

TAM 212 – Dynamics

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Recap

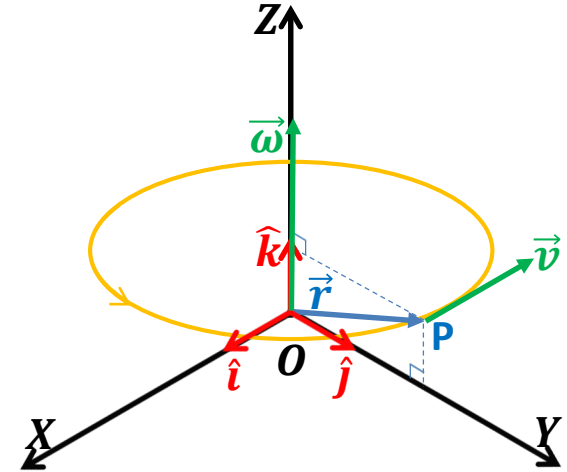
- Angular velocity and acceleration

Today

- Solving vector equations
- Tangential-normal basis

Angular velocity is a vector $\vec{\omega} = \omega \hat{\omega}$

$$\vec{\omega} = \begin{cases} \text{magnitude} & \omega = \text{speed of rotation} \\ \text{direction} & \hat{\omega} = \text{axis of rotation (RH rule)} \end{cases}$$



Point **P** rotating about origin

Position vector $\vec{r}$

Velocity vector $\vec{v} = \vec{\omega} \times \vec{r}$

Acceleration vector

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

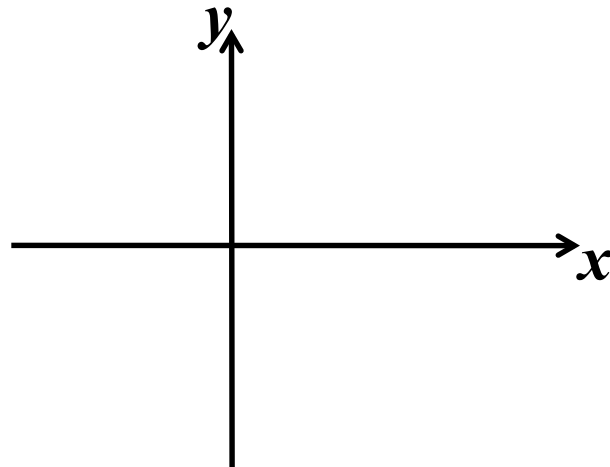
For 2D x-y plane: $\vec{v} = \vec{\omega} \times \vec{r} = \omega_z \vec{r}^\perp = \omega_z (-r_y, r_x)$

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \alpha_z \vec{r}^\perp - \omega_z^2 \vec{r}$$

Example - Solving vector equations

A particle in the x-y plane is rotating about the origin with an angular velocity $\vec{\omega} = -2\hat{k}$ rad/s. The particle has a velocity $\vec{v} = 12\hat{i} - 2\hat{j}$ m/s.

What is the position vector $\vec{r}$ of the particle?



Solving vector equations

1. Write vector unknowns in terms of scalar variables
2. Expand vector equations and equate components to get scalar equations
3. Solve scalar equations for scalar unknowns
4. Reconstruct vector unknowns

Example

A particle is rotating in the x-y plane with variable angular velocity. At an instant, it is observed to have: $\vec{r} = 2\hat{i} - \hat{j}$ m and $\vec{a} = -4\hat{i} - 3\hat{j}$ m/s²

What are $\vec{\omega}$ and $\vec{\alpha}$?

Example - Solving vector equations

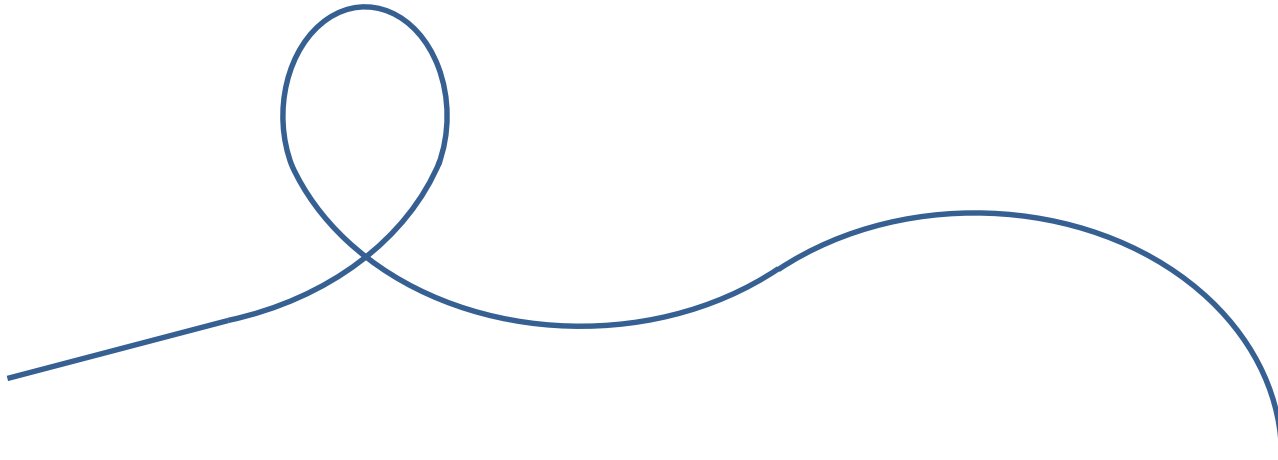
Given $\vec{r} = (2, -1)$ and $\vec{a} = (-4, -3)$. Want $\vec{\omega}$ and $\vec{\alpha}$

1. Scalar variables:
2. Expand vector equations:
3. Solve scalar equations:
4. Reconstruct vector unknowns:

Do we need to have the same numbers of vector variables as the vector equations? A. Yes B. No

Do we need to have the same numbers of scalar variables as the scalar equations? A. Yes B. No

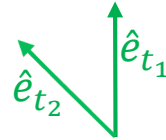
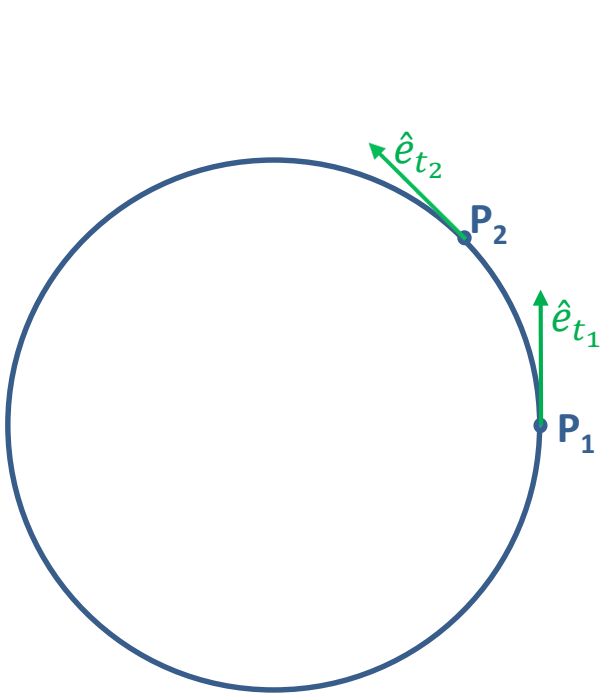
Tangential-Normal (TN) basis



TN basis $\begin{cases} \hat{e}_t = \text{tangential basis vector in the direction of motion} \\ \hat{e}_n = \text{normal basis vector, points inward, } \hat{e}_t \perp \hat{e}_n \end{cases}$

TN basis defined for a point on a path

What is $\dot{\hat{e}}_t$?

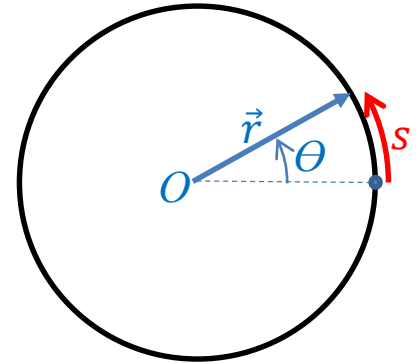


$$\dot{\hat{e}}_t = \frac{d}{dt} \hat{e}_t$$

What is κ ?

κ (kappa) = curvature

$$\vec{a} = \ddot{s}\hat{e}_t + \dot{s}^2\kappa\hat{e}_n$$



What is $\ddot{s}$?

$$\vec{v} = \dot{s}\hat{e}_t \quad \dot{s} = v = \text{speed of P}$$

$$\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

