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Monday, April 1, 2019 9:07 AM

Announcements

- First day of TAM 211 only class!
- Cumulative exam this week (Thursday-Saturday)

❑ Upcoming deadlines:

- Monday (4/1–TODAY!): PrairieLearn HW9/11

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Goals and Objectives

- Understand the concepts of center of gravity, center of mass, and centroid.
- Be able to determine the location of these points for a body.

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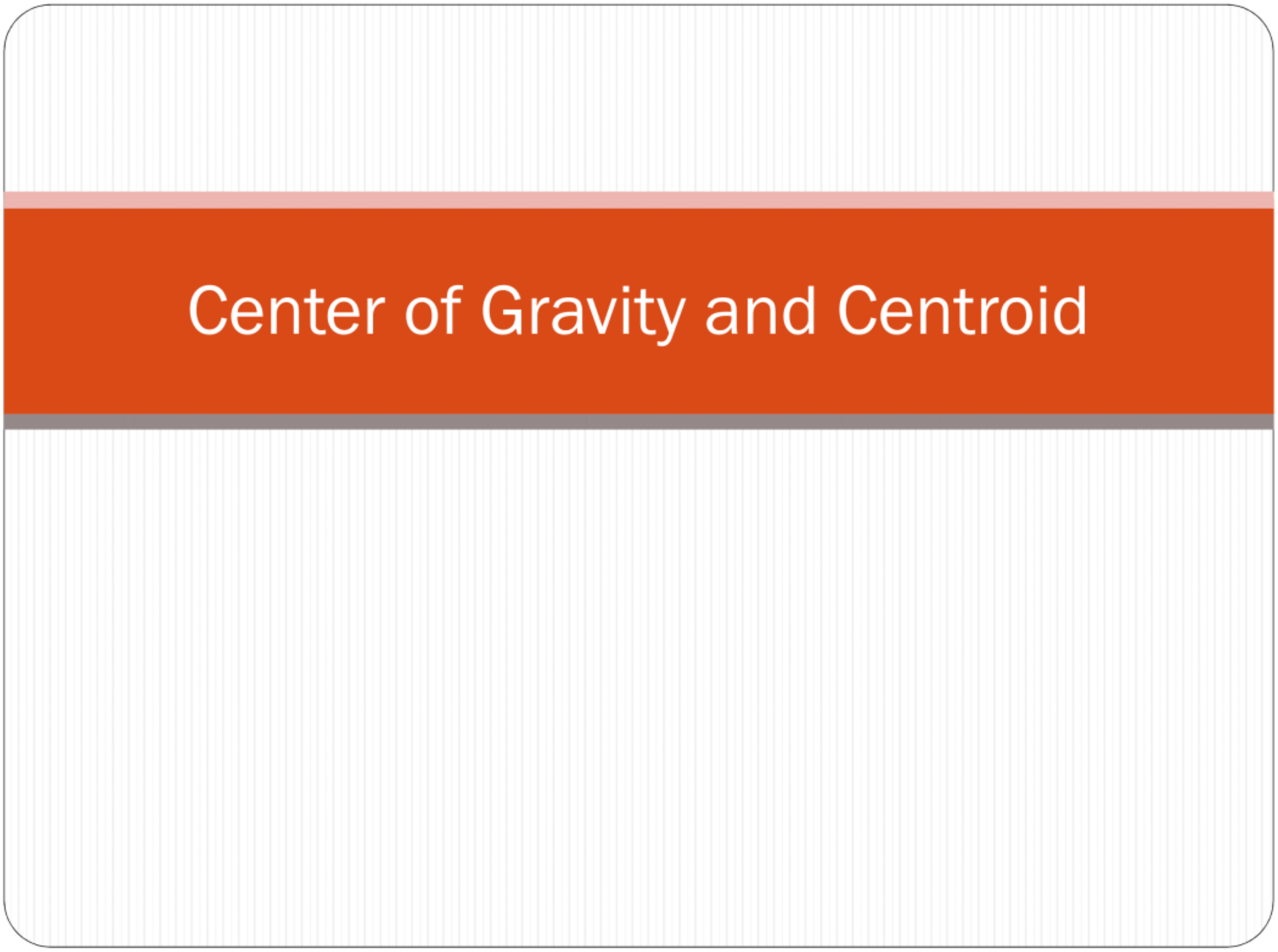

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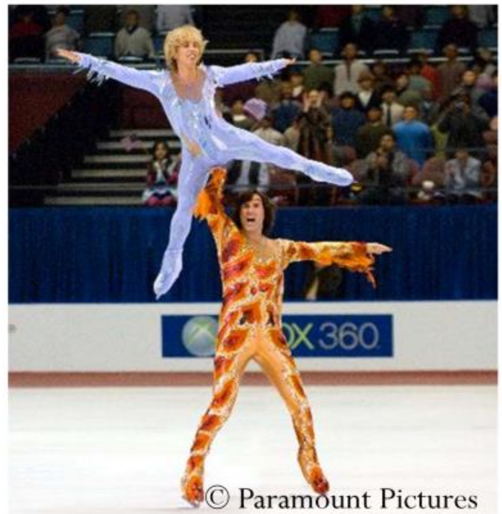
Center of gravity

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

How can we determine these resultant weights and their lines of action?

where to place this?

Center of gravity



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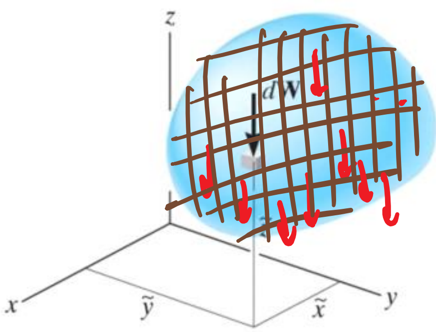
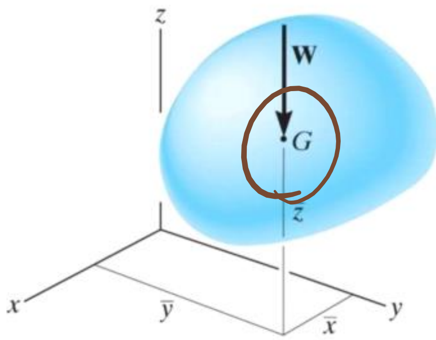
Center of gravity



A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight dW.



The **center of gravity (CG)** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.

From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

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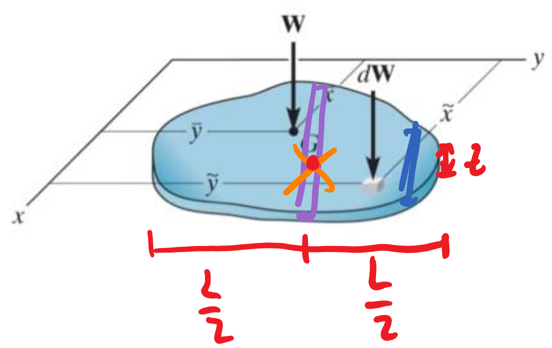
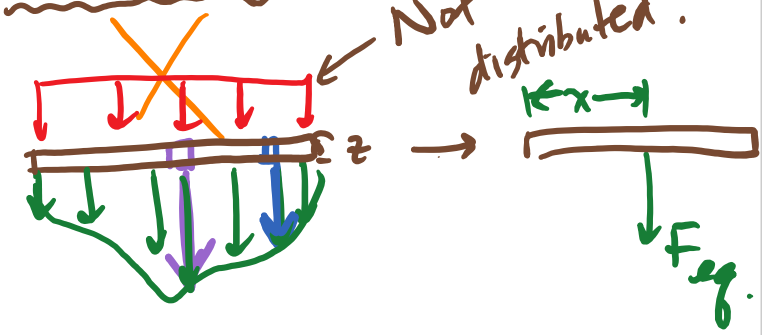
Center of gravity



Side view:

Not uniformly distributed.



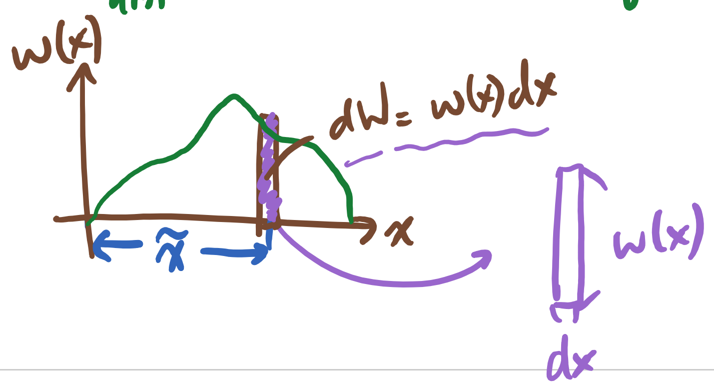
$$M_{Dist} = M_{eq.}$$

$$\int \tilde{x}\, dW = x W_{eq}$$

$$x = \frac{\int \tilde{x}\, dW}{W_{eq.}}$$

$$= \frac{\int \hat{x}\, d}{\int dW}$$

$$M_{dist} = \int \tilde{x}\, dW \qquad M_{eq} = x W_{eq.}$$



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Centroid

Center of Mass	Center of Volume	Center of Area
$\bar{x} = \dfrac{\int \tilde{x}\,dm}{\int dm}$	$\bar{x} = \dfrac{\int \tilde{x}\,dV}{\int dV}$	$\bar{x} = \dfrac{\int \tilde{x}\,dA}{\int dA}$
$\bar{y} = \dfrac{\int \tilde{y}\,dm}{\int dm}$	$\bar{y} = \dfrac{\int \tilde{y}\,dV}{\int dV}$	$\bar{y} = \dfrac{\int \tilde{y}\,dA}{\int dA}$
$\bar{z} = \dfrac{\int \tilde{z}\,dm}{\int dm}$	$\bar{z} = \dfrac{\int \tilde{z}\,dV}{\int dV}$	$\bar{z} = \dfrac{\int \tilde{z}\,dA}{\int dA}$

$$F = W = \underline{\rho V g} = mg$$

$$\bar{x} = \frac{\int \tilde{x}\,dF}{\int dF} = \frac{\int \tilde{x}\,dW}{\int dW} = \frac{\int \tilde{x}\,d(\rho V g)}{\int d(\rho V g)}$$

- if g is constant: $\bar{x} = \dfrac{\cancel{g}\int \tilde{x}\,d(\rho V)}{\cancel{g}\int d(\rho V)} = \dfrac{\int \tilde{x}\,dm}{\int dm}$

→ center of gravity & center of mass are the same.

- if ρ is constant: $\bar{x} = \dfrac{\cancel{g\rho}\int \tilde{x}\,dV}{\ldots} = \dfrac{\int \tilde{x}\,dV}{\int dV}$

$\cancel{g\rho}\int dV \qquad \int dV$

→ center of g & V are the same.

• if z is constant, where $V = Az$.

$$\bar{x} = \frac{\int \tilde{x}\, d(Az)}{\int d(Az)} = \frac{\cancel{z}\int \tilde{x}\, dA}{\cancel{z}\int dA} = \frac{\int \tilde{x}\, dA}{\int dA}.$$

→ center of g & area are the same

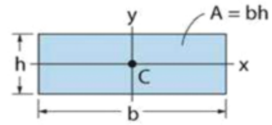
Centroid

The centroid, C, is a point defining the geometric center of an object.

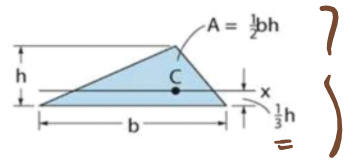
The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogeneous (density or specific weight is constant throughout the body).

If an object has an axis of symmetry, then the centroid of object lies on that axis.

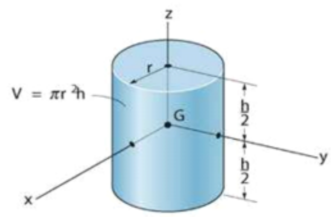
In some cases, the centroid may not be located on the object.



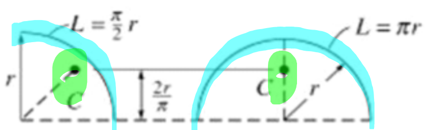
Rectangular area



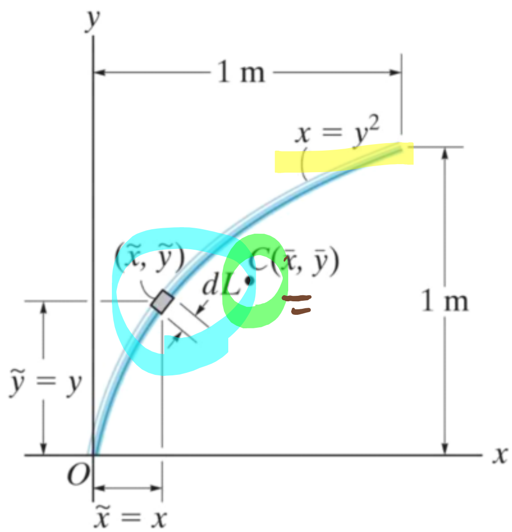
Triangular area



Cylinder



Quarter and semicircle arcs



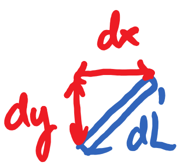
Locate the centroid of the rod bent into the shape of a parabolic arc.

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL}$$

• Assume the width of the rod relatively small.

• Define $dL = f(x,y)$

$$\int dL = \int \sqrt{dx^2 + dy^2} \; ?$$



$$dL = \sqrt{dx^2 + dy^2}$$

$$= \sqrt{dx^2\left(1 + \frac{dy^2}{dx^2}\right)} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

or

$$= \sqrt{dy^2\left(\frac{dx^2}{dy^2} + 1\right)} = dy\sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

$$= \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad ?$$

or

$$= \int \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy \quad ?$$

for $x = y^2$

$$\frac{dx}{dy} = 2y \;\to\; \int dL = \int \sqrt{(2y)^2 + 1}\, dy$$

$$\frac{dy}{dx} = \frac{d}{dx}(y) = \frac{d}{dx}(\sqrt{x}) = \frac{1}{2}x^{-1/2}$$

• Putting everything together:

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL} = \frac{\int x\sqrt{(2y)^2+1}\, dy}{\int \sqrt{(2y)^2+1}\, dy} \;\to\; \boxed{\bar{x} = \frac{\int y^2\sqrt{(2y)^2+1}\, dy}{\int \sqrt{(2y)^2+1}\, dy}}$$

$$x = \frac{\text{[illegible]}}{\int dL} = \frac{\text{[illegible]}}{\int \sqrt{(2y)^2+1}\, dy} \Rightarrow \boxed{x = \frac{\text{[illegible]}}{\int \sqrt{2y^2+1}\, dy}}$$

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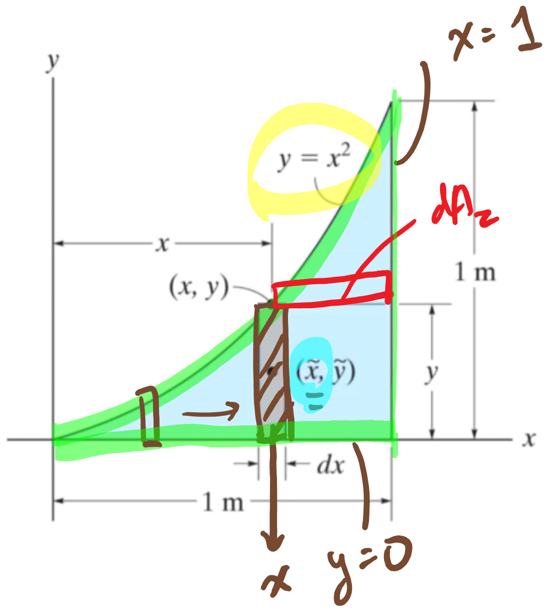
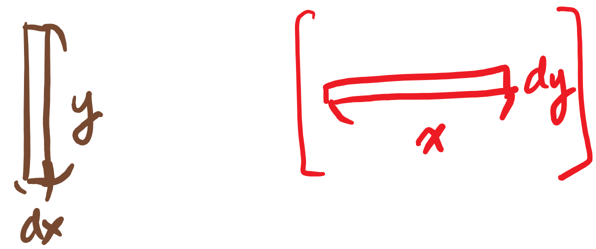


Locate the centroid of the area.

Define $dA = y\,dx = x^2\,dx$



$\tilde{x}$ = centroid of the slice.

x = location of the slice. → $\tilde{x} = x$ for this case.

$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} = \frac{\int_0^1 x\,x^2\,dx}{\int_0^1 x^2\,dx} = \frac{\int_0^1 x^3\,dx}{\int_0^1 x^2\,dx} = \frac{\left.\frac{x^4}{4}\right|_0^1}{\left.\frac{x^3}{3}\right|_0^1} = \frac{3}{4}\text{ m}$$