TAM 212 – Dynamics

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Lecture Objectives

- Rotation Motion

Recap

- Derivative of in polar coordinates – acceleration
- Names of acceleration terms

Today

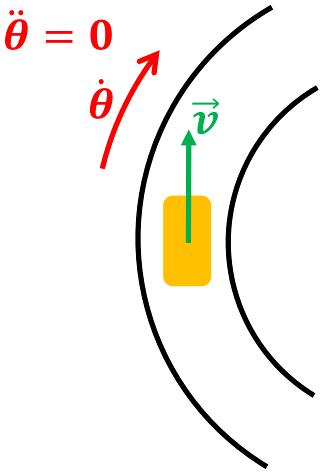
- Acceleration terms
- Integrating

	Cartesian	Polar
position	$\vec{r} = x\hat{\imath} + y\hat{\jmath}$	$\vec{r} = r\hat{e}_r$
velocity	$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$
acceleration	$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$	$\vec{a} = \left(\boldsymbol{\ddot{r} - r\dot{\theta}^2}\right)\hat{e}_r + \left(\boldsymbol{r\ddot{\theta} + 2\dot{r}\dot{\theta}}\right)\hat{e}_\theta$

In Polar coordinates, acceleration includes **radial** acceleration, **centripetal** acceleration, **angular** acceleration and **Coriolis** acceleration.

Centripetal acceleration

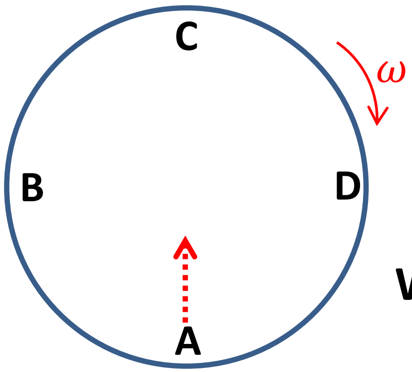
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$



Example – Merry-go round

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

A tries to roll a ball to **C**.



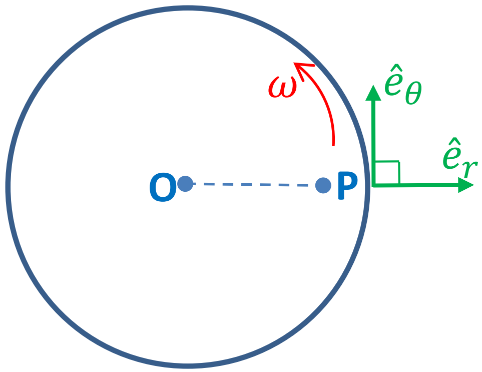
Who will get the ball?

E – it depends

Example – Merry-go round

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

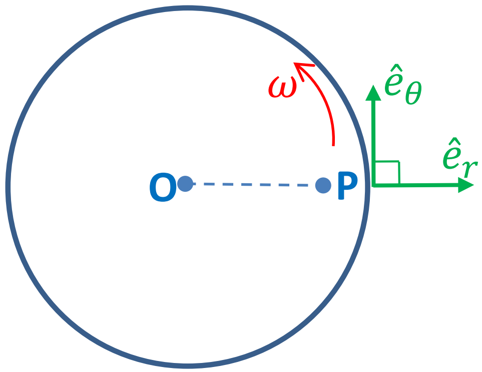
Top view of merry-go round, rotating about O:



Example – Merry-go round

$$\vec{\boldsymbol{a}} = (\ddot{\boldsymbol{r}} - \boldsymbol{r}\dot{\boldsymbol{\theta}}^2)\hat{\boldsymbol{e}}_{\boldsymbol{r}} + (\boldsymbol{r}\ddot{\boldsymbol{\theta}} + 2\dot{\boldsymbol{r}}\dot{\boldsymbol{\theta}})\hat{\boldsymbol{e}}_{\boldsymbol{\theta}}$$

Top view of merry-go round, rotating about O:



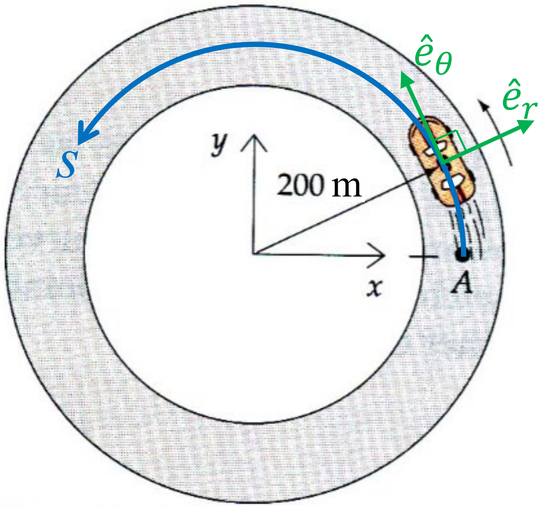
Example - Integrating

A particle P has a constant velocity $\vec{v} = 6t\hat{\imath} - 2\hat{\jmath}$ m/s
At time $t = 1$ s, it is at $\vec{r} = 4\hat{\jmath}$ m.

Example – Polar coordination integrating

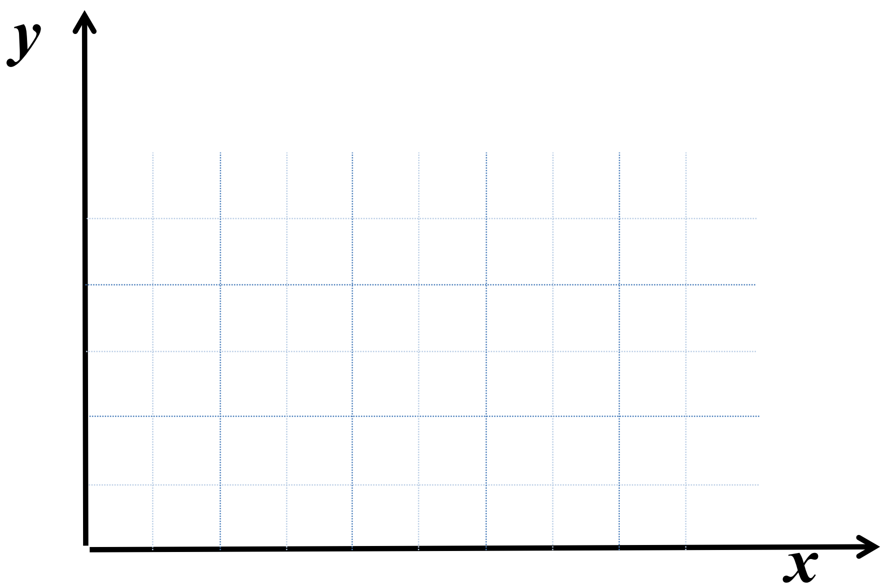
A car starts at A and increase its speed around the track at $6\ \mathrm{m/s^2}$ traveling counterclockwise. Determine the position Q and time at which the car's acceleration magnitude reaches $20\ \mathrm{m/s^2}$. (s = tangential path length)

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$



Example – Polar coordination integrating

A particle P starts at time $t = 0$ s, it is at $\vec{r} = \hat{\imath}$ m and always has a velocity $\vec{v} = 2\hat{e}_r + \hat{e}_\theta$ m/s.
Where is P at $t = 1$ s?



How would we actually compute this?

Coordinates (components)

$\vec{v} = 2\hat{e}_r + \hat{e}_\theta$

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

At $t = 0$: $\vec{r}(0) = \hat{\imath}$

Integrating

Cartesian: $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$

integrate to find $x(t)$ and $y(t)$

Polar: $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

reduce to $\dot{r}(t)$ and $\dot{\theta}(t)$ before integrating

Base idea: turn vector integrals into scalar integrals in coordinates