TAM 212 – Dynamics

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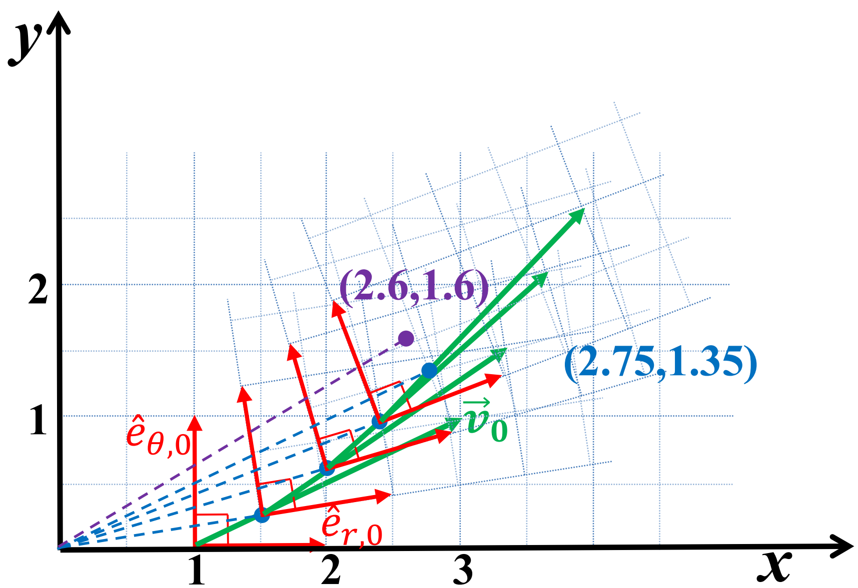
Recap

- Polar coordination Integrating
- Acceleration terms – centripetal acceleration
- Angular velocity

Today

- Angular velocity and acceleration



Cartesian: $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$
integrate to find $x(t)$ and $y(t)$

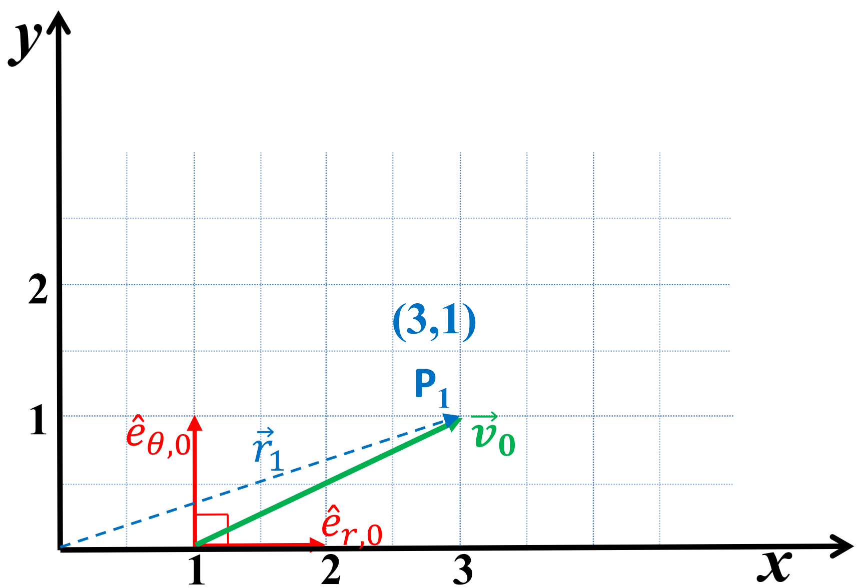
Polar: $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$
reduce to $\dot{r}(t)$ and $\dot{\theta}(t)$ before integrating

Base idea: turn vector integrals into scalar integrals in coordinates

Example – Polar coordination integrating

A particle P starts at time $t = 0$ s, it is at $\vec{r} = \hat{\imath}$ m and always has a velocity $\vec{v} = 2\hat{e}_r + \hat{e}_\theta$ m/s.
Where is P at $t = 1$ s?

$\vec{\boldsymbol{r}}(\mathbf{1}) =$
A. $\mathbf{3\hat{\imath}}$ **m**
B. $\mathbf{-3\hat{\imath}}$ **m**
C. $\mathbf{3\hat{\imath} + \hat{\jmath}}$ **m**
D. $\mathbf{3\hat{\imath} - \hat{\jmath}}$ **m**
E. **none of the above**



$(1,0) + (2,1) = (3,1)$ incorrect!

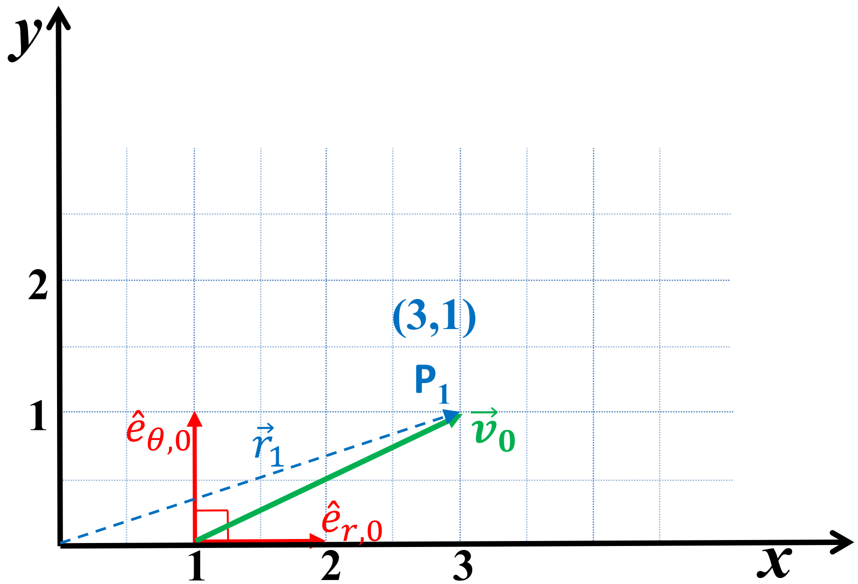
$$v = \dot{x} = \frac{x(t + dt) - x(t)}{dt}$$

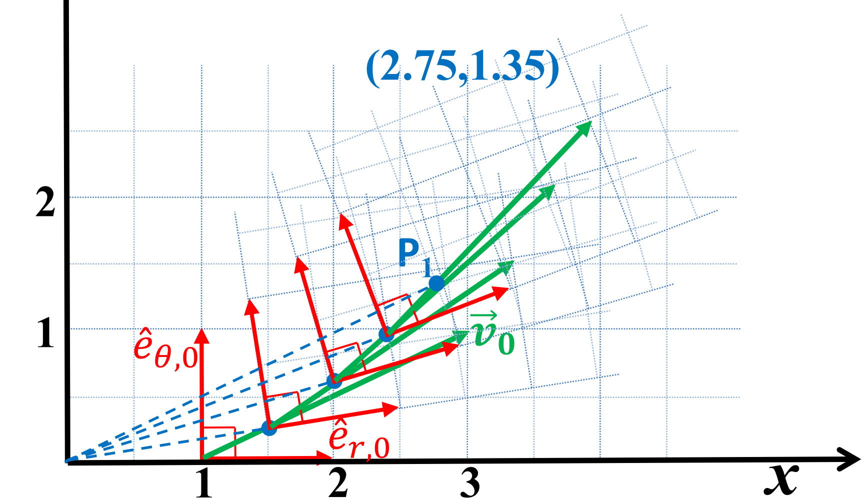
$$x(t + \Delta t) \approx x(t) + v\Delta t$$

Example – Polar coordination integrating

A particle P starts at time $t = 0$ s, it is at $\vec{r} = \hat{\imath}$ m and always has a velocity $\vec{v} = 2\hat{e}_r + \hat{e}_\theta$ m/s.
Where is P at $t = 1$ s?

$\vec{\boldsymbol{r}}(\mathbf{1}) =$
A. $\mathbf{3}\hat{\boldsymbol{\imath}}$ **m**
B. $-\mathbf{3}\hat{\boldsymbol{\imath}}$ **m**
C. $\mathbf{3}\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$ **m**
D. $\mathbf{3}\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}}$ **m**
E. **none of the above**





Example – Polar coordination integrating

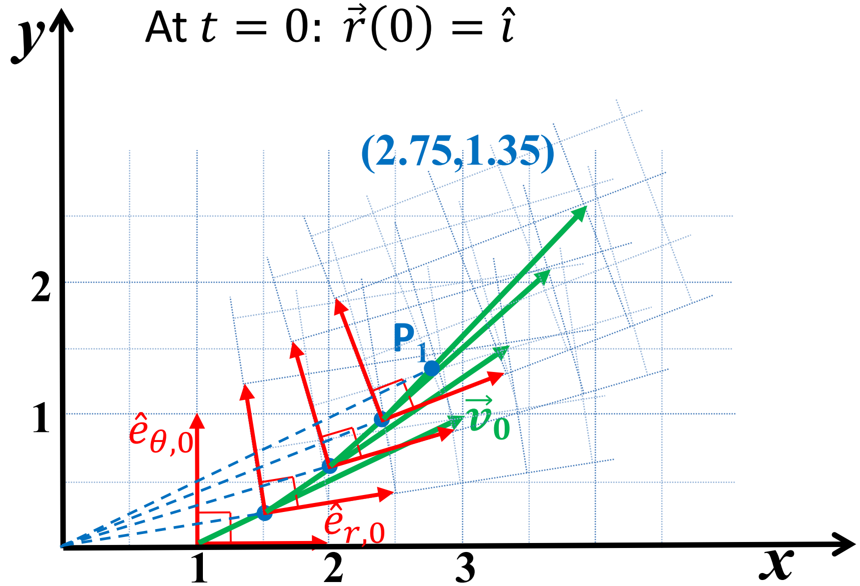
A particle P starts at time $t = 0$ s, it is at $\vec{r} = \hat{\imath}$ m and always has a velocity $\vec{v} = 2\hat{e}_r + \hat{e}_\theta$ m/s.
Where is P at $t = 1$ s?

Actual computation:

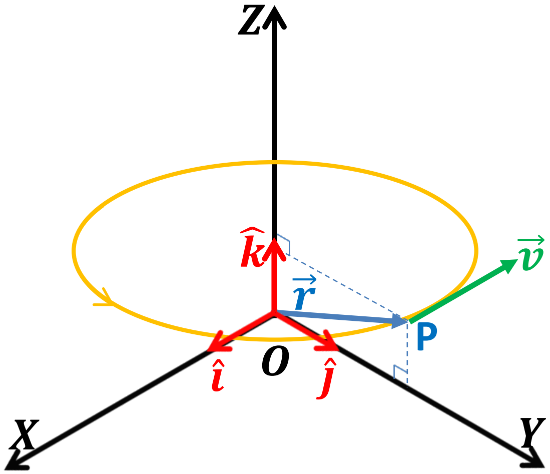
$$\vec{v} = 2\hat{e}_r + \hat{e}_\theta$$

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

At $t = 0$: $\vec{r}(0) = \hat{\imath}$



Angular velocity for particles (rotating about axis through origin)



$\omega =$ speed of rotation

$\hat{\omega} =$ axis of rotation

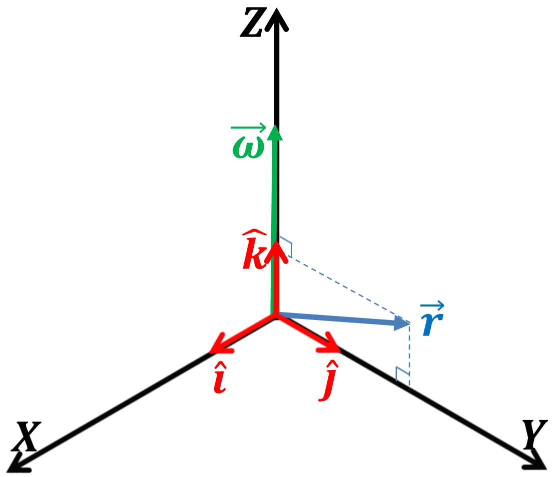
$\vec{\omega} = \omega\hat{\omega} =$ angular velocity

Ex. $\vec{\omega} = \omega\hat{k}$. What is $\vec{v}$ of **P**?

Example

$$\vec{r} = 4\hat{j} + 3\hat{k} \text{ m}$$

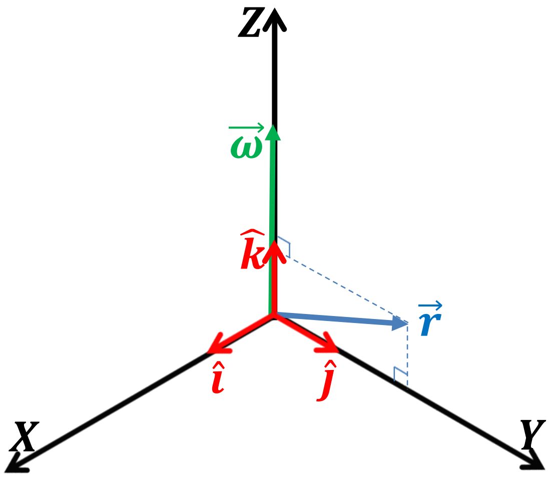
$$\vec{\omega} = 3\hat{k} \text{ rad/s}$$



Example

$\vec{r} = 4\hat{j} + 3\hat{k}$ m

$\vec{\omega} = 3\hat{k}$ rad/s



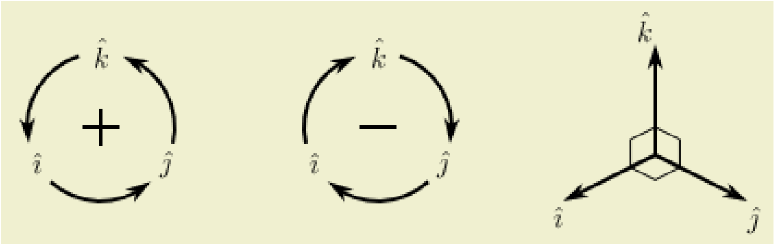
What is $\vec{v}$?

Angular acceleration

$\vec{\alpha} = \dot{\vec{\omega}} =$ angular acceleration – time derivative of angular velocity

$\vec{v} = \vec{\omega} \times \vec{r} = \dot{\vec{r}}$

$\vec{a} = \dot{\vec{v}}$



Angular acceleration

$\vec{\alpha} = \dot{\vec{\omega}} =$ angular acceleration – time derivative of angular velocity

$\vec{v} = \vec{\omega} \times \vec{r} = \dot{\vec{r}}$

$$\vec{a} = \dot{\vec{v}} = \frac{d}{dt}\vec{v} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

$$= \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}}$$

Important！

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

Order matters in cross product

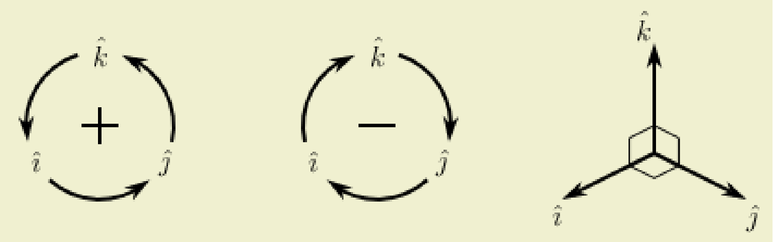
Exmaple $\vec{\alpha} = 0,\ \vec{\omega} = \vec{k},\ \vec{r} = \vec{i}$

$$\vec{a} = 0 + \vec{k} \times \vec{k} \times \vec{i} = 0 + 0$$

$$\vec{a} = 0 + \vec{k} \times \left(\vec{k} \times \vec{i}\right)$$

$$= \vec{k} \times (\vec{j})$$

$$= -\vec{i}$$



Example

For 2D x-y plane: $\vec{\boldsymbol{v}} = \vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}} = \boldsymbol{\omega_z} \vec{\boldsymbol{r}}^{\perp}$

$$\vec{\boldsymbol{a}} = \vec{\boldsymbol{\alpha}} \times \vec{\boldsymbol{r}} + \vec{\boldsymbol{\omega}} \times (\vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}}) = \boldsymbol{\alpha_z} \vec{\boldsymbol{r}}^{\perp} - \boldsymbol{\omega_z^2} \vec{\boldsymbol{r}}$$

A particle at position $\vec{r}$ is rotating about the z-axis with $\dot{\theta} = 3$ rad/s and $\ddot{\theta} = 2$ rad/s². What are $\vec{\omega}$ and $\vec{\alpha}$?

Example - Solving vector equations

A particle in the x-y plane is rotating about the origin with an angular velocity $\vec{\omega} = -2\hat{k}$ rad/s. The particle has a velocity $\vec{v} = 12\hat{\imath} - 2\hat{\jmath}$ m/s. What is the position vector $\vec{r}$ of the particle?

$r_x =$

A. -2 m
B. -1 m
C. 0 m
D. 1 m
E. 2 m