Announcements

- Cumulative exam continues (through Saturday, April 6)

- Upcoming deadlines:
- Tuesday (4/9): PL HW12

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Objectives

- Composite body method for finding centroid. (center of gravity)

→ apply to EoE.

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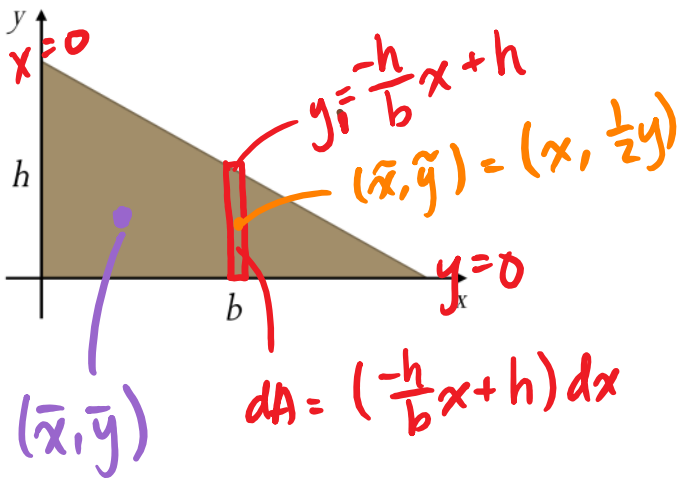
Determine the centroid of a triangle with a base of b and a height of h.



$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA}.$$

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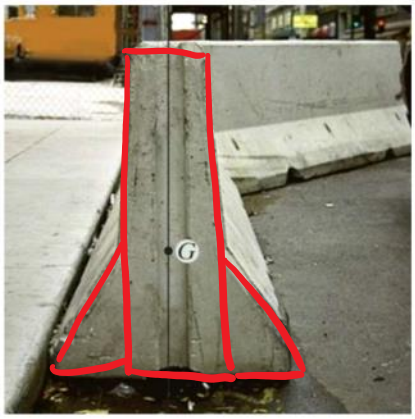
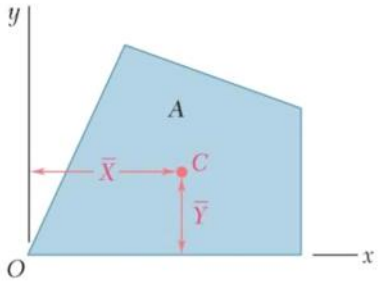
Composite bodies

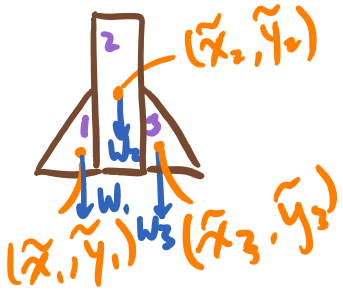
A composite body consists of a series of connected simpler shaped bodies.

Such body can be sectioned or divided into its composite parts and, provided the weight and location of the center of gravity of each of these parts are known, we can then eliminate the need for integration to determine the center of gravity of the entire body.





$$x = \frac{x_1F_1 + x_2F_2 + x_3F_3}{F_R}$$



$$\bar{x} = \frac{\Sigma \tilde{x}_i W_i}{\Sigma W_i} \qquad \bar{y} = \frac{\Sigma \tilde{y}_i W_i}{\Sigma W_i}$$

$$\left(\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \right)$$

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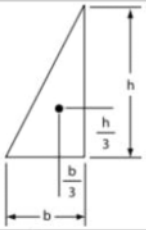
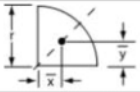
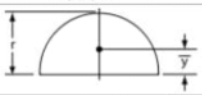
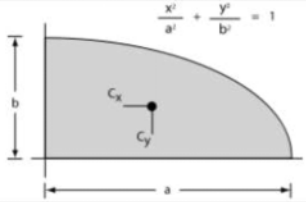
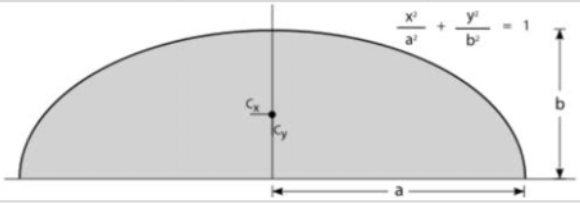
Centroid of typical 2D shapes

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

http://en.wikipedia.org/wiki/List_of_centroids

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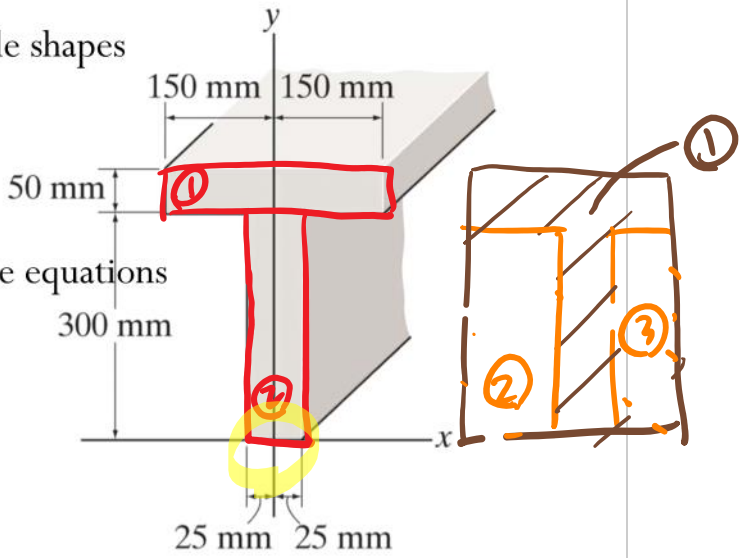
Composite bodies – Analysis Procedure

1. Divide the body into finite number of simple shapes
2. Consider "holes" as "negative" parts
3. Establish coordinate axes
4. Determine centroid location by applying the equations

$$\bar{x} = \frac{\sum \tilde{x} W}{\sum W} \qquad \bar{x} = \frac{\sum \tilde{x} A}{\sum A}$$

$$\bar{y} = \frac{\sum \tilde{y} W}{\sum W} \qquad \bar{y} = \frac{\sum \tilde{y} A}{\sum A}$$

$$\bar{z} = \frac{\sum \tilde{z} W}{\sum W} \qquad \bar{z} = \frac{\sum \tilde{z} A}{\sum A}$$



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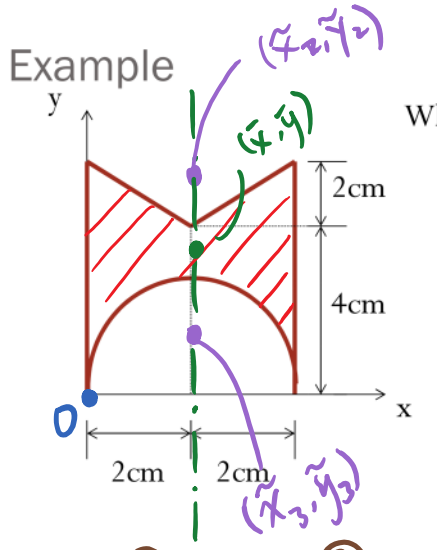
Example



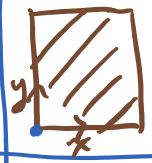
What is the centroid of the resultant area?

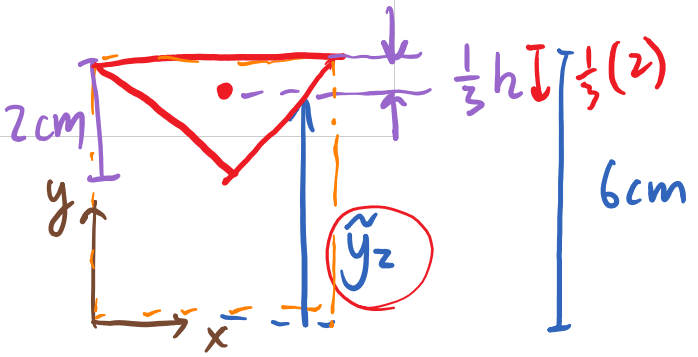
Find: $\bar{x}$ and $\bar{y}$

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i} = 2\text{ cm by symmetry}$$

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{y_1 A_1 + y_2 A_2 + y_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(3\text{cm})(24\text{cm}^2) - \left(\frac{16}{3}\text{cm}\right)(4\text{cm}^2) - \left(\frac{8}{3\pi}\right)(2\pi)\text{ cm}^3}{(24 - 4 - 2\pi)\text{ cm}^2}$$

	①	②	③
			
A_i	24 cm^2	4 cm^2	$\frac{1}{2}(2\text{cm})^2\pi$
y_i	3cm	$\left[6 - \frac{1}{3}(2)\right]$ cm	$\frac{4(2)}{3\pi}$ cm



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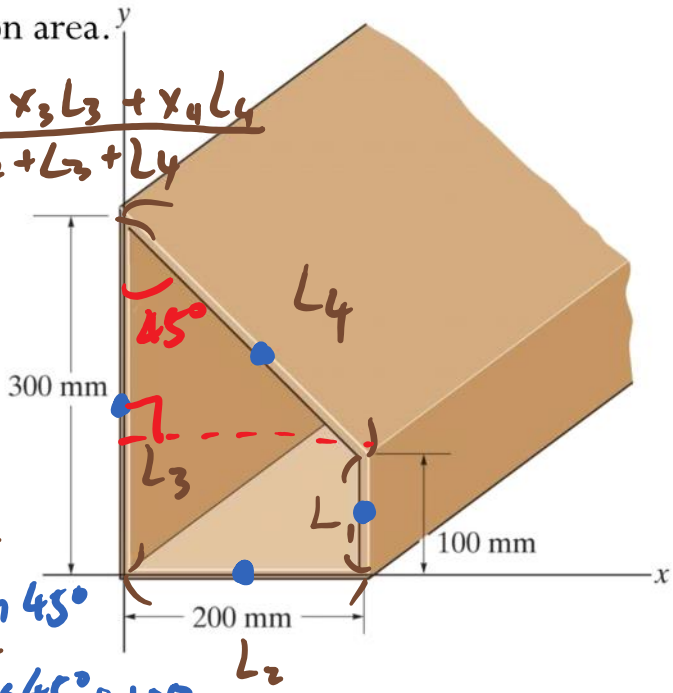
Example

Locate the centroid of the cross section area.

$$\bar{x} = \frac{\sum x_i L_i}{\sum L_i} = \frac{x_1 L_1 + x_2 L_2 + x_3 L_3 + x_4 L_4}{L_1 + L_2 + L_3 + L_4}$$

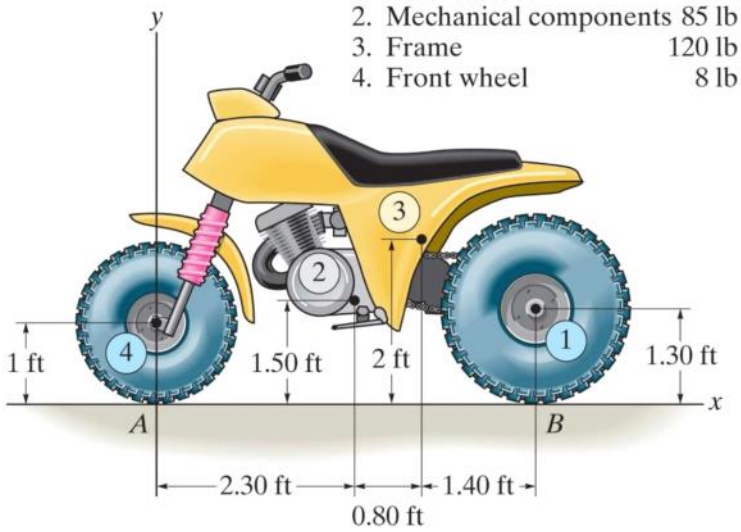
$$\bar{y} = \frac{\sum y_i L_i}{\sum L_i}$$



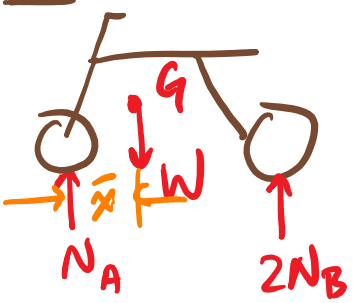
	①	②	③	④
x_i	200	100	0	$\frac{1}{2}L_4 \sin 45°$
y_i	50	0	150	$\frac{1}{2}L_4 \cos 45° + 100$
L_i	100	200	300	$200\sqrt{2}$

Determine the location of the center of gravity of the three-wheeler. If the three-wheeler is symmetrical with respect to the x-y plane, determine the normal reaction each of its wheels exerts on the ground.

1.	Rear wheels	18 lb
2.	Mechanical components	85 lb
3.	Frame	120 lb
4.	Front wheel	8 lb



FBD



$$\bar{x} = \frac{x_1 W_1 + x_2 W_2 + x_3 W_3 + x_4 W_4}{W_1 + W_2 + W_3 + W_4}$$

e.g. $x_1 = (2.3 + 0.8 + 1.4)\,\text{ft}$

EoE

$$\Sigma M_A = -W\bar{x} + 2N_B x_1 = 0 .$$