TAM 212 – Dynamics

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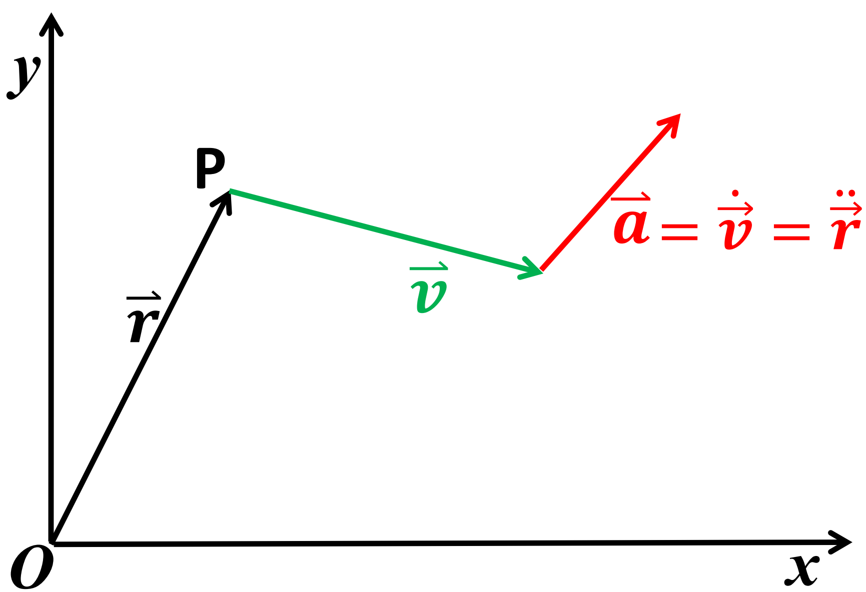
Lecture Objectives

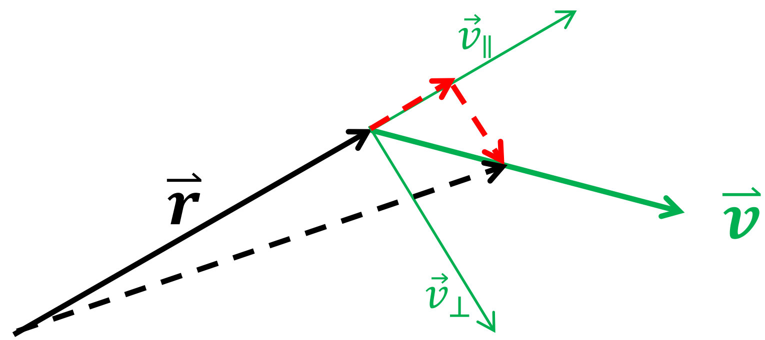
- Non-Cartesian vector calculus
  - Derivation
  - Polar basis

- Polar position, velocity and acceleration
  - Special cases
  - Create system of scalar equations from vector notation

Recap

- Vector derivatives & graphical estimation
- Chain rule

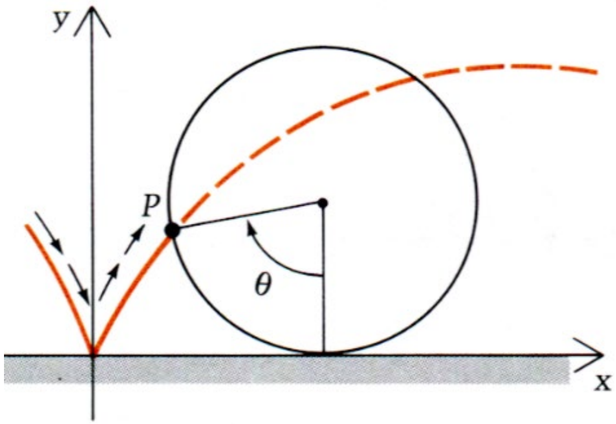




unit vectors $\dot{\hat{a}} \perp \hat{a}$

$$\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\hat{r}}$$

Cycloid Example



A cycloid is the curve traced by a point (such as P) on the rim of a rolling wheel. In terms of the parameter θ, the equations of the cycloid are:

$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$

$$\vec{r} = a(\theta - \sin\theta)\hat{\imath} + a(1 - \cos\theta)\hat{\jmath}$$

$\theta = bt^2$ rad

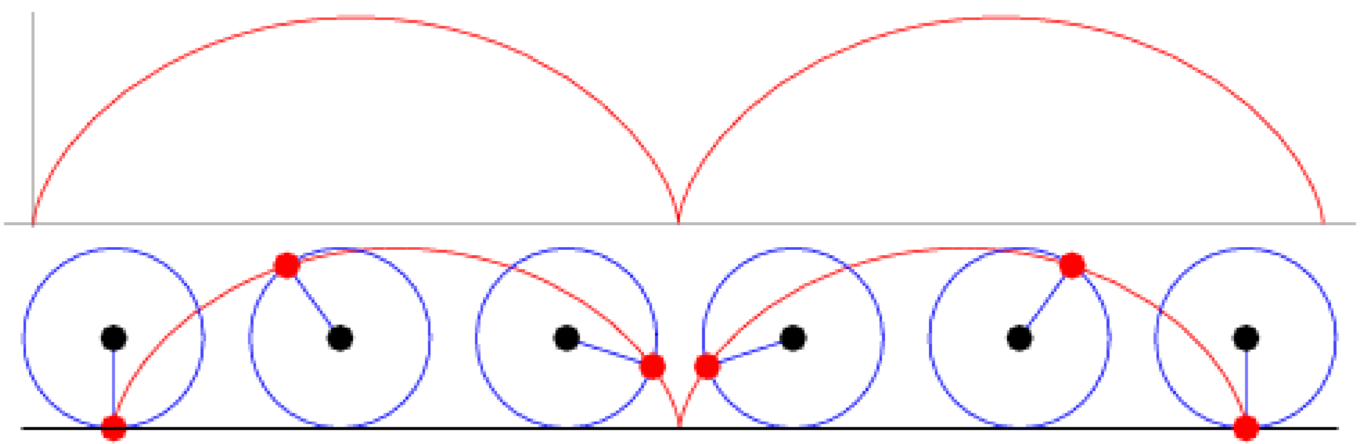
$a = 1$ m

$b = \pi/_4$ rad/s^2

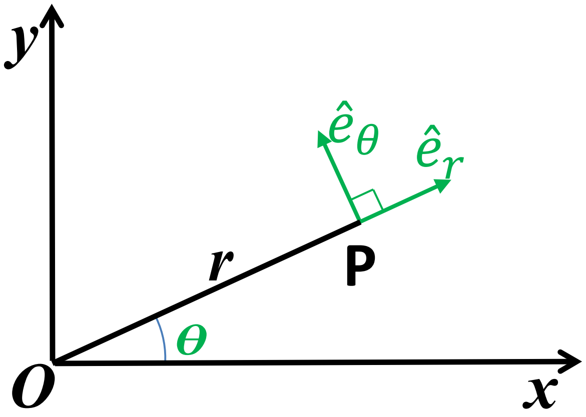
Cycloid Example

$\theta = bt^2$ rad
$a = 1$ m
$b = {}^{\pi}/_{4}$ rad/s^2

At $t = 2$ s



Derivative of in polar coordinates



$$\hat{e}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$
$$\hat{e}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$$

$$\vec{r} = r\hat{e}_r$$

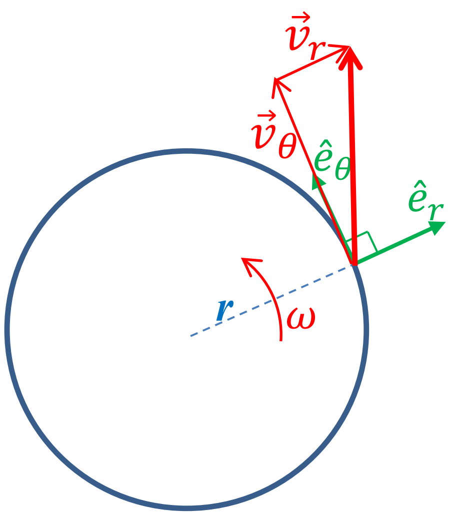
Velocity

$$\dot{\vec{r}} = \vec{v}$$

	Cartesian	Polar
position	$\vec{r} = x\hat{\imath} + y\hat{\jmath}$	$\vec{r} = r\hat{e}_r$
velocity	$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$$\vec{v} = \dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$$

$$\vec{v} = \vec{v}_r + \vec{v}_\theta$$



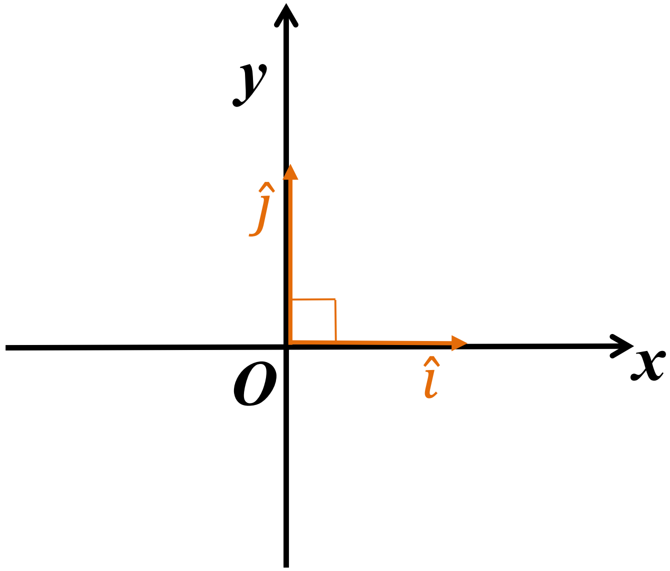
$\dot{\theta} = \omega \rightarrow$ angular velocity

$v_\theta = r\dot{\theta} \rightarrow$ tangential velocity

$= r\omega$

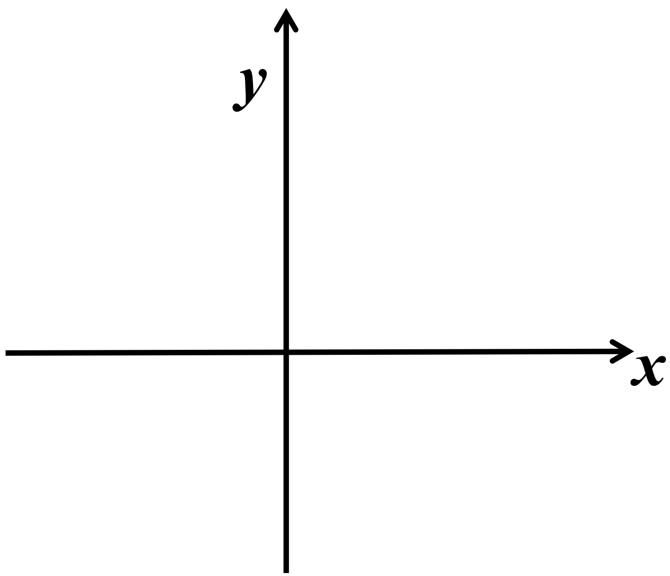
Example

$$\vec{r} = (\sin t)(\hat{\imath} + \hat{\jmath})$$



Example

$$r(t) = 4$$
$$\theta(t) = \omega t \qquad \omega \text{ is constant}$$



Example

A particle observed at a certain instant has:

$v = 5$ m/s

$\dot{\theta} = 1$ rad/s

$\dot{r} = -3$ m/s

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

Example

If particle P is moving with a constant speed, can it be accelerating?

A. **Yes**
B. **No**

If particle P is moving with a constant velocity, can it be accelerating?

A. **Yes**
B. **No**

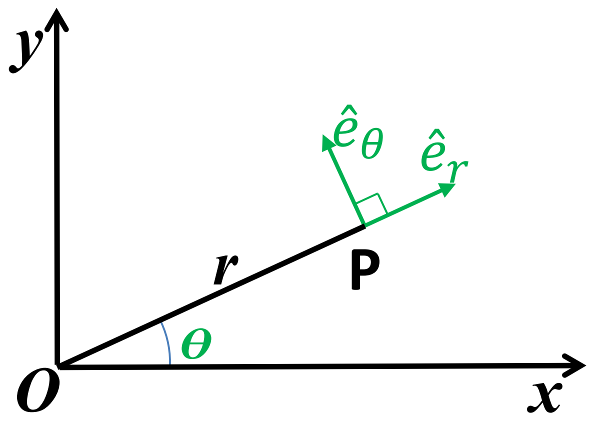
Acceleration

$$\dot{\vec{\boldsymbol{v}}} = \vec{\boldsymbol{a}}$$

	Cartesian	Polar
position	$\vec{r} = x\hat{\imath} + y\hat{\jmath}$	$\vec{r} = r\hat{e}_r$
velocity	$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$
acceleration		

Acceleration in polar coordinates

$$\dot{\vec{v}} = \vec{a}$$



$$\dot{\hat{e}}_r = \dot{\theta}\hat{e}_\theta \qquad \dot{\hat{e}}_\theta = -\dot{\theta}\hat{e}_r$$

$$\vec{r} = r\hat{e}_r$$

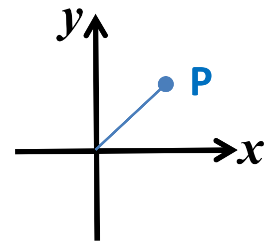
$$\vec{v} = \dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$$

Special cases of $\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$

$t = 0: r = 1,\ \theta = \frac{\pi}{4}$

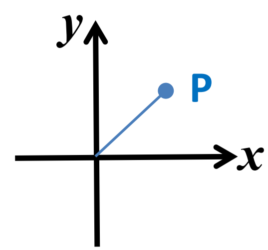
1. $\vec{a} = \ddot{r}\hat{e}_r$



What is constant?

A. r B. $\dot{r}$
C. θ D. $\dot{\theta}$

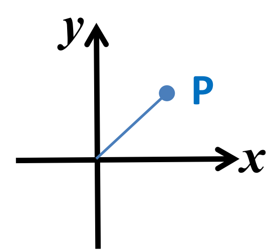
2. $\vec{a} = -r\dot{\theta}^2\hat{e}_r$



What is constant?

A. r B. $\dot{r}$
C. θ D. $\dot{\theta}$

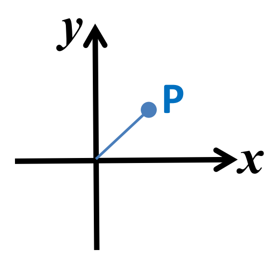
3. $\vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta$



What is constant?

A. r B. $\dot{r}$
C. θ D. $\dot{\theta}$

4. $\vec{a} = -r\dot{\theta}^2\hat{e}_r + 2\dot{r}\dot{\theta}\hat{e}_\theta$



What is constant?

A. r B. $\dot{r}$
C. θ D. $\dot{\theta}$

Acceleration

$$\dot{\vec{v}} = \vec{a}$$

	Cartesian	Polar
position	$\vec{r} = x\hat{\imath} + y\hat{\jmath}$	$\vec{r} = r\hat{e}_r$
velocity	$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$	$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$
acceleration	$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$	$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$

Though the **Coriolis force** is useful in mathematical equations, there is actually **no physical force involved**. Instead, it is just the ground moving at a different speed than an moving object in the air.

Example – Merry-go round

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

Top view of merry-go round, rotating about O: