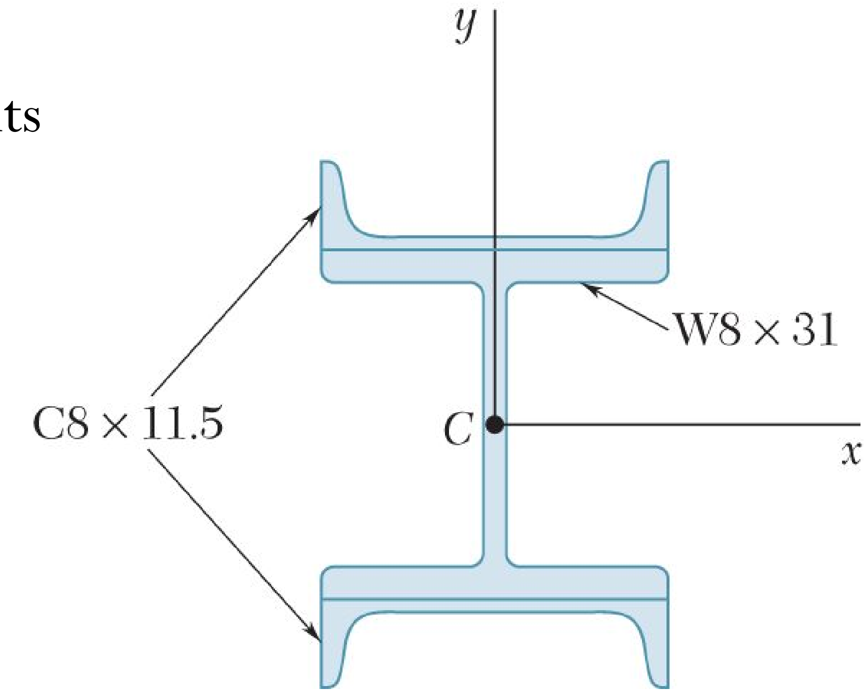
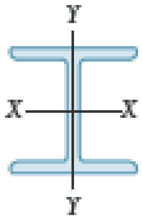
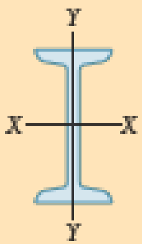
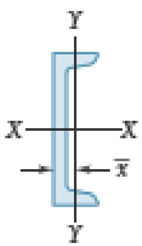
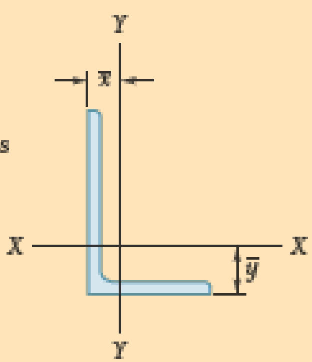
Announcements

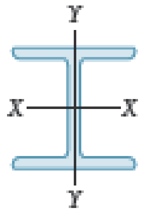
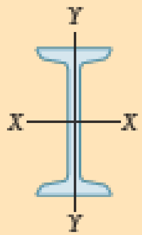
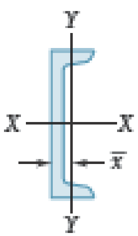
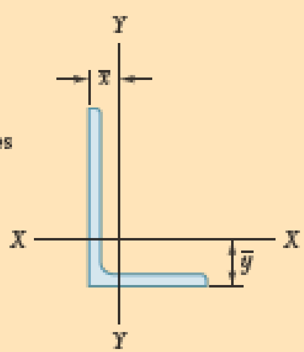
- CBTF Quiz 5 continues.

- Upcoming deadlines:
- Friday (4/19): Written Assignment
- Tuesday (4/23): PL HW

Two channels are welded to a rolled W section as shown. Determine the moments of inertia of the combined section with respect to the centroidal x-axis.



		Designation	Area, in^2	Depth, in.	Width, in.	Axis X-X			Axis Y-Y		
						$\bar{I}_x$, in^4	$\bar{k}_x$, in.	$\bar{y}$, in.	$\bar{I}_y$, in^4	$\bar{k}_y$, in.	$\bar{x}$, in.
W Shapes (Wide-Flange Shapes)		W18 × 76†	22.3	18.2	11.0	1330	7.73		152	2.61	
		W16 × 57	16.8	16.4	7.12	758	6.72		43.1	1.60	
		W14 × 38	11.2	14.1	6.77	385	5.87		26.7	1.55	
		W8 × 31	9.12	8.00	8.00	110	3.47		37.1	2.02	
S Shapes (American Standard Shapes)		S18 × 54.7†	16.0	18.0	6.00	801	7.07		20.7	1.14	
		S12 × 31.8	9.31	12.0	5.00	217	4.83		9.33	1.00	
		S10 × 25.4	7.45	10.0	4.66	123	4.07		6.73	0.980	
		S6 × 12.5	3.66	6.00	3.33	22.0	2.45		1.80	0.702	
C Shapes (American Standard Channels)		C12 × 20.7†	6.08	12.0	2.94	129	4.61		3.86	0.797	0.698
		C10 × 15.3	4.48	10.0	2.60	67.3	3.87		2.27	0.711	0.634
		C8 × 11.5	3.37	8.00	2.26	32.5	3.11		1.31	0.623	0.572
		C6 × 8.2	2.39	6.00	1.92	13.1	2.34		0.687	0.536	0.512
Angles		L6 × 6 × 1‡	11.0			35.4	1.79	1.86	35.4	1.79	1.86
		L4 × 4 × $\frac{1}{2}$	3.75			5.52	1.21	1.18	5.52	1.21	1.18
		L3 × 3 × $\frac{1}{4}$	1.44			1.23	0.926	0.836	1.23	0.926	0.836
		L6 × 4 × $\frac{1}{2}$	4.75			17.3	1.91	1.98	6.22	1.14	0.981
		L5 × 3 × $\frac{1}{2}$	3.75			9.43	1.58	1.74	2.55	0.824	0.746
		L3 × 2 × $\frac{1}{4}$	1.19			1.09	0.953	0.980	0.390	0.569	0.487

	Designation	Area mm²	Depth mm	Width mm	Axis X-X $\bar{I}_x$ 10^6 mm⁴	Axis X-X $\bar{k}_x$ mm	Axis X-X $\bar{y}$ mm	Axis Y-Y $\bar{I}_y$ 10^6 mm⁴	Axis Y-Y $\bar{k}_y$ mm	Axis Y-Y $\bar{x}$ mm
W Shapes (Wide-Flange Shapes) 	W460 × 113†	14400	462	279	554	196		63.3	66.3	
	W410 × 85	10800	417	181	316	171		17.9	40.6	
	W360 × 57.8	7230	358	172	160	149		11.1	39.4	
	W200 × 46.1	5880	203	203	45.8	88.1		15.4	51.3	
S Shapes (American Standard Shapes) 	S460 × 81.4†	10300	457	152	333	180		8.62	29.0	
	S310 × 47.3	6010	305	127	90.3	123		3.88	25.4	
	S250 × 37.8	4810	254	118	51.2	103		2.80	24.1	
	S150 × 18.6	2360	152	84.6	9.16	62.2		0.749	17.8	
C Shapes (American Standard Channels) 	C310 × 30.8†	3920	305	74.7	53.7	117		1.61	20.2	17.7
	C250 × 22.8	2890	254	66.0	28.0	98.3		0.945	18.1	16.1
	C200 × 17.1	2170	203	57.4	13.5	79.0		0.545	15.8	14.5
	C150 × 12.2	1540	152	48.8	5.45	59.4		0.286	13.6	13.0
Angles 	L152 × 152 × 25.4‡	7100			14.7	45.5	47.2	14.7	45.5	47.2
	L102 × 102 × 12.7	2420			2.30	30.7	30.0	2.30	30.7	30.0
	L76 × 76 × 6.4	929			0.512	23.5	21.2	0.512	23.5	21.2
	L152 × 102 × 12.7	3060			7.20	48.5	50.3	2.59	29.0	24.9
	L127 × 76 × 12.7	2420			3.93	40.1	44.2	1.06	20.9	18.9
	L76 × 51 × 6.4	768			0.454	24.2	24.9	0.162	14.5	12.4

Determine the moments of inertia of the bracket with respect to the x- and y-axes.

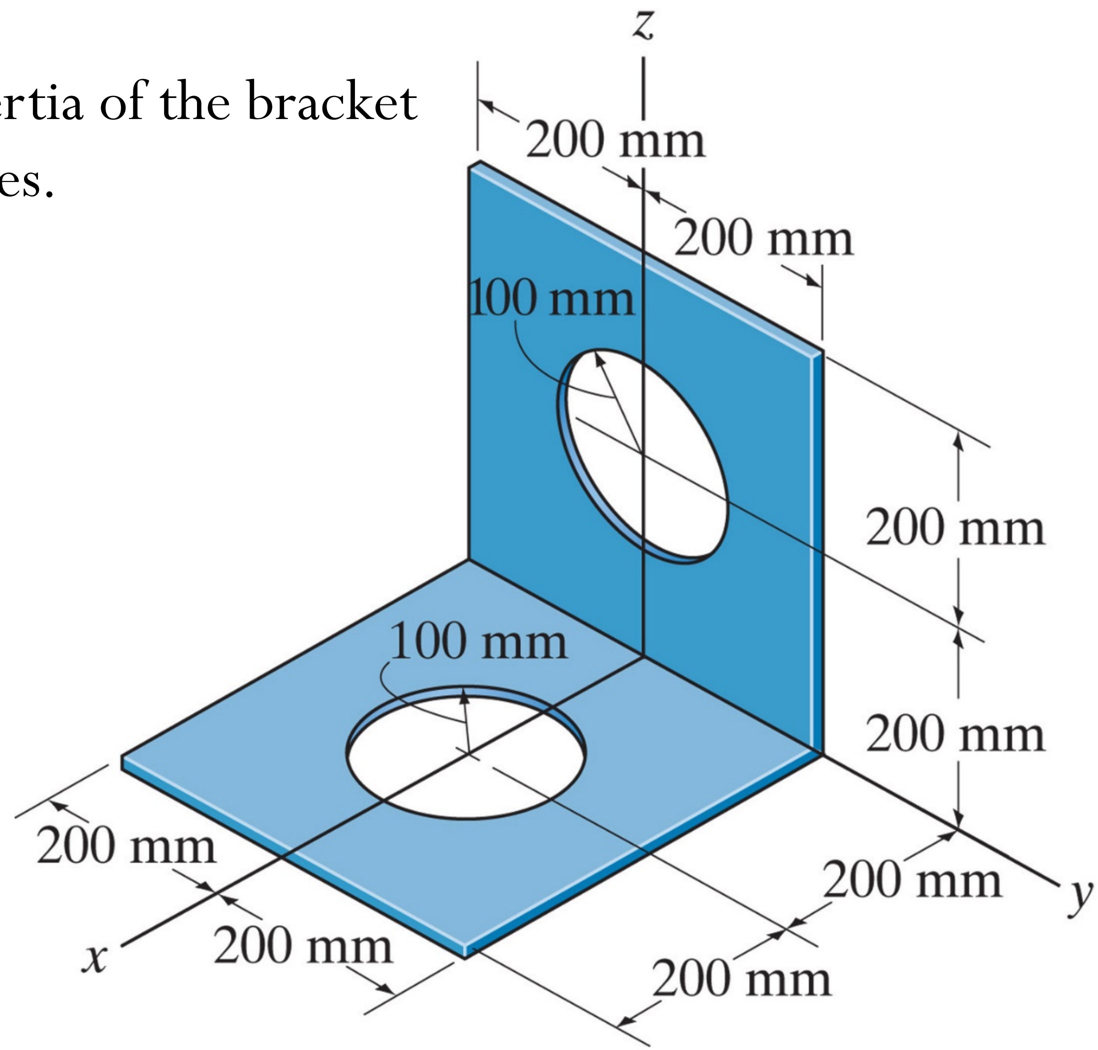
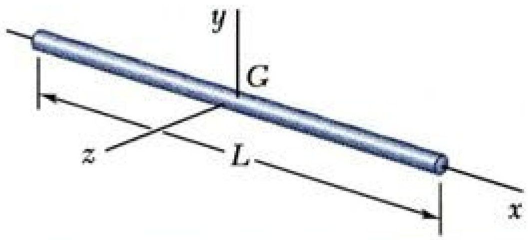
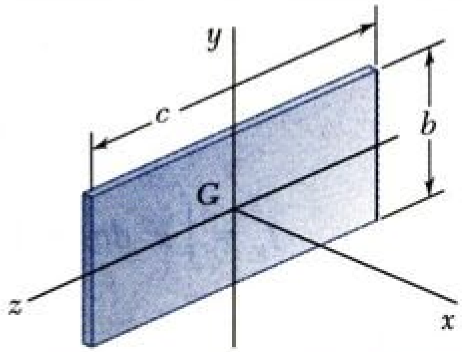


Figure: 10_P106-107

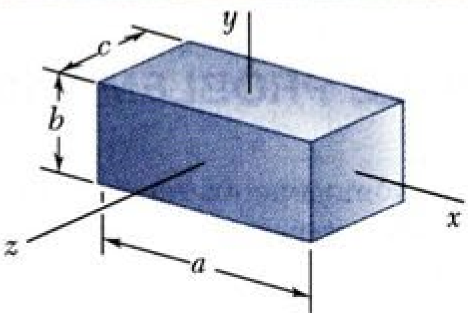
Moments of Inertia of Common Geometric Shapes



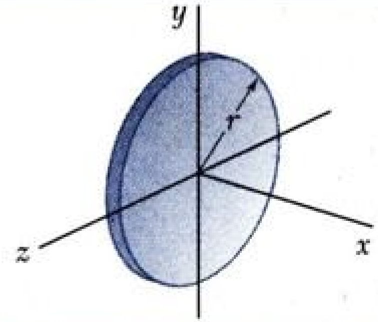
$$I_y = I_z = \frac{1}{12} mL^2$$



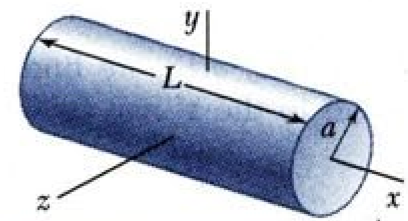
$$I_x = \frac{1}{12} m(b^2 + c^2)$$
$$I_y = \frac{1}{12} mc^2$$
$$I_z = \frac{1}{12} mb^2$$



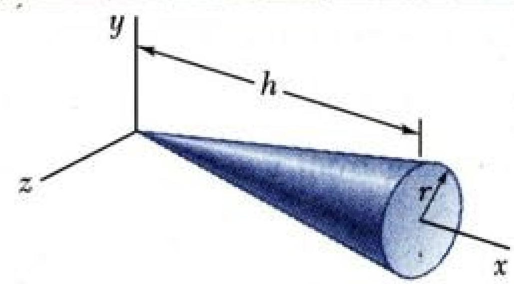
$$I_x = \frac{1}{12} m(b^2 + c^2)$$
$$I_y = \frac{1}{12} m(c^2 + a^2)$$
$$I_z = \frac{1}{12} m(a^2 + b^2)$$



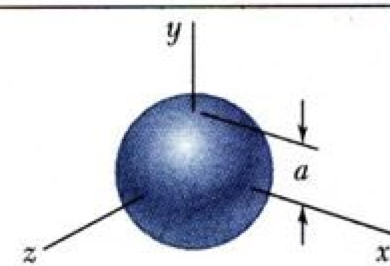
$$I_x = \frac{1}{2} mr^2$$
$$I_y = I_z = \frac{1}{4} mr^2$$



$$I_x = \frac{1}{2} ma^2$$
$$I_y = I_z = \frac{1}{12} m(3a^2 + L^2)$$



$$I_x = \frac{3}{10} ma^2$$
$$I_y = I_z = \frac{3}{5} m(\frac{1}{4} a^2 + h^2)$$



$$I_x = I_y = I_z = \frac{2}{5} ma^2$$

Virtual Work

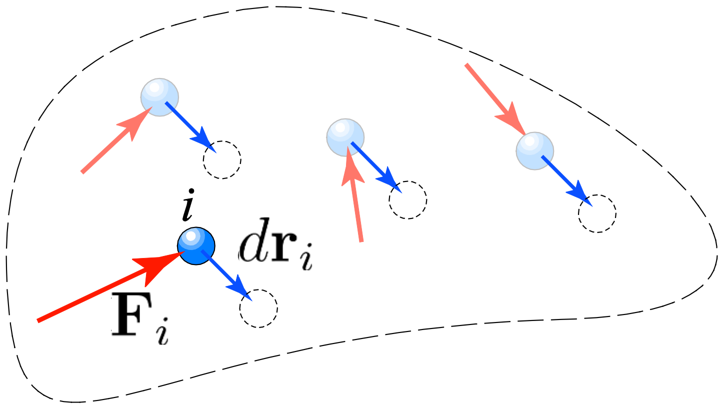
Main goals and learning objectives

- Introduce the principle of virtual work

- Show how it applies to determining the equilibrium configuration of a series of pin-connected members

Definition of Work

Work of a force



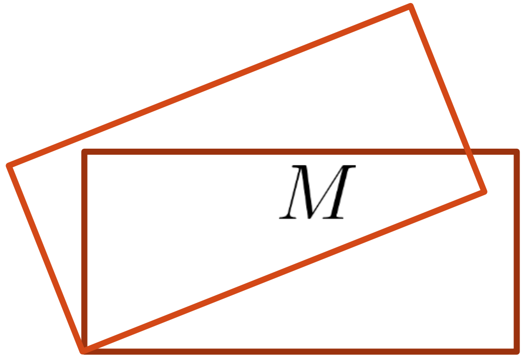
A force does work when it undergoes a displacement in the direction of the line of action.

The work dU produced by the force $\boldsymbol{F}$ when it undergoes a differential displacement $d\boldsymbol{r}$ is given by

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

Definition of Work

Work of a couple $$dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$$

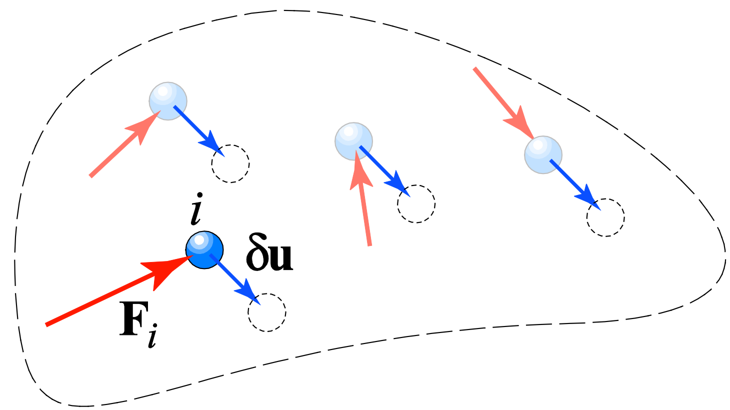


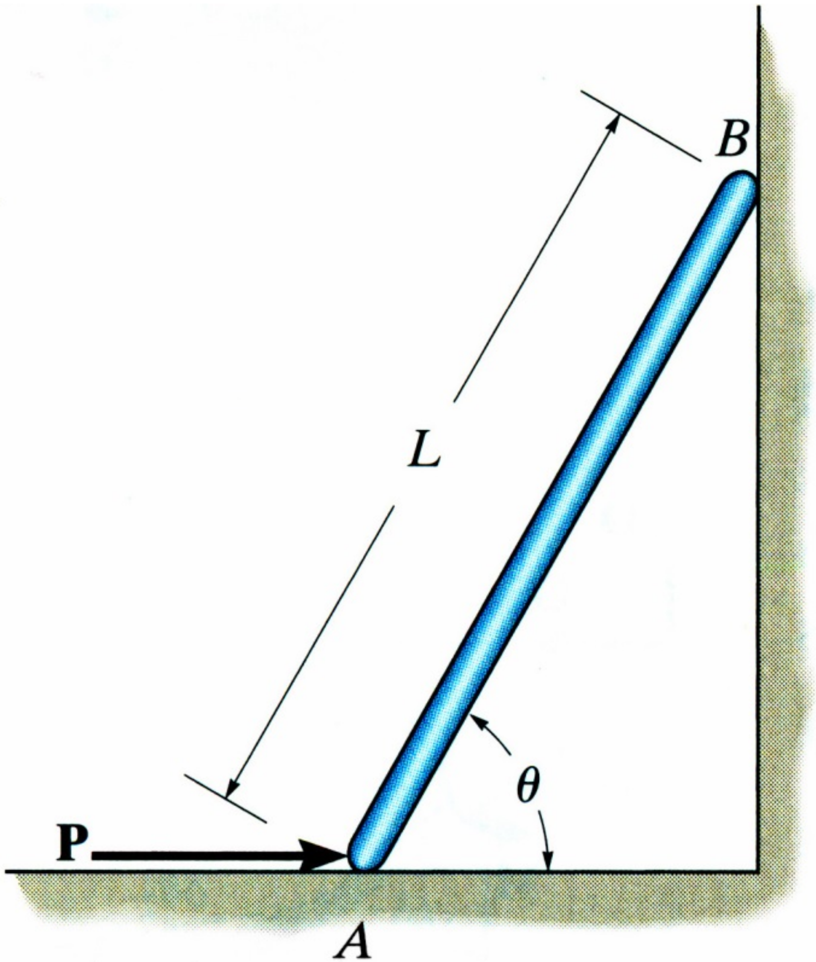
Virtual Displacements

A *virtual displacement* is a conceptually possible displacement *or* rotation of all *or* part of a system of particles. The movement is assumed to be possible, but actually does not exist.

Principle of Virtual Work

The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Thus,





The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium.