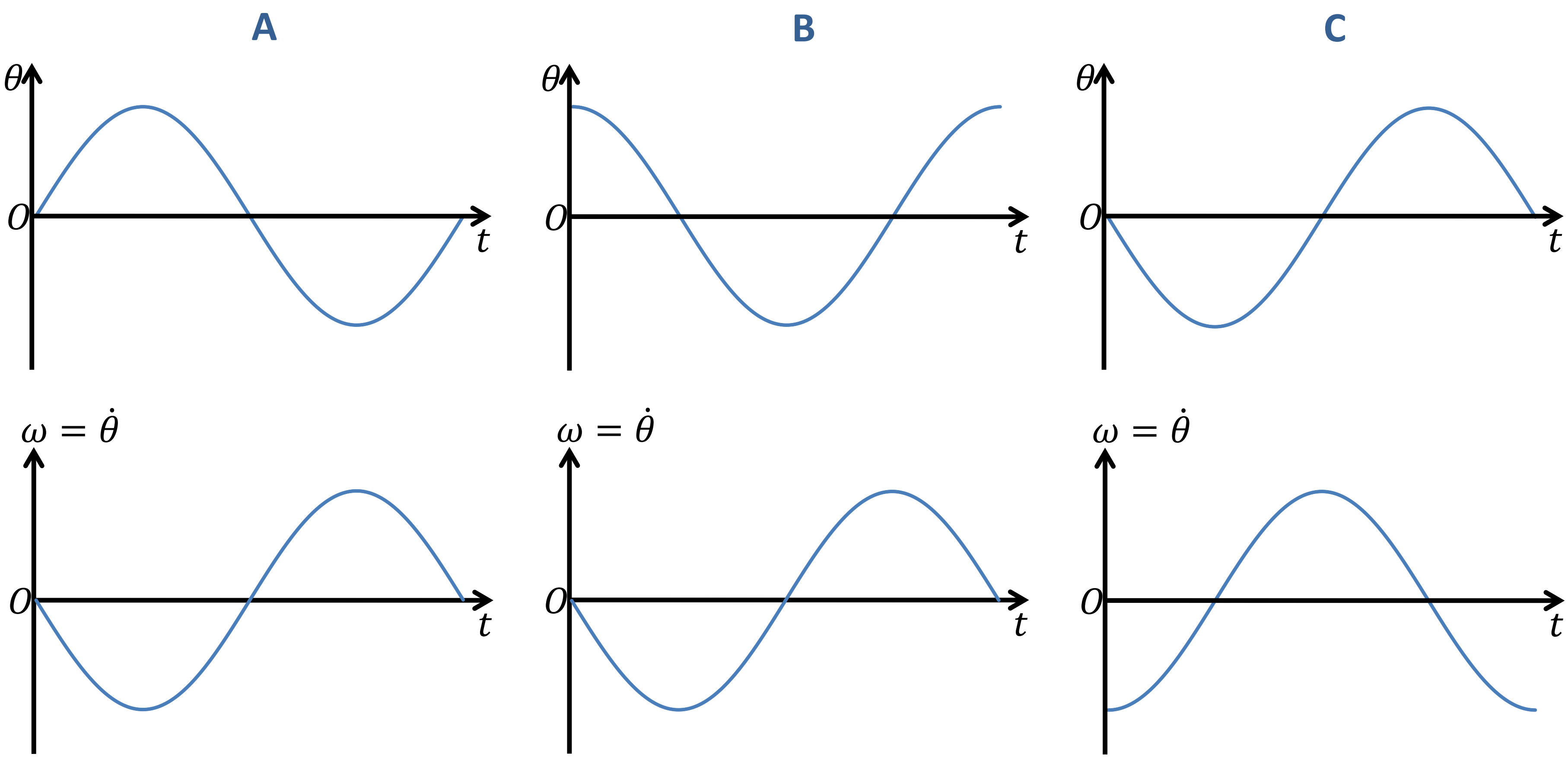
1. System diagram



Which coordinate system is most suitable for this?

- **A. Cartesian**
- **B. Polar**
- **C. Tangential-Normal**

Kiiking



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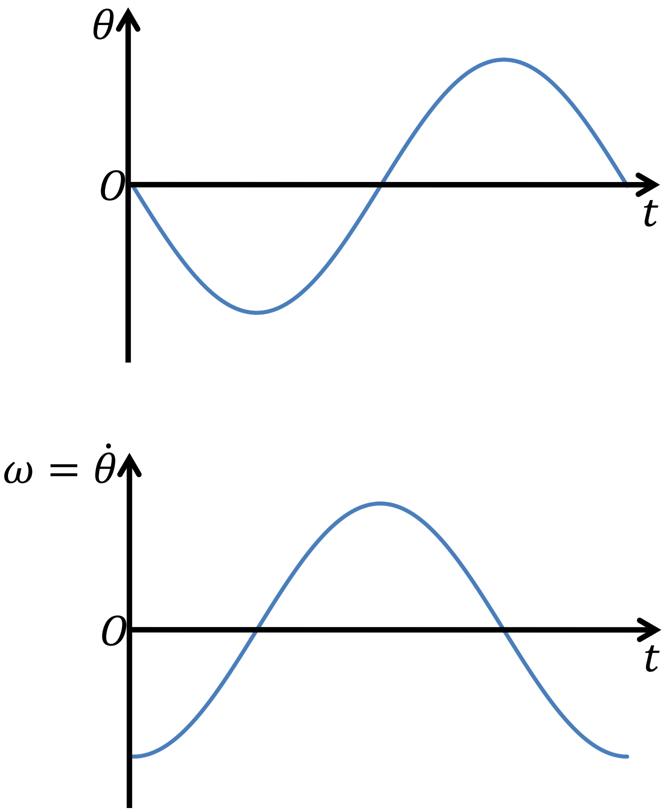


Numerical Integration Simulation

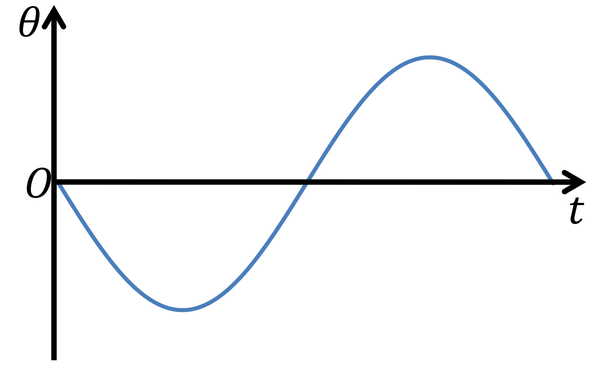
Independent variables:

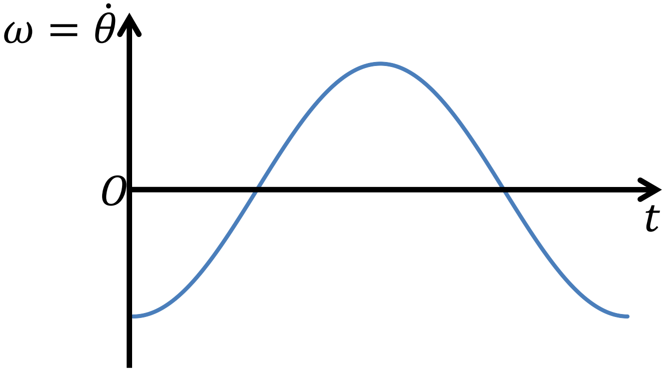
Initial conditions:

State variables:

Time step:

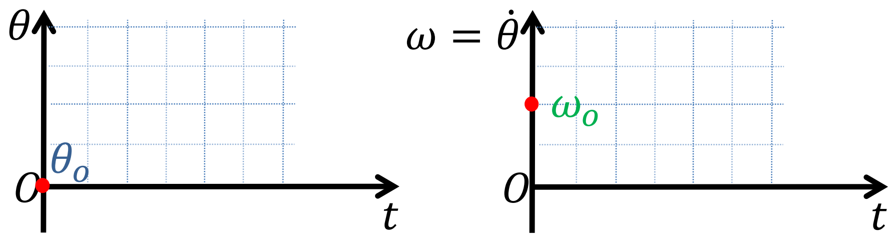
Update rule:



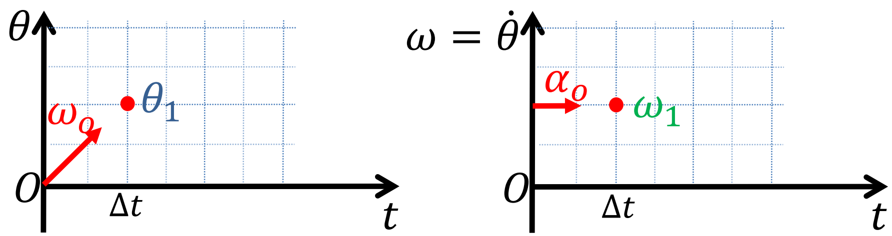


Numerical Integration Diagram

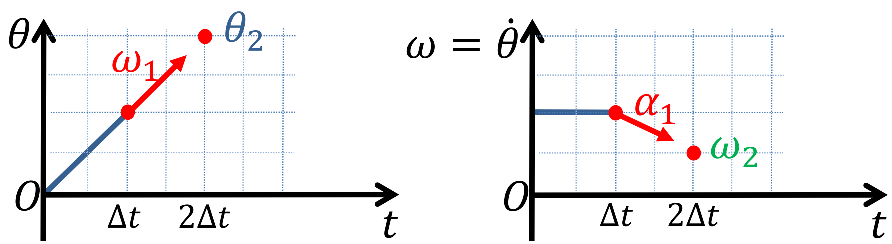
Initial Conditions:



First Step:



Second Step:



θ_o given

ω_o given

calculate α_o

$\theta_1 = \theta_o + \Delta t \omega_o$

$\omega_1 = \omega_o + \Delta t \alpha_o$

calculate α_1

$\theta_2 = \theta_1 + \Delta t \omega_1$

$\omega_2 = \omega_1 + \Delta t \alpha_1$

Numerical Integration Summary

1. Independent variables for us is time.

2. State variables come in pairs: position, velocity (not acceleration).

$$\theta(t) \qquad \omega(t)$$
$$r_x(t) \qquad v_x(t) = \dot{r}_x(t)$$

3. Initial conditions are the state variables at $t = 0$:

$$\theta_o = \theta(0) \qquad \omega_o = \omega(0)$$
$$r_{xo} = r_x(0) \qquad v_{xo} = v_x(0)$$

4. Time step Δt is how to jump forward in time.

5. Update rule: (need acceleration here)

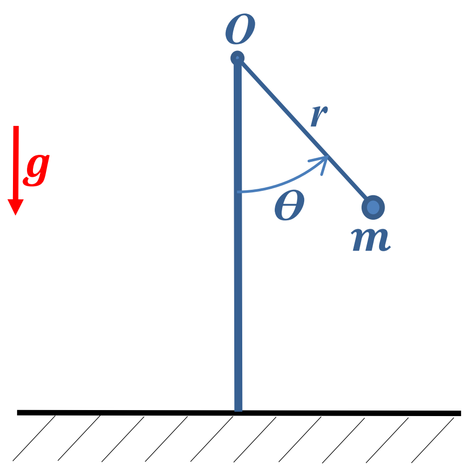
$$t_{n+1} = t_n + \Delta t = n\Delta t$$

$$\theta_{n+1} = \theta_n + \Delta t \dot{\theta}_n = \theta_n + \Delta t \omega_n$$
$$\omega_{n+1} = \omega_n + \Delta t \ddot{\theta}_n = \omega_n + \Delta t \alpha_n$$
$$r_{x,n+1} = r_{x,n} + \Delta t \dot{r}_{x,n} = r_{x,n} + \Delta t v_{x,n}$$
$$v_{x,n+1} = v_{x,n} + \Delta t \ddot{r}_{x,n} = v_{x,n} + \Delta t a_{x,n}$$

6. Need to compute second derivatives $\left(\ddot{\theta} = \alpha, \ddot{r}_x = a_x\right)$ at each time step.

Kiiking – How do we calculate α?

1. System diagram



Solution steps

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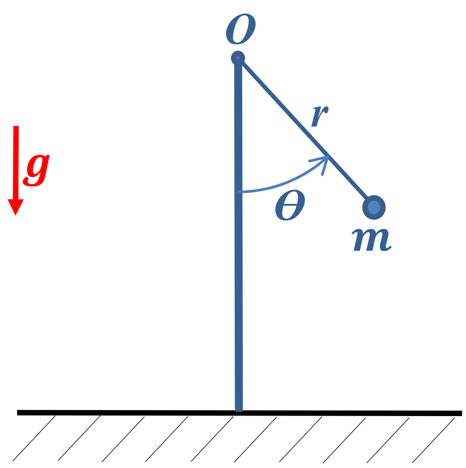
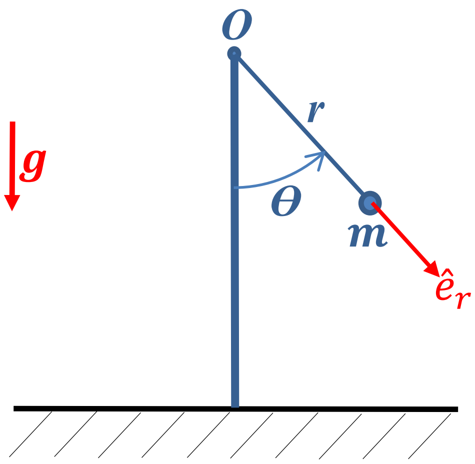


2. FBD

Kiiking



The tension in the swing arm is T